

# Multinomials and the CJS Model

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To gain familiarity with the multinomial, it is helpful to work with drawing values from it. We use the function ‘`rmultinom`’ to generate several datasets. We start using the example from the textbook for passes, where the probabilities are for a completion, incompleteness, and interception. The code generates 10 replications of releases (trials) of 100 individuals on occasion 1.

```
x <- rmultinom(10, size = 100, prob = c(0.6, 0.38, (1 - 0.6 - 0.38)))
row.names(x) <- c("complete", "incomplete", "intercept")
x
```

```
##           [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## complete    59    62    58    59    58    61    60    60    60
## incomplete   40    37    39    40    39    38    40    39    37
## intercept    1     1     3     1     3     1     0     1     3
```

## CJS Example Using Multinomial

We’ll set constant parameter values for  $\phi_i$  and  $p_i$  and use them to (1) calculate the probability of encounter history that’s possible in a 3-year study, (2) see 2 versions of probability statements for encounter histories that end in 0’s, and (3) check that the probabilities sum to 1.0. Examine the probability statement for the 100 encounter history: an animal can have this history by living through 2, 1, or 0 of the intervals, i.e., this type of history doesn’t provide a lot of information. If you have low values of  $p$ , you can end up with a lot of these, and your precision will suffer.

```
phi <- 0.8
p <- 0.3
(pr111 <- phi * p * phi * p)

## [1] 0.0576
(pr110 <- phi * p * ((1 - phi) + phi * (1 - p)))

## [1] 0.1824
(pr110.alt <- phi * p * (1 - (phi * p)))

## [1] 0.1824
(pr101 <- phi * (1 - p) * phi * p)

## [1] 0.1344
(pr100 <- (phi * (1 - p) * phi * (1 - p) +
           phi * (1 - p) * (1 - phi) +
           (1 - phi)))

## [1] 0.6256
(pr100.alt <- 1 - (pr111 + pr110 + pr101))

## [1] 0.6256
round(c(pr111, pr110, pr101, pr100, sum(pr111, pr110, pr101, pr100)), 3)

## [1] 0.058 0.182 0.134 0.626 1.000
```

## Generate CJS Capture Histories with Random Multinomials

You can quickly generate summarized capture history information for a CJS study using the `rmultinom` function.

```
set.seed(502) # you can set the seed to make the results repeatable
(rm <- rmultinom(10, size = 250, prob = c(pr111, pr110, pr101, pr100)))
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]  17  17  14  10  12  12  15  16  19  21
## [2,]  45  51  43  41  50  47  44  48  43  43
## [3,]  33  30  31  37  30  34  33  28  26  32
## [4,] 155 152 162 162 158 157 158 158 162 154
```

```
# Check proportions
round(rm/250, 3)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,] 0.068 0.068 0.056 0.040 0.048 0.048 0.060 0.064 0.076 0.084
## [2,] 0.180 0.204 0.172 0.164 0.200 0.188 0.176 0.192 0.172 0.172
## [3,] 0.132 0.120 0.124 0.148 0.120 0.136 0.132 0.112 0.104 0.128
## [4,] 0.620 0.608 0.648 0.648 0.632 0.628 0.632 0.632 0.648 0.616
```

```
# compare average proportions against original data
round(apply(rm, 1, mean)/250,3)
```

```
## [1] 0.061 0.182 0.126 0.631
```

```
round(c(pr111, pr110, pr101, pr100),3)
```

```
## [1] 0.058 0.182 0.134 0.626
```

## Analyze Data from Each Simulation in MARK

phi	se.phi	p	se.p
0.771	0.092	0.328	0.061
0.719	0.082	0.372	0.064
0.765	0.100	0.302	0.061
0.934	0.136	0.217	0.051
0.750	0.097	0.316	0.062
0.826	0.109	0.278	0.057
0.788	0.100	0.304	0.060
0.695	0.083	0.367	0.066
0.654	0.076	0.397	0.069
0.734	0.082	0.369	0.063

## Summarize Results of Many Simulated Datasets

The above example only used 10 simulated datasets, which is nice for displaying the steps but inadequate for properly assessing the expected outcomes, which would require many simulated datasets, e.g., thousands. Ignoring the fact that we've only done 10 simulations, let's check the average performance and the variability in performance of the CJS model for the data and estimates generated above. Notice that here because we have multiple estimates of  $\phi$  and  $p$ , we can empirically estimate the SE of each by calculating the SD of all the estimates of  $\phi$  and  $p$ .

parameter	mean	sd	min	max
phi	0.764	0.077	0.654	0.934
se.phi	0.096	0.017	0.076	0.136
p	0.325	0.054	0.217	0.397
se.p	0.062	0.005	0.051	0.069

## Simulations in Program MARK

You can conduct simulations for various types of capture-recapture studies in Program MARK. Appendix A of the on-line Cooch and White book has really helpful material on the topic, which we can review later in the semester if you like. Conducting simulations can be very helpful in understanding how models operate, evaluating the impacts of violating assumptions, and designing studies. Having simulation code becomes especially helpful when you have many occasions where working out all the probability statements can be tedious at best, and/or when covariates are involved.