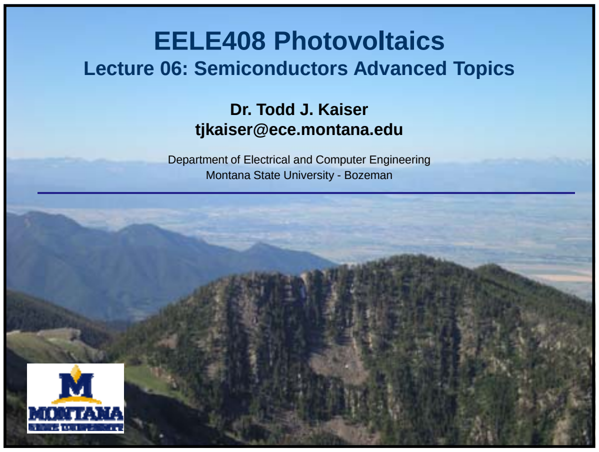

EELE408 Photovoltaics
Lecture 06: Semiconductors Advanced Topics

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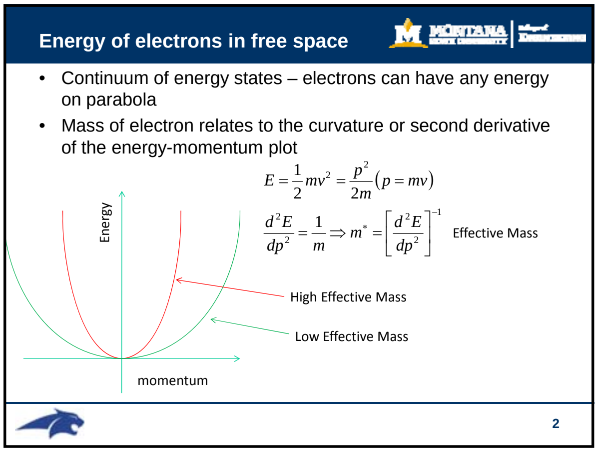
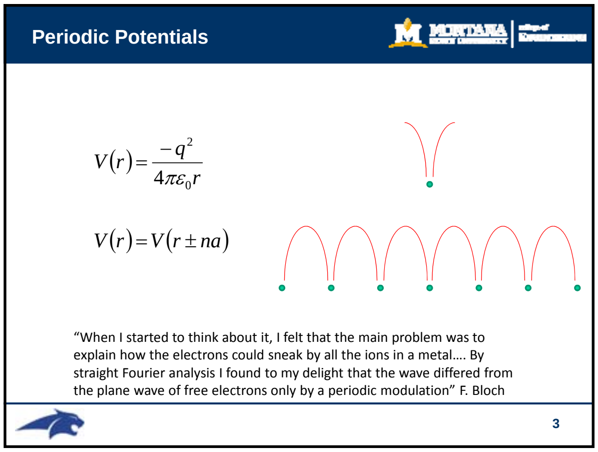


Energy of electrons in free space

- Continuum of energy states – electrons can have any energy on parabola
- Mass of electron relates to the curvature or second derivative of the energy-momentum plot

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}\left(p = mv\right)$$

$$\frac{d^2E}{dp^2} = \frac{1}{m} \Rightarrow m^* = \left[\frac{d^2E}{dp^2}\right]^{-1}$$ Effective Mass

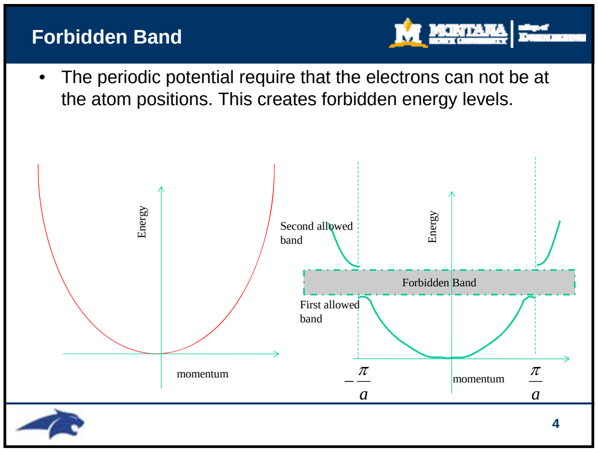


Periodic Potentials

$$V(r) = \frac{-q^2}{4\pi\varepsilon_0 r}$$

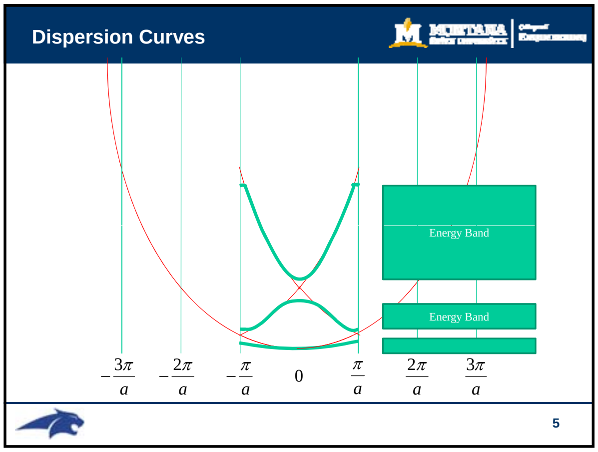
$$V(r) = V(r \pm na)$$

"When I started to think about it, I felt that the main problem was to explain how the electrons could sneak by all the ions in a metal…. By straight Fourier analysis I found to my delight that the wave differed from the plane wave of free electrons only by a periodic modulation" F. Bloch



Forbidden Band

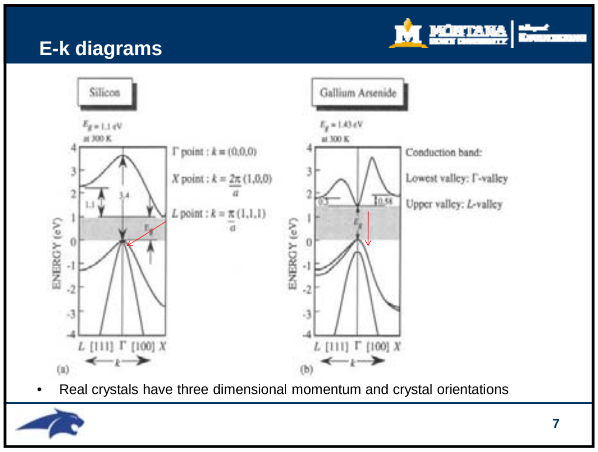
- The periodic potential require that the electrons can not be at the atom positions. This creates forbidden energy levels.



Dispersion Curves

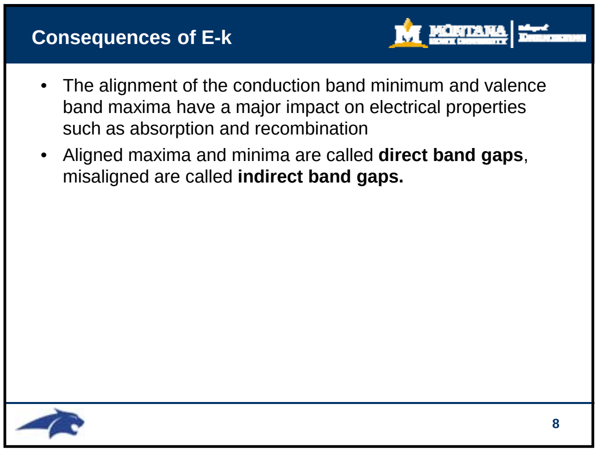
Schrodinger Equation for Crystals

- **Solution shows some energies have multiple energy states while some energies are disallowed.**
  - **Electrons can only occupy places "on" the E-k diagram**
  - **The band gap is minimum difference between the two uppermost bands (conduction and valence band)**
- **Near the minima or maxima, bands are approximately parabolic, and hence can be approximated as "free" electrons**
- **Effective mass of each band near zone center varies, depending on its curvature**
- **Effective mass varies with k.**



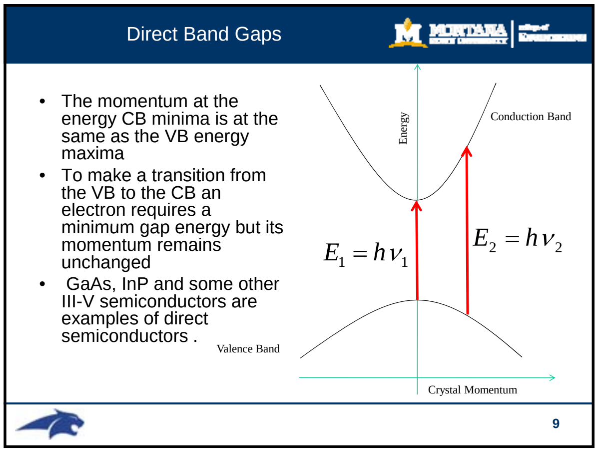
E-k diagrams

- Real crystals have three dimensional momentum and crystal orientations



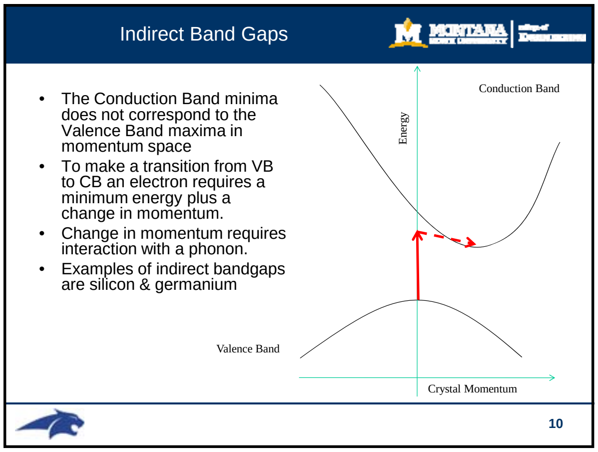
Consequences of E-k

- The alignment of the conduction band minimum and valence band maxima have a major impact on electrical properties such as absorption and recombination
- Aligned maxima and minima are called **direct band gaps**, misaligned are called **indirect band gaps.**



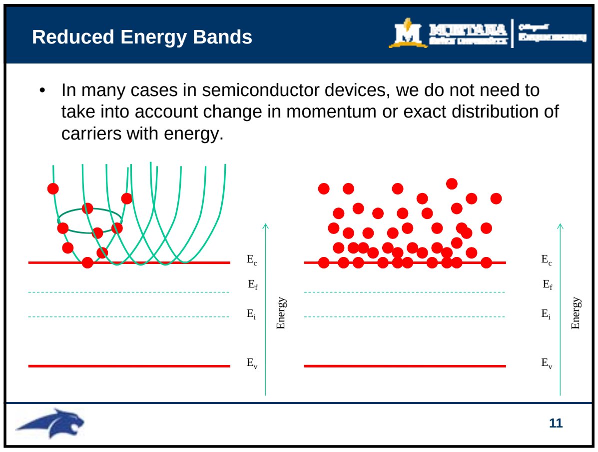
Direct Band Gaps

- The momentum at the energy CB minima is at the same as the VB energy maxima
- To make a transition from the VB to the CB an electron requires a minimum gap energy but its momentum remains unchanged
- GaAs, InP and some other III-V semiconductors are examples of direct semiconductors .



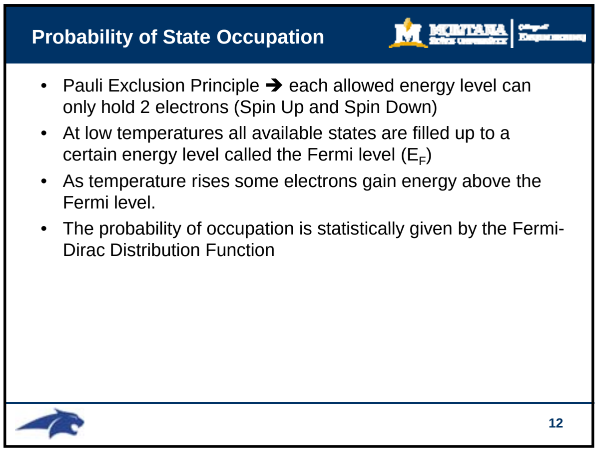
Indirect Band Gaps

- The Conduction Band minima does not correspond to the Valence Band maxima in momentum space
- To make a transition from VB to CB an electron requires a minimum energy plus a change in momentum.
- Change in momentum requires interaction with a phonon.
- Examples of indirect bandgaps are silicon & germanium



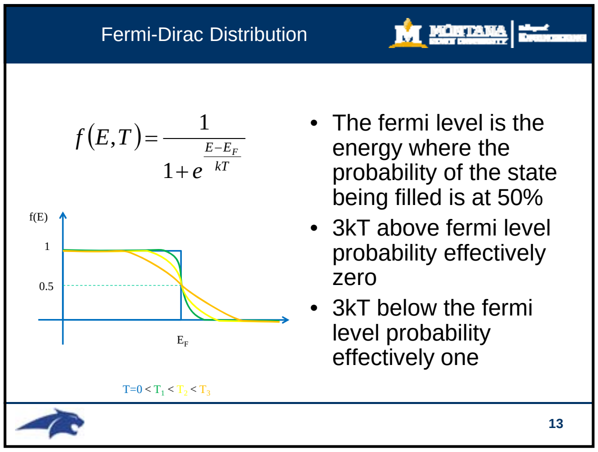
Reduced Energy Bands

- In many cases in semiconductor devices, we do not need to take into account change in momentum or exact distribution of carriers with energy.



Probability of State Occupation

- Pauli Exclusion Principle ➔ each allowed energy level can only hold 2 electrons (Spin Up and Spin Down)
- At low temperatures all available states are filled up to a certain energy level called the Fermi level (E_F)
- As temperature rises some electrons gain energy above the Fermi level.
- The probability of occupation is statistically given by the Fermi-Dirac Distribution Function

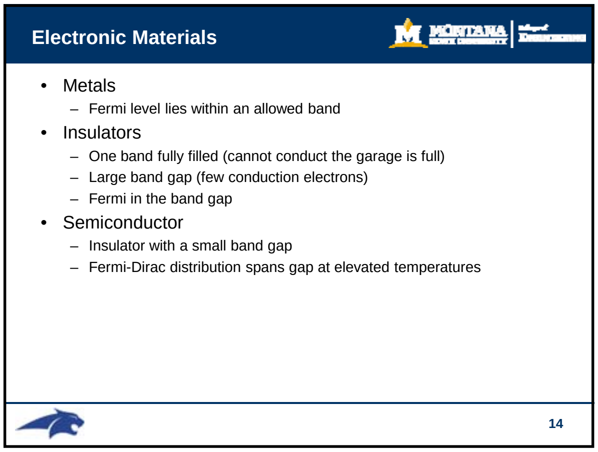


Fermi-Dirac Distribution

$$f(E,T)=\frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

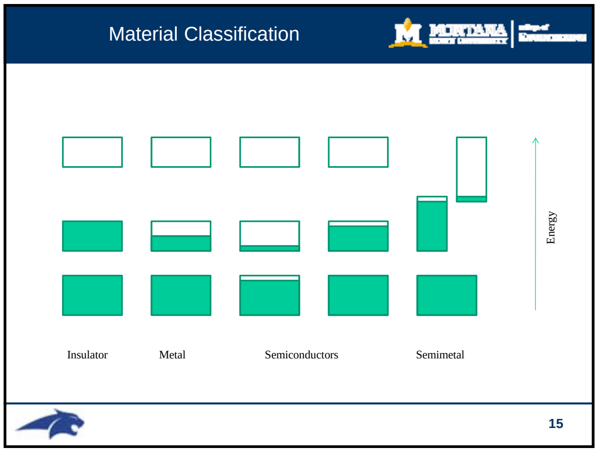
$T=0 < T_1 < T_2 < T_3$

- The fermi level is the energy where the probability of the state being filled is at 50%
- 3kT above fermi level probability effectively zero
- 3kT below the fermi level probability effectively one

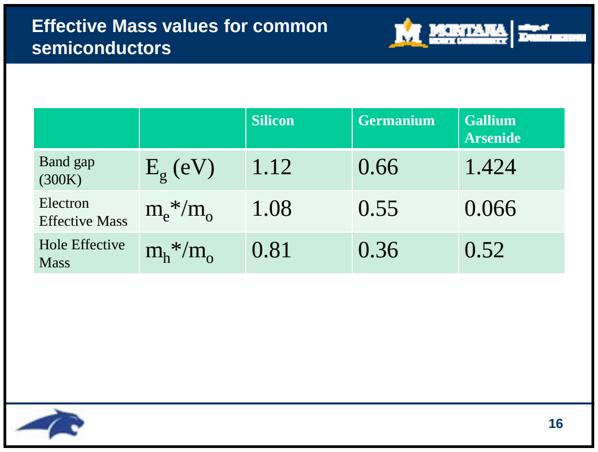


Electronic Materials

- Metals
  - Fermi level lies within an allowed band
- Insulators
  - One band fully filled (cannot conduct the garage is full)
  - Large band gap (few conduction electrons)
  - Fermi in the band gap
- Semiconductor
  - Insulator with a small band gap
  - Fermi-Dirac distribution spans gap at elevated temperatures

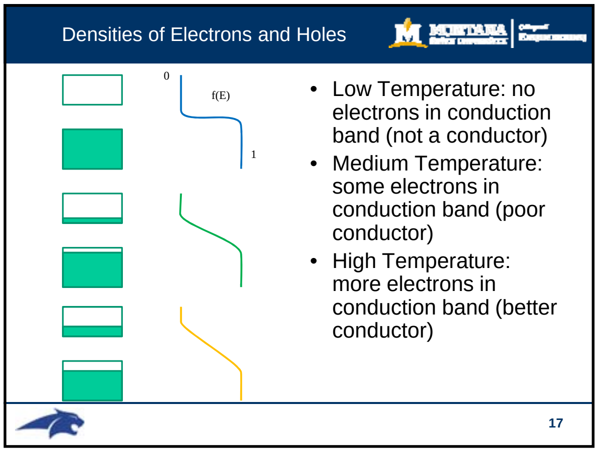


Material Classification



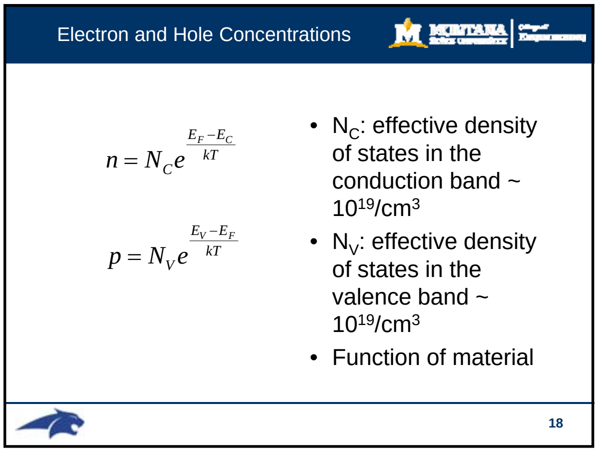
Effective Mass values for common semiconductors

		Silicon	Germanium	Gallium Arsenide
Band gap (300K)	E_g (eV)	1.12	0.66	1.424
Electron Effective Mass	m_e^*/m_o	1.08	0.55	0.066
Hole Effective Mass	m_h^*/m_o	0.81	0.36	0.52



Densities of Electrons and Holes

- Low Temperature: no electrons in conduction band (not a conductor)
- Medium Temperature: some electrons in conduction band (poor conductor)
- High Temperature: more electrons in conduction band (better conductor)



Electron and Hole Concentrations

$$n = N_C e^{\frac{E_F - E_C}{kT}}$$

$$p = N_V e^{\frac{E_V - E_F}{kT}}$$

- N_C: effective density of states in the conduction band ~ $10^{19}/\text{cm}^3$
- N_V: effective density of states in the valence band ~ $10^{19}/\text{cm}^3$
- Function of material

Fermi Level of Intrinsic Semiconductors

$$n = p = n_i$$

$$np = n_i^2 = N_C N_V e^{\frac{E_V - E_C}{kT}} = N_C N_V e^{-\frac{E_g}{kT}}$$

$$N_C e^{\frac{E_F - E_C}{kT}} = N_V e^{\frac{E_V - E_F}{kT}}$$

$$E_F = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln\left(\frac{N_V}{N_C}\right)$$

Small

- Concentration of holes equals the concentration of electrons
- A function of the effective density of states and gap energy
- Fermi levels lies close the midgap

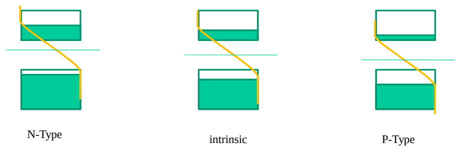
Fermi Level in Doped Semiconductors

$$n \approx N_D = N_C e^{\frac{E_F - E_C}{kT}}$$

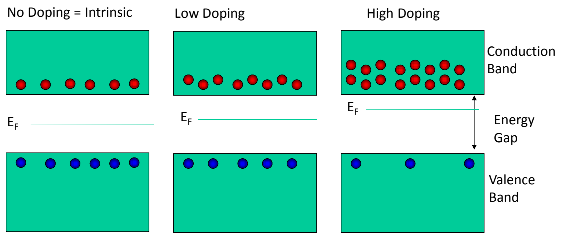
$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right)$$

$$p \approx N_A = N_V e^{\frac{E_V - E_F}{kT}}$$

$$E_V - E_F = kT \ln\left(\frac{N_A}{N_V}\right)$$



Fermi Level : N-type



As the donor doping concentration increases the fermi level moves towards the conduction band

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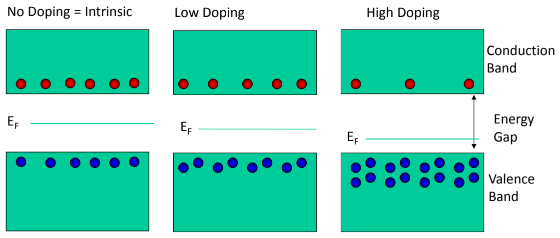
Fermi Level : P-type



As the donor doping concentration increases the fermi level moves towards the conduction band

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Review for Test #1 on Sunlight

- Energy of a photon (E_{ph})
- Flux (Φ)
- Radiant Power Density (H)
- Spectral Irradiance
- Blackbody Radiation
- Peak Radiation Wavelength (λ_{peak})
- Solar Constant = 1367
- Air Mass (AM)
- Direct Radiation
- Diffuse Radiation
- Declination Angle (δ)
- Elevation Angle (α)
- Azimuth Angle (ψ)
- Zenith Angle (θ)
- Hour Angle (HRA)
- Latitude Angle (ϕ)
- Tilt Angle (β)
- Solar Noon
- Radiation on Tilted Surface
- Peak Sun Hours (PSH)
- Isoflux Contours