

For the reasons described in last lecture, fixed effort (FE) harvest ('proportional harvest') is safer than fixed quota (FQ) harvest ('constant harvest') for offtake from a density dependent population.

This conclusion comes from identifying stable vs unstable equilibrium densities, which are population sizes at which harvest equals recruitment, so N stays constant.

Further important points from the fixed effort model.

A. Quantitatively, what will the offtake be at MSY?

Recall that MSY occurs at $N=K/2$, for a population with *linear* density dependence.

$$\frac{dN}{dt} = rN_t \left(1 - \frac{N_t}{K}\right)$$

equilibrium: recruitment = harvest = $\frac{dN}{dt}$

recall :

$$MSY : N = \frac{K}{2}$$

so

$$\frac{N}{K} = \frac{1}{2}$$

substituting :

$$H_{MSY} = \frac{dN}{dt} = rN_t \left(1 - \frac{1}{2}\right) = \frac{rN_t}{2}$$

This is the maximum number of individuals that can be sustainably harvested, H_{MSY} .

Harvest = Exploitation _ rate _ per _ cap * Popsiz

$$H = EN$$

$$E = \frac{H}{N}$$

$$E_{MSY} = \frac{rN}{2} * \frac{1}{N} = \frac{r}{2}$$

This is the per capita exploitation rate at MSY, E_{MSY} . It is not the maximum sustainable per capita exploitation rate, E_{MAX} .

Major points:

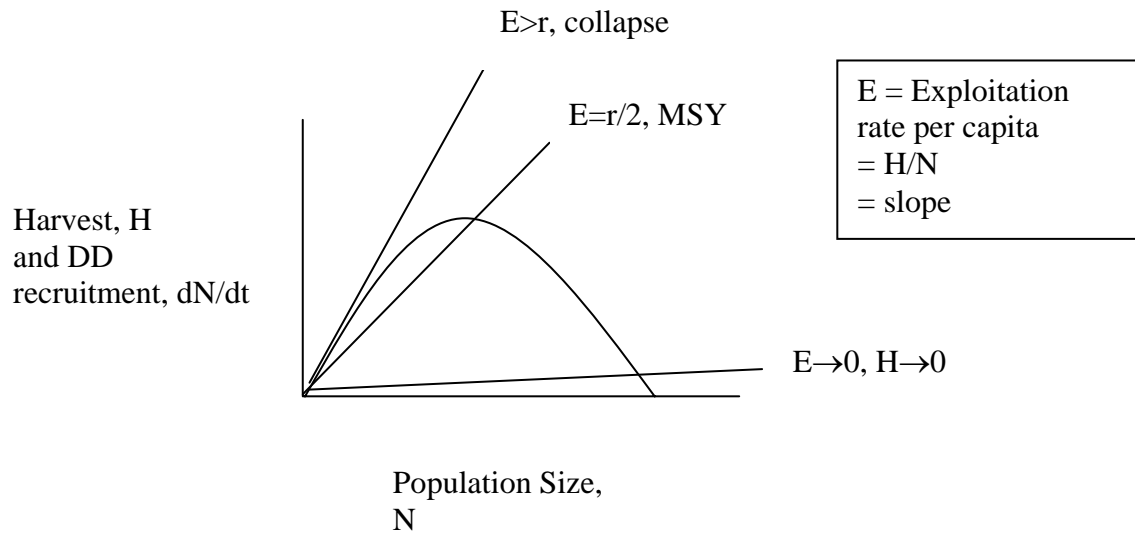
- 1) In $E = r/2$ and $H = Nr/2$, the two is not some sort of fudge factor to be safe!!
- 1) You will see equations for conservative sustainable yield, CSY

$$CSY = \frac{r}{2} * N * F$$

Here, F = fudge factor with value <1 to be safe

- 2) $E > r$ is the point of collapse where $H > dN/dt$. (Logical: the maximum recruitment with a logistically growing population is rN , (when N is small and $[1-N/K]$ is nearly one). $E=H/N$, so $E > r$ is where E exceeds maximum recruitment rate possible. This defines E_{MAX} .
- 3) So $r/2 < E < r$ defines the range of diminishing returns (reduced harvest with increased effort) between N_{MSY} and $N_{collapse}$

4) Graphically:



B. What is the equilibrium population size for any given exploitation rate?

Start with equation for density dependent population growth

$$\frac{dN}{dt} = rN_t \left(1 - \frac{N_t}{K}\right)$$

We will have an equilibrium if the harvest, H, exactly equals the recruitment.

So to maintain a constant population size over time, set harvest equal to recruitment:

$$H = rN_t \left(1 - \frac{N_t}{K}\right)$$

Recall that harvest (H) equals the exploitation rate per capita (E), which is the probability that any individual will be harvested, multiplied by population size.

$$H = EN$$

So, dividing both sides of the equation by N_t gives:

$$\frac{H}{N_t} = E = r\left(1 - \frac{N_t}{K}\right)$$

This is an equation that gives the exploitation rate, E , as a function of population size, N . That is what we need to answer the original question, what is the expected population size for any given exploitation rate.

All we have to do is solve the equation for N .

$$E = r\left(1 - \frac{N_t}{K}\right)$$

divide each side by r

$$\frac{E}{r} = 1 - \frac{N_t}{K}$$

add N_t/K to each side, subtract E/r from each side

$$\frac{N_t}{K} = 1 - \frac{E}{r}$$

multiply each side by K

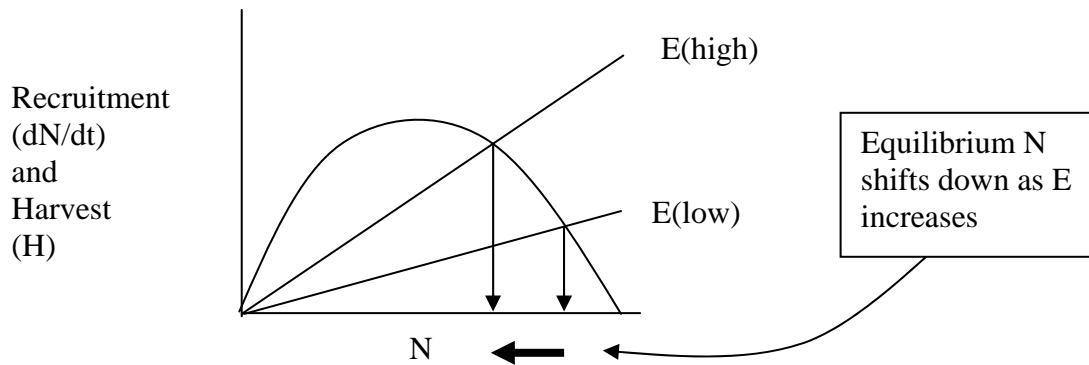
$$N_t = K\left(1 - \frac{E}{r}\right)$$

This is the equation for population size, N as a function of exploitation rate, E , for a population that grows in a (linear) density dependent manner.

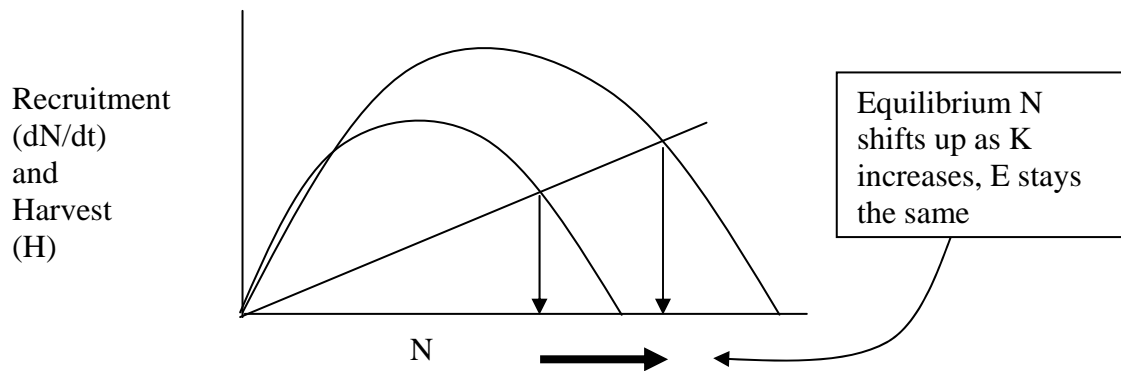
$$N_t = K\left(1 - \frac{E}{r}\right)$$

Three major points from this equation:

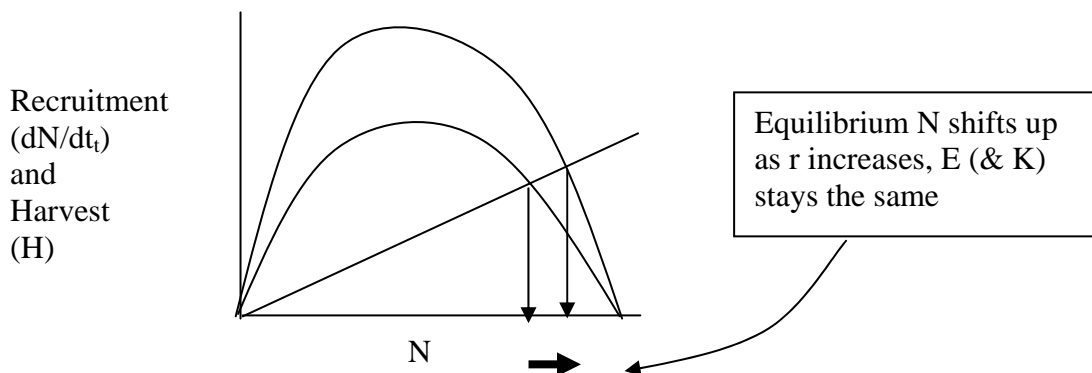
1. As exploitation rate increases, equilibrium population size decreases. ($E \uparrow$, $N \downarrow$).



2. As the carrying capacity increases, equilibrium population size increases, for any given level of exploitation ($K \uparrow$, $N \uparrow$, if $E = \text{constant}$)



3. As the population growth rate increases, equilibrium population size increases, for any given level of exploitation ($r \uparrow$, $N \uparrow$, if $E = \text{constant}$).



C. A final point about fixed effort or proportional harvest.

FE is preferable to FQ for the reasons explained before → FE avoids unstable low density equilibrium, therefore less likely to cause a population crash.

HOWEVER, fixed effort harvest is regulated by laws that control the exploitation rate E, while fixed quota harvesting directly regulates the harvest, H.

Per capita exploitation rate = harvest/population size

$$E = H/N$$

1. The harvest H and the population size N are both variables that have variances (uncertainties) associated with them. These uncertainties compound to make the variance of E quite large. In other words, we face a lot of uncertainty in knowing what the exploitation rate truly is.
2. Incidental kill that is not harvested, poaching, and intentional under-reporting of legal harvest will all tend to make the reported estimate of E lower than the true E. (These issues also cause underestimation of H.)
3. Harvesters will try to improve their efficiency (modifying equipment, methods of hunting), so that with no change in laws or regulation (E apparently constant), the probability of catching an individual can increase (E actually increases). Regulation of exploitation rate is tracking a moving target.