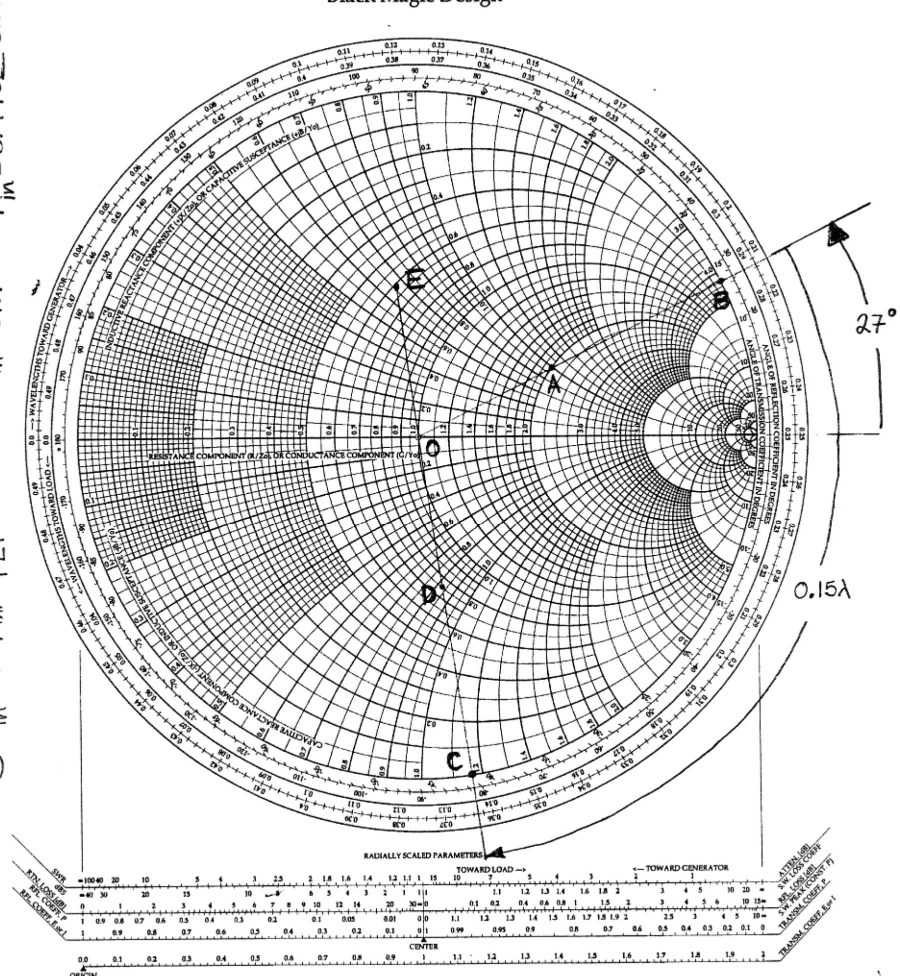
Lecture 12

Example: A load of $Z_L = 100 + j50\,\Omega$ terminates a $50\,\Omega$ line. What is the input impedance, input admittance, and reflection coefficient 0.15λ from the load? What is the reflection coefficient at the load?

The Complete Smith Chart
Black Magic Design



⑤ Rotate $\lambda/4$ on constant SWR circle (radius = 28.5mm) to locate $\hat{y}_{in}$ → E $\hat{y}_{in} = 0.62 + j0.67$

$y_{in} = \frac{1}{Z_0}\hat{y}_{in} = \frac{1}{50}(0.62 + j0.67)$ $y_{in} = 0.012 + j0.013$ S

⑥ $\Gamma_{in} = ?$ $|\Gamma_{in}| = |\Gamma_L| = 0.445$ $\angle\Gamma_{in} \sim -81.7°$ $\Gamma_{in} = 0.445\angle{-81.7°}$

Solution

① $\hat{z}_L = \frac{Z_L}{Z_0} = 2 + j\,\Omega \rightarrow$ pt. A

② $\Gamma_L = ?$ $|\Gamma_L| = \frac{OA}{OB} = \frac{28.5\text{mm}}{64\text{ mm}} = 0.445$

$\angle\Gamma_L \sim 27°$

③ Rotate 0.15λ toward generator
$\rightarrow$ rotate CW $\rightarrow 0.213\lambda + 0.15\lambda = 0.363\lambda$

④ Either draw constant SWR circle through A and take intersection of circle with OC or measure 28.5mm from O along OC
↳ pt D $\rightarrow \hat{z}_{in} = 0.75 - j0.82$

$Z_{in} = 50\,(0.75 - j0.82)$

$Z_{in} = 37.5 - j41\,\Omega$

Solution of the previous example via analytical expression

▷ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 50}{100 + j50 + 50} = \frac{50 + j50}{150 + j50}$

$$\boxed{\Gamma_L = 0.447 \angle 26.6^\circ}$$

▷ $\Gamma_{in} \triangleq \Gamma(\ell = 0.15\lambda) = \Gamma_L e^{-j2\beta\ell} = 0.447 e^{j26.6} e^{-j2\frac{2\pi}{\lambda}(0.15\lambda)}$

$$\Gamma_{in} = 0.447 e^{j26.6} e^{-j108}$$

$$\boxed{\Gamma_{in} = 0.447 e^{-j81.4^\circ}}$$

▷ $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$ or $Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$

$$Z_{in} = 50 \frac{1 + 0.447e^{-j81.4}}{1 - 0.447e^{-j81.4}} = \boxed{37.53 - j41.46\ \Omega = Z_{in}}$$

$$\boxed{Y_{in} = \frac{1}{Z_{in}} = 0.012 + j0.013\ S}$$

<end lecture 12>

Lecture 13 was a general discussion of homework one and other issues. No notes poseted

Lecture 14 Microstrip and Stripline
Notes from the text:

3.7 STRIPLINE

We now consider stripline, a planar-type of transmission line that lends itself well to microwave integrated circuitry and photolithographic fabrication. The geometry of a stripline is shown in Figure 3.22a. A thin conducting strip of width W is centered between two wide conducting ground planes of separation b, and the entire region between the ground planes is filled with a dielectric. In practice, stripline is usually constructed by etching the center conductor on a grounded substrate of thickness $b/2$, and then covering with another grounded substrate of the same thickness. An example of a stripline circuit is shown in Figure 3.23.

Since stripline has two conductors and a homogeneous dielectric, it can support a TEM wave, and this is the usual mode of operation. Like the parallel plate guide and coaxial lines, however, the stripline can also support higher order TM and TE modes, but these are usually avoided in practice (such modes can be suppressed with shorting screws between the ground

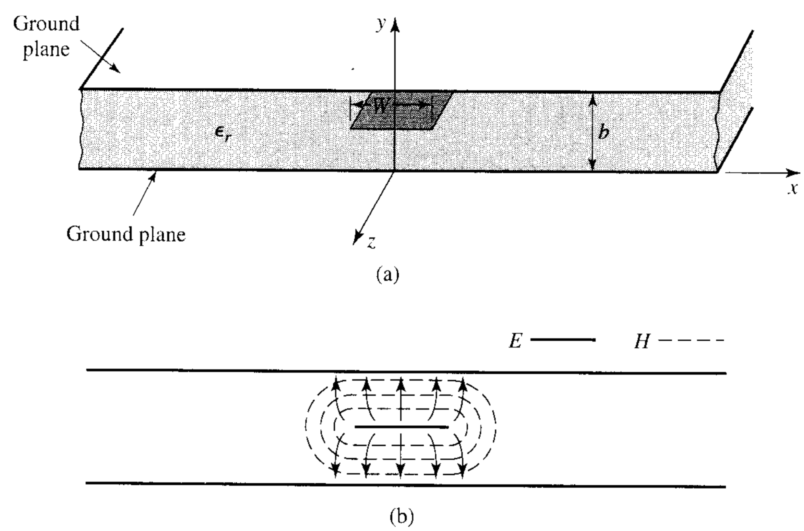


FIGURE 3.22 Stripline transmission line. (a) Geometry. (b) Electric and magnetic field lines.

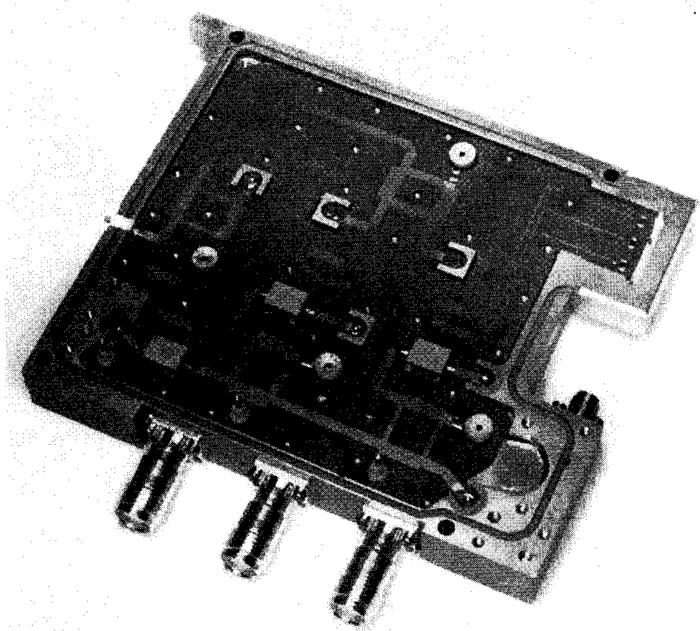


FIGURE 3.23 Photograph of a stripline circuit assembly, showing four quadrature hybrids, open-circuit tuning stubs, and coaxial transitions.
Courtesy of Harlan Howe, Jr., M/A-COM Inc.

planes and by restricting the ground plane spacing to less than $\lambda/4$). Intuitively, one can think of stripline as a sort of "flattened out" coax—both have a center conductor completely enclosed by an outer conductor and are uniformly filled with a dielectric medium. A sketch of the field lines for stripline is shown in Figure 3.22b. The main difficulty we will have with stripline is that it does not lend itself to a simple analysis, as did the transmission lines and waveguides that we have previously discussed. Since we will be concerned primarily with the TEM mode of the stripline, an electrostatic analysis is sufficient to give the propagation constant and characteristic impedance. An exact solution of Laplace's equation is possible by a conformal mapping approach [6], but the procedure and results are cumbersome. Thus, we will present closed-form expressions that give good approximations to the exact results and then discuss an approximate numerical technique for solving Laplace's equation for a geometry similar to stripline; this technique will also be applied to microstrip line in the following section.

Formulas for Propagation Constant, Characteristic Impedance, and Attenuation

From Section 3.1 we know that the phase velocity of a TEM mode is given by thus the propagation constant of the stripline is

$$\beta = \frac{\omega}{v_p} = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \sqrt{\epsilon_r}k_0. \tag{3.177}$$

In (3.176), $c = 3 \times 10^8$ m/sec is the speed of light in free-space. The characteristic impedance of a transmission line is given by

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C}, \tag{3.178}$$

where L and C are the inductance and capacitance per unit length of the line. Thus, we can find Z_0 if we know C. As mentioned above, Laplace's equation can be solved by conformal mapping to find the capacitance per unit length of the stripline. The resulting solution, however, involves complicated special functions [6], so for practical computations simple formulas have been developed by curve fitting to the exact solution [6], [7]. The resulting formula for characteristic impedance is

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}, \tag{3.179a}$$

where W_e is the effective width of the center conductor given by

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \frac{W}{b} > 0.35 \\ (0.35 - W/b)^2 & \text{for } \frac{W}{b} < 0.35. \end{cases} \tag{3.179b}$$

These formulas assume a zero strip thickness, and are quoted as being accurate to about 1% of the exact results. It is seen from (3.179) that the characteristic impedance decreases as the strip width W increases.

When designing stripline circuits, one usually needs to find the strip width, given the characteristic impedance (and height b and permittivity ϵ_r), which requires the inverse of the formulas in (3.179). Such formulas have been derived as

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r}\, Z_0 < 120 \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r}\, Z_0 > 120, \end{cases} \tag{3.180a}$$

where

$$x = \frac{30\pi}{\sqrt{\epsilon_r} Z_0} - 0.441. \tag{3.180b}$$

Since stripline is a TEM type of line, the attenuation due to dielectric loss is of the same form as that for other TEM lines and is given in (3.30). The attenuation due to conductor loss can be found by the perturbation method or Wheeler's incremental inductance rule. An approximate result is

$$\alpha_c = \begin{cases} \dfrac{2.7 \times 10^{-3} R_s \epsilon_r Z_0}{30\pi(b - t)} A & \text{for } \sqrt{\epsilon_r} Z_0 < 120 \\ \dfrac{0.16 R_s}{Z_0 b} B & \text{for } \sqrt{\epsilon_r} Z_0 > 120 \end{cases} \quad \text{Np/m}, \tag{3.181}$$

with

$$A = 1 + \frac{2W}{b - t} + \frac{1}{\pi}\frac{b + t}{b - t} \ln\left(\frac{2b - t}{t}\right),$$

$$B = 1 + \frac{b}{(0.5W + 0.7t)}\left(0.5 + \frac{0.414t}{W} + \frac{1}{2\pi} \ln \frac{4\pi W}{t}\right),$$

where t is the thickness of the strip.

EXAMPLE 3.5 STRIPLINE DESIGN

Find the width for a 50 Ω copper stripline conductor, with $b = 0.32$ cm and $\epsilon_r = 2.20$. If the dielectric loss tangent is 0.001 and the operating frequency is 10 GHz, calculate the attenuation in dB/λ. Assume a conductor thickness of $t = 0.01$ mm.

Solution

Since $\sqrt{\epsilon_r}Z_0 = \sqrt{2.2}(50) = 74.2 < 120$, and $x = 30\pi/(\sqrt{\epsilon_r}Z_0) - 0.441 = 0.830$, (3.180) gives the width as $W = bx = (0.32)(0.830) = 0.266$ cm. At 10 GHz, the wavenumber is

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = 310.6 \text{ m}^{-1}.$$

From (3.30) the dielectric attenuation is

$$\alpha_d = \frac{k\tan\delta}{2} = \frac{(310.6)(0.001)}{2} = 0.155 \text{ Np/m}.$$

The surface resistance of copper at 10 GHz is $R_s = 0.026$ Ω. Then from (3.181) the conductor attenuation is

$$\alpha_c = \frac{2.7\times 10^{-3}R_s\epsilon_r Z_0 A}{30\pi(b-t)} = 0.122 \text{ Np/m},$$

since $A = 4.74$. The total attenuation constant is

$$\alpha = \alpha_d + \alpha_c = 0.277 \text{ Np/m}.$$

In dB,

$$\alpha(\text{dB}) = 20\log e^{\alpha} = 2.41 \text{ dB/m}.$$

At 10 GHz, the wavelength on the stripline is

$$\lambda = \frac{c}{\sqrt{\epsilon_r}f} = 2.02 \text{ cm},$$

so in terms of wavelength the attenuation is

$$\alpha(\text{dB}) = (2.41)(0.0202) = 0.049 \text{ dB}/\lambda. \qquad \blacksquare$$

3.8 MICROSTRIP

Microstrip line is one of the most popular types of planar transmission lines, primarily because it can be fabricated by photolithographic processes and is easily integrated with other passive and active microwave devices. The geometry of a microstrip line is shown in Figure 3.25a. A conductor of width W is printed on a thin, grounded dielectric substrate of thickness d and relative permittivity ϵ_r; a sketch of the field lines is shown in Figure 3.25b.

If the dielectric were not present ($\epsilon_r = 1$), we could think of the line as a two-wire line consisting of two flat strip conductors of width W, separated by a distance $2d$ (the

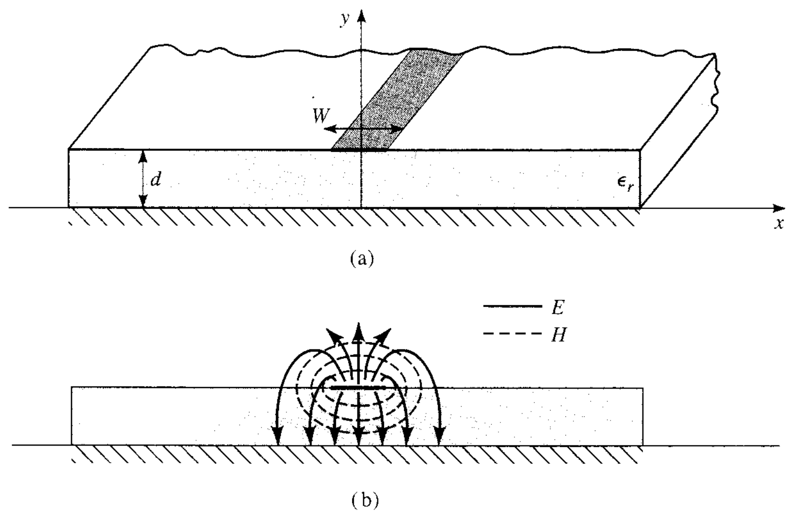


FIGURE 3.25 Microstrip transmission line. (a) Geometry. (b) Electric and magnetic field lines.

ground plane can be removed via image theory). In this case we would have a simple TEM transmission line, with $v_p = c$ and $\beta = k_0$.

The presence of the dielectric, and particularly the fact that the dielectric does not fill the air region above the strip ($y > d$), complicates the behavior and analysis of microstrip line. Unlike stripline, where all the fields are contained within a homogeneous dielectric region, microstrip has some (usually most) of its field lines in the dielectric region, concentrated between the strip conductor and the ground plane, and some fraction in the air region above the substrate. For this reason the microstrip line cannot support a pure TEM wave, since the phase velocity of TEM fields in the dielectric region would be $c/\sqrt{\epsilon_r}$, but the phase velocity of TEM fields in the air region would be c. Thus, a phase match at the dielectric-air interface would be impossible to attain for a TEM-type wave.

In actuality, the exact fields of a microstrip line constitute a hybrid TM-TE wave, and require more advanced analysis techniques than we are prepared to deal with here. In most practical applications, however, the dielectric substrate is electrically very thin ($d \ll \lambda$), and so the fields are quasi-TEM. In other words, the fields are essentially the same as those of the static case. Thus, good approximations for the phase velocity, propagation constant, and characteristic impedance can be obtained from static or quasi-static solutions. Then the phase velocity and propagation constant can be expressed as

$$v_p = \frac{c}{\sqrt{\epsilon_e}}, \quad (3.193)$$

$$\beta = k_0\sqrt{\epsilon_e}, \quad (3.194)$$

where ϵ_e is the effective dielectric constant of the microstrip line. Since some of the field lines are in the dielectric region and some are in air, the effective dielectric constant satisfies the relation

$$1 < \epsilon_e < \epsilon_r,$$

and is dependent on the substrate thickness, d, and conductor width, W.

We will first present design formulas for the effective dielectric constant and characteristic impedance of microstrip line; these results are curve-fit approximations to rigorous quasi-static solutions [8], [9]. Then we will outline a numerical method of solution (similar to that used in the previous section for stripline) for the capacitance per unit length of microstrip line.

Formulas for Effective Dielectric Constant, Characteristic Impedance, and Attenuation

The effective dielectric constant of a microstrip line is given approximately by

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12d/W}}. \quad (3.195)$$

The effective dielectric constant can be interpreted as the dielectric constant of a homogeneous medium that replaces the air and dielectric regions of the microstrip, as shown in Figure 3.26. The phase velocity and propagation constant are then given by (3.193) and (3.194).

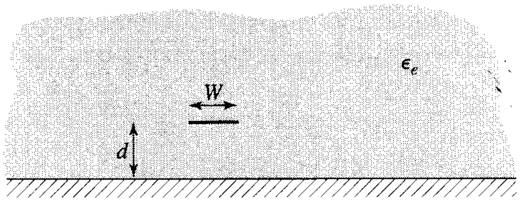


FIGURE 3.26 Equivalent geometry of quasi-TEM microstrip line, where the dielectric slab of thickness d and relative permittivity ϵ_r has been replaced with a homogeneous medium of effective relative permittivity, ϵ_e.

Given the dimensions of the microstrip line, the characteristic impedance can be calculated as

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \leq 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_e}\,[W/d + 1.393 + 0.667 \ln (W/d + 1.444)]} & \text{for } W/d \geq 1. \end{cases} \tag{3.196}$$

For a given characteristic impedance Z_0 and dielectric constant ϵ_r, the W/d ratio can be found as

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\epsilon_r}\right\}\right] & \text{for } W/d > 2, \end{cases} \tag{3.197}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}.$$

Considering microstrip as a quasi-TEM line, the attenuation due to dielectric loss can be determined as

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1) \tan \delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} \text{ Np/m}, \tag{3.198}$$

where $\tan \delta$ is the loss tangent of the dielectric. This result is derived from (3.30) by multiplying by a "filling factor,"

$$\frac{\epsilon_r(\epsilon_e - 1)}{\epsilon_e(\epsilon_r - 1)},$$

which accounts for the fact that the fields around the microstrip line are partly in air (lossless) and partly in the dielectric. The attenuation due to conductor loss is given approximately by [8]

$$\alpha_c = \frac{R_s}{Z_0 W} \text{ Np/m}, \tag{3.199}$$

where $R_s = \sqrt{\omega\mu_0/2\sigma}$ is the surface resistivity of the conductor. For most microstrip substrates, conductor loss is much more significant than dielectric loss; exceptions may occur with some semiconductor substrates, however.

EXAMPLE 3.7 MICROSTRIP DESIGN

Calculate the width and length of a microstrip line for a 50 Ω characteristic impedance and a 90° phase shift at 2.5 GHz. The substrate thickness is $d = 0.127$ cm, with $\epsilon_r = 2.20$.

Solution

We first find W/d for $Z_0 = 50\ \Omega$, and initially guess that $W/d > 2$. From (3.197),

$$B = 7.985, \qquad W/d = 3.081.$$

So $W/d > 2$; otherwise we would use the expression for $W/d < 2$. Then $W = 3.081d = 0.391$ cm. From (3.195) the effective dielectric constant is

$$\epsilon_e = 1.87.$$

The line length, ℓ, for a 90° phase shift is found as

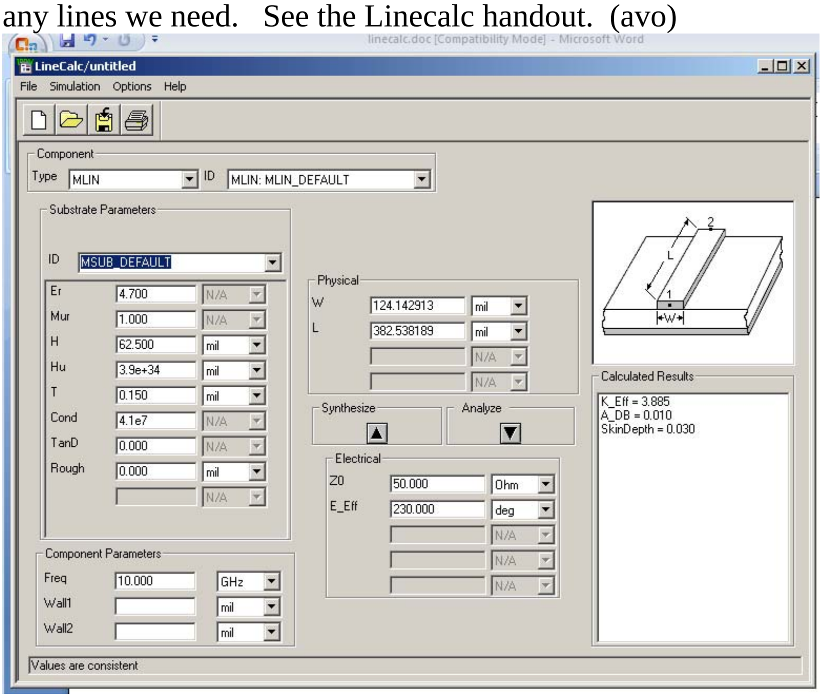
$$\phi = 90^\circ = \beta\ell = \sqrt{\epsilon_e}k_0\ell,$$

$$k_0 = \frac{2\pi f}{c} = 52.35\ \text{m}^{-1},$$

$$\ell = \frac{90^\circ(\pi/180^\circ)}{\sqrt{\epsilon_e}k_0} = 2.19\ \text{cm}.$$

■

Note that ADS has the Linecalc tool to design this type of line. We will use it to design any lines we need. See the Linecalc handout. (avo)



< end Lecture 14 >