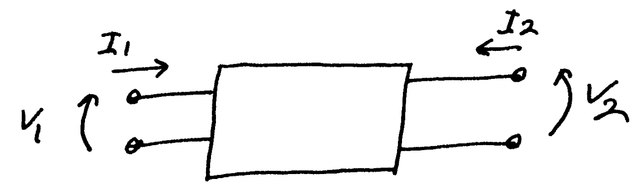
EE 43308 - Lecture 15

Microwave Network Analysis → chapter 4

→ the N-port model ←

Consider a 2-port Network:

we have an input V_1 and I_1,
and an output V_2 and I_2



we may represent this by Z, Y, H, C, or S parameters. → (and there are more!)

Z parameters:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left[\bar{\bar{Z}}\right] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Rightarrow \text{output open ckt}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Rightarrow \text{"transfer impedance"}$$

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$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

H parameters → "Hybrid" parameters.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

used for transistors.

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \quad \text{input impedance}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} \qquad \underbrace{\frac{I_2}{I_1}\bigg|_{V_2=0} = \beta}_{\text{transistor current gain.}}$$

Similar ~~de~~ derivations for Y + C

Problem at microwave frequencies →

we can't get good shorts or opens ⇒

shorts & opens cause reflected waves ⇒ $Z(\ell)$

~~so~~ the reference plane is frequency dependent

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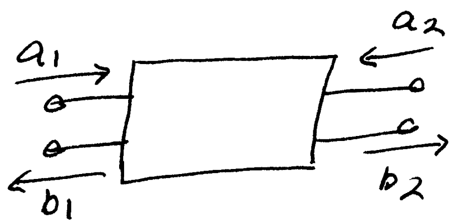
the solution is to use S-parameters.
"Scattering" parameters are measured with matched source & load → eliminates reflections

Consider a 2 port with normalized waves a_1, a_2 incident, and b_1, b_2 reflected



the equations are:

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

a_1^2 = input power
b_1^2 = reflected power.

this may be expanded to an n-port network.

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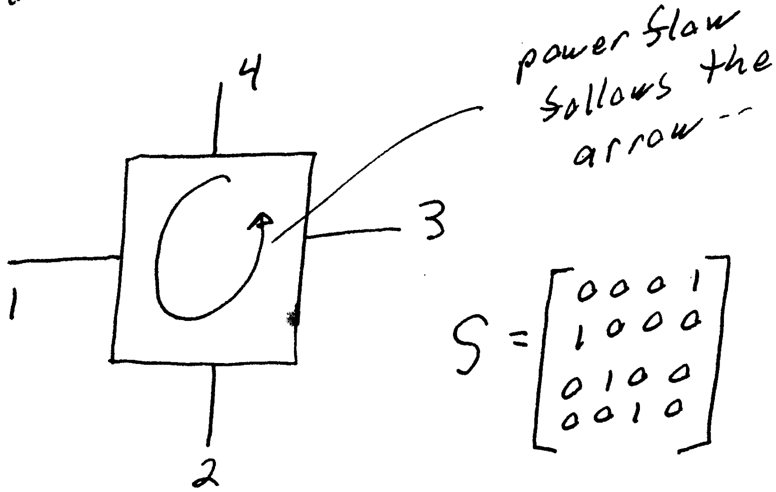
$$\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & & \\ S_{m1} & \cdots & S_{mn} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$b_n = \sum_K S_{nK} a_K .$$

output n

example - A microwave Circulator.



$$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

note: we have referred to V^+ & V^- as the peak voltages. as input voltages.

$$P_1 = a_1^2 = \frac{(V_1^+)^2}{2 Z_0} \Rightarrow a_1 = \frac{V_1^+}{\sqrt{2 Z_0}}$$

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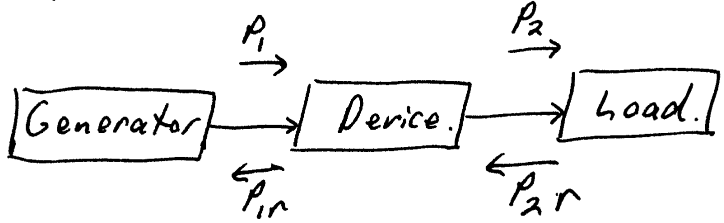
some texts show $a_1 = \frac{v_1^+}{\sqrt{Z_0}}$ where

v_1^+ is the rms sinewave voltage. – We will use this

S-parameters + power gains:



Transducer gain $G_T = |S_{21}|^2 = \frac{\text{Power delivered}}{\text{Power available}}$

(also called insertion gain.

$$= \frac{P_2 - P_2 r}{P_1}$$

Power Gain.

$$G_P = \frac{|S_{21}|^2}{1 - |S_{11}|^2} = \frac{\text{Power delv to load}}{\text{Power available from gen}}$$

$$= \frac{P_2 - P_2 r}{P_1 - P_1 r}$$

Available gain

$$G_a = \frac{|S_{21}|^2}{1 - |S_{22}|^2} = \frac{\text{Power available at output}}{\text{Power available at gener}} = \frac{P_2}{P_1}$$

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max available gain.

$$MAG = \frac{|S_{21}|^2}{\left((1-|S_{11}|^2)(1-|S_{22}|)\right)^2}$$

$$= \frac{\text{Power available at output}}{\text{power into the input port}} = \frac{P_2}{P_1 - P_{1r}}$$

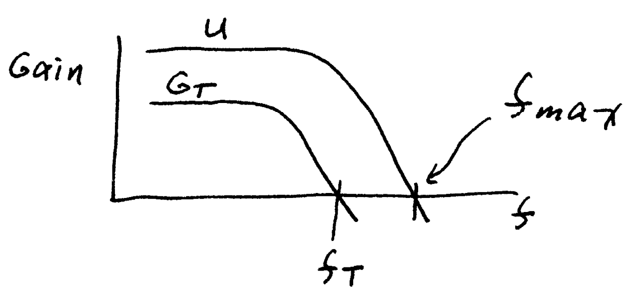
unilateral Power Gain.

$$U = \frac{|S_{11}||S_{22}||S_{12}S_{21}|}{(1-|S_{11}|^2)|1-|S_{22}|^2|}$$

$f_{max} \Rightarrow$ point where $U = 1$

$f_T \Rightarrow$ point where $G_T = 1$

more on these gains Later in the course.



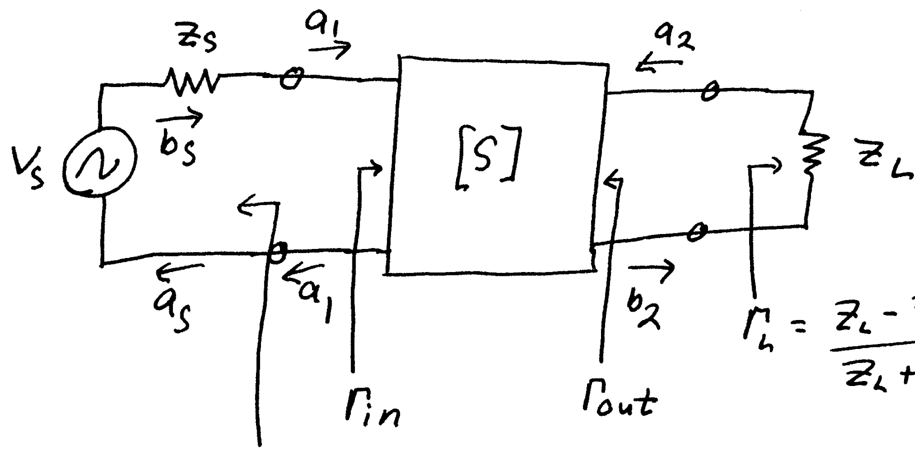
$1-|S_{11}|^2 \Rightarrow$ input miss match Loss

$1-|S_{22}|^2 \Rightarrow$ output miss match Loss

$\frac{1}{1-|S_{11}|^2} \Rightarrow$ gain increase.

<end lecture 15>

43308-~~16~~ Lecture 16 → S-parameters in a circuit:



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$\Gamma_{in} = S_{11} + \underbrace{\frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}}_{\text{input impedance change due to } \Gamma_L \text{ \& } S_{12}} \qquad \Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

S_{12} ← reverse gain.

$$\boxed{\begin{array}{l}\underline{\text{Unilateral}}\ \ \underline{\text{2-port}} \Rightarrow S_{12} = 0 \\ \quad \text{then } \Gamma_{in} = S_{11} \Rightarrow \text{Load does not change the input impedance.}\end{array}}$$

2.

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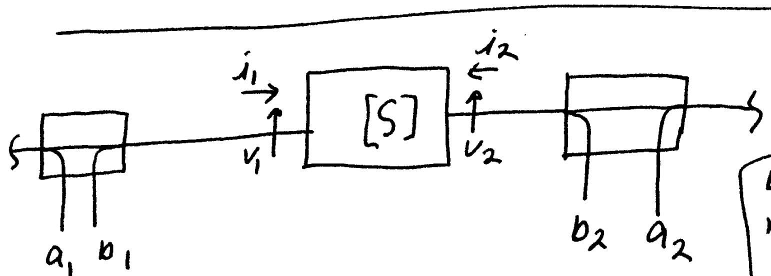
it can be shown that

$$G_a = \underbrace{\frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}}_{\text{input miss-match Loss}} |S_{21}|^2 \underbrace{\frac{1}{1-|\Gamma_{out}|^2}}_{\text{output miss-match Loss.}}$$

(power gain.

Voltage gain.

$$A_v = \frac{V_{out}}{V_{in}} = \frac{S_{21}(1+\Gamma_L)}{(1-S_{22}\Gamma_L)+S_{11}(1-S_{22}\Gamma_L)+S_{12}S_{21}\Gamma_L}$$



a VNA measures a_1, a_2, b_1, b_2

Let v_1^+ + v_1^- be <u>rms</u> voltages.

$$V_1 = v_1^+ + v_1^- \quad (1)$$

$$i_1 = i_1^+ - i_1^- = \frac{v_1^+}{Z_0} - \frac{v_1^-}{Z_0} \quad (2)$$

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$$a_1 = \frac{v_1^+}{\sqrt{Z_0}} = i_1^+ \sqrt{Z_0} \Rightarrow a_1^2 = \frac{(V_1^+)^2}{Z_0} = \underline{(i_1^+)^2 Z_0} \Rightarrow \text{input power.}$$

same approach for other terms.

Can show:

$$a_1 = \frac{1}{2\sqrt{Z_0}}\left[V_1 + Z_0 i_1\right] \quad (3)$$

minus

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = i_1^- \sqrt{Z_0} = \frac{1}{2\sqrt{Z_0}}\left[v_1 \cancel{+} Z_0 i_1\right] \quad (4)$$

$$a_2 = \frac{v_2^+}{\sqrt{Z_0}} = i_2^+ \sqrt{Z_0} = \frac{1}{2\sqrt{Z_0}}\left[v_2 + Z_0 i_2\right] \quad (5)$$

$$b_2 = \frac{v_2^-}{\sqrt{Z_0}} = i_2^- \sqrt{Z_0} = \frac{1}{2\sqrt{Z_0}}\left[v_2 - Z_0 i_2\right] \quad (6)$$

using (3), (4):

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{v_2 - i_1 Z_0}{v_1 + i_1 Z_0} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$\Rightarrow$ so $\underline{S_{11} = \Gamma_1}$ with $Z_L = 50$, (ie $\Gamma_L = 0$)

This is how a VNA works!

Properties of the Scattering Matrix

- Reciprocal Networks: Passive networks containing only linear elements including resistors, capacitors, inductors and transformers.

$$\Rightarrow Z_{mn} = Z_{nm}$$

$$\Rightarrow S_{ij} = S_{ji} \quad \text{for } i \neq j$$

- Lossless Networks: A passive network containing no resistors.

Under the lossless case, the [S] is unitary. That is,

$$\sum_{i=1}^{N} S_{ij} S_{ij}^{*} = 1 \qquad j = 1, 2, \ldots, N$$

In words: the sum of the products of each term of any one column (or row if network is reciprocal) multiplied by its own conjugate is unity.

For a two-port:

Column 1

$$S_{11} S_{11}^{*} + S_{21} S_{21}^{*} = 1$$

Column 2

$$S_{12} S_{12}^{*} + S_{22} S_{22}^{*} = 1$$

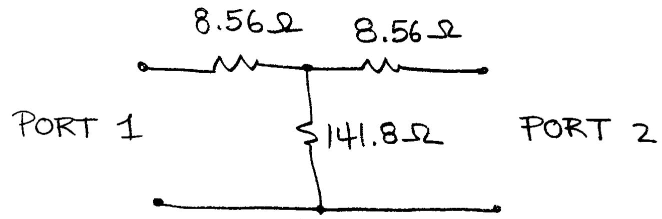
If the lossless network is reciprocal

$$S_{12} = S_{21} \qquad |S_{11}| = |S_{22}|$$

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

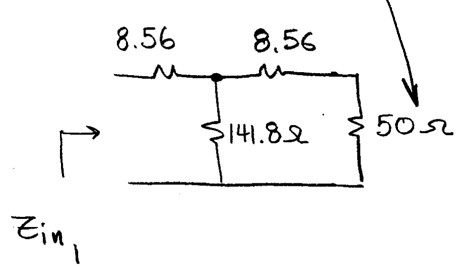
While the [S] is the predominant descriptor of microwave networks, other matricies such as the impedance, admittance, and transmission matrix are also used.

Evaluation of S-parameters – ex. 4.4 Pozar



With 50Ω ports.

- $S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{\text{PORT 2 matched}} = \Gamma_1 = \dfrac{Z_{in_1} - Z_0}{Z_{in_1} + Z_0}$

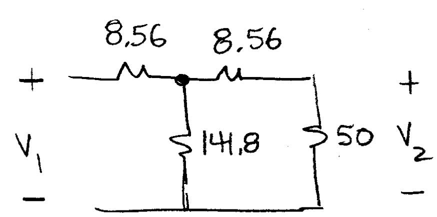


$$Z_{in_1} = 8.56 + 141.8 \,//\, 58.56 = 8.56 + 41.44 = 50\Omega$$

$$S_{11} = \frac{50-50}{50+50} = 0$$

By symmetry $S_{22} = S_{11} = 0$

- $S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{\text{PORT 2 matched}}$



Important : Since $S_{11} = 0$

V_1 total $= V_1^+$ (no V_1^-), since port 2 is matched

V_2 total $= V_2^-$ (no V_2^+)

Using Voltage division

$$V_2 = \left(\frac{50}{50+8.56}\right)\left(\frac{58.56 // 141.8}{58.56 // 141.8 + 8.56}\right)V_1 = \left(\frac{50}{58.56}\right)\left(\frac{41.44}{50}\right)V_1$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_2^-}{V_1^+} = 0.707$$

The network is reciprocal and thus $S_{12} = S_{21} = 0.707$

$$\begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

We should realize that, due to the existence of resistive elements, the network is not lossless. Let's verify

Does $|S_{11}|^2 + |S_{21}|^2 = 1$?

$$|0|^2 + |0.707|^2 = 0.5$$

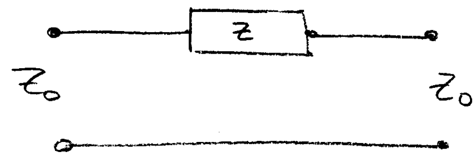
$\Rightarrow$ Half the power $(10 \times \log 0.5 = -3\text{dB})$ is lost.
The network acts as a 3dB attenuator.

In general, loss in the passive network is given by

$$P_{LOSS} = 1 - |S_{11}|^2 - |S_{21}|^2$$

Another example

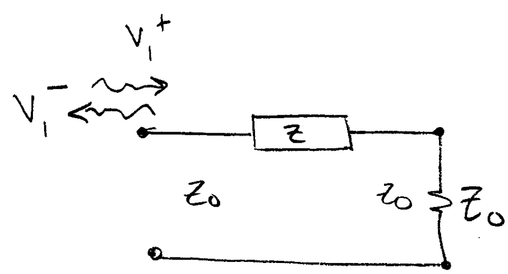
What are the s-parameters for a series element?



$Z = R + jX_L$

$\Rightarrow$ Clearly reciprocal $\Rightarrow$ $S_{21} = S_{12}$

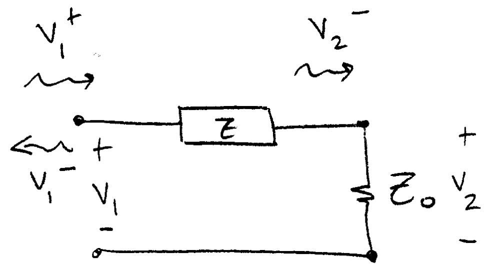
- $S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{\text{Port 2 matched}}$ $(V_2^+ = 0)$



$Z_{in} = Z + Z_o$

$$S_{11} = \frac{(Z + Z_o) - Z_o}{(Z + Z_o) + Z_o} \qquad S_{11} = S_{22} = \frac{Z}{2Z_o + Z}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{\text{Port 2 matched}}$$



Since port 2 is matched, no $V_2^+ \Rightarrow V_2 = V_2^-$

$$\frac{V_2}{V_1} = \frac{Z_0}{Z+Z_0} \quad \text{but} \quad V_1 = V_1^+ + V_1^- \quad (S_{11} \neq 0 \,!)$$

$$V_2 = V_2^- = (V_1^+ + V_1^-)\,\frac{Z_0}{Z+Z_0} \qquad (1)$$

from before, $V_1^- = S_{11} V_1^+ = \dfrac{Z}{2Z_0+Z} V_1^+$ (2)

(2) → (1)

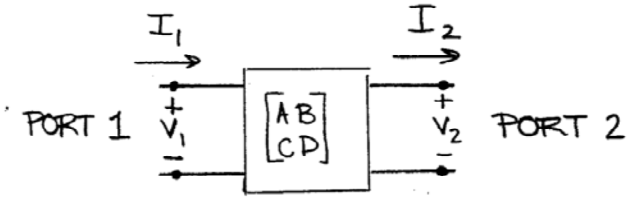
$$V_2^- = V_1^+\left[1 + \frac{Z}{2Z_0+Z}\right]\frac{Z_0}{Z+Z_0} = V_1^+\left[\frac{2Z_0+2Z}{2Z_0+Z}\cdot\frac{Z_0}{Z+Z_0}\right]$$

$$S_{21} = \frac{V_2^-}{V_1^+} = \frac{2Z_0}{2Z_0+Z} = S_{12}$$

Additional information on the ABCD 2-port:

The Transmission Matrix

- Also known as the "ABCD matrix" and "chain scattering parameters"
- Particularly useful when the overall network consists of a cascade of two-port networks

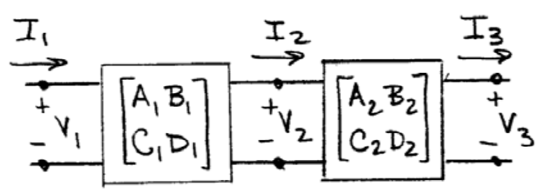


$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$\rightarrow \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Consider a cascade of two-ports



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

The ABCD matrix of the cascade is equal to the product of the individual ABCD matricies.

TABLE 4.1 The *ABCD* Parameters of Some Useful Two-Port Circuits

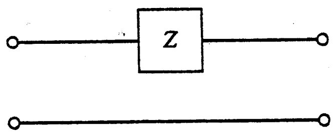
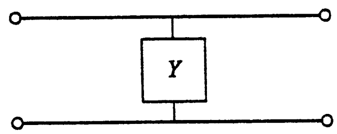
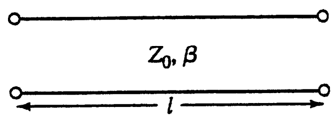
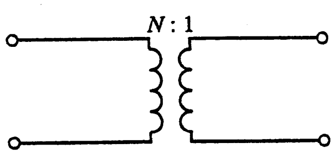
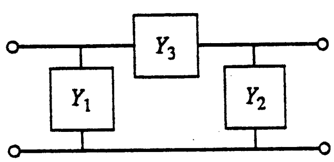
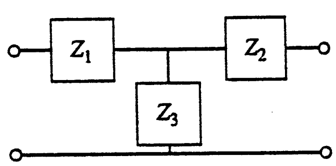
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = jY_0 \sin \beta l$	$B = jZ_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

TABLE 4.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\dfrac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\dfrac{2Z_{12}Z_0}{\Delta Z}$	$\dfrac{-2Y_{12}Y_0}{\Delta Y}$	$\dfrac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\dfrac{2Z_{21}Z_0}{\Delta Z}$	$\dfrac{-2Y_{21}Y_0}{\Delta Y}$	$\dfrac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\dfrac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\dfrac{Y_{22}}{\|Y\|}$	$\dfrac{A}{C}$
Z_{12}	$Z_0\dfrac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\dfrac{-Y_{12}}{\|Y\|}$	$\dfrac{AD-BC}{C}$
Z_{21}	$Z_0\dfrac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\dfrac{-Y_{21}}{\|Y\|}$	$\dfrac{1}{C}$
Z_{22}	$Z_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\dfrac{Y_{11}}{\|Y\|}$	$\dfrac{D}{C}$
Y_{11}	$Y_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{22}}{\|Z\|}$	Y_{11}	$\dfrac{D}{B}$
Y_{12}	$Y_0\dfrac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{12}}{\|Z\|}$	Y_{12}	$\dfrac{BC-AD}{B}$
Y_{21}	$Y_0\dfrac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{21}}{\|Z\|}$	Y_{21}	$\dfrac{-1}{B}$
Y_{22}	$Y_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{11}}{\|Z\|}$	Y_{22}	$\dfrac{A}{B}$
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{11}}{Z_{21}}$	$\dfrac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{\|Z\|}{Z_{21}}$	$\dfrac{-1}{Y_{21}}$	B
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{1}{Z_{21}}$	$\dfrac{-\|Y\|}{Y_{21}}$	C
D	$\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{22}}{Z_{21}}$	$\dfrac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

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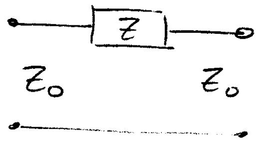
An example using the above conversation table:



$$[ABCD] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

from table 4.2

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{1 + Z/Z_0 - 1}{1 + Z/Z_0 + 1} = \frac{Z/Z_0}{2 + Z/Z_0}$$

$$S_{11} = \frac{Z}{2Z_0 + Z} \checkmark \qquad S_{22} = S_{11}$$

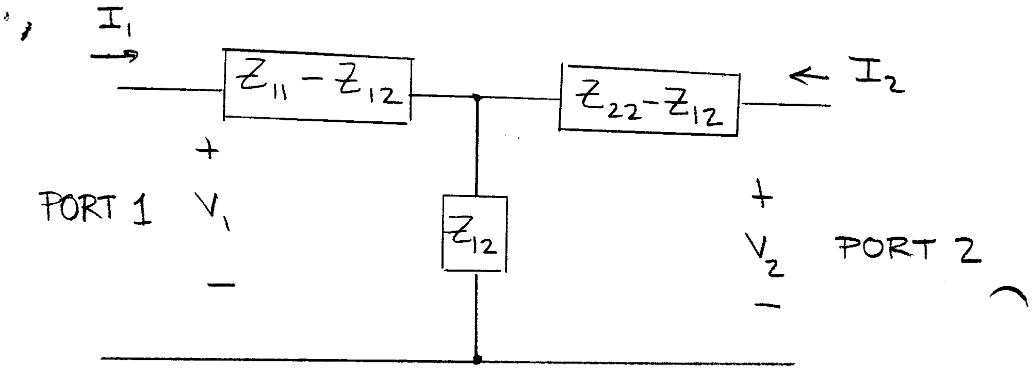
$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{2 + Z/Z_0}$$

$$S_{21} = \frac{2Z_0}{2Z_0 + Z} \checkmark \qquad S_{21} = S_{12}$$

Equivalent Circuits For Two-Port Networks

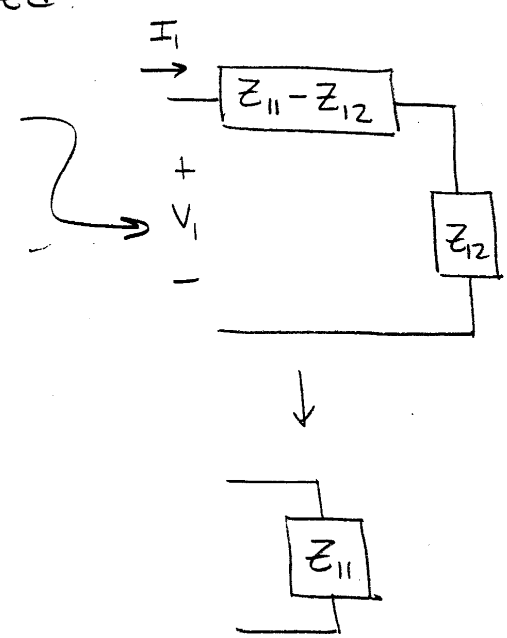
It is often handy to construct equivalent circuits for two-port networks. The most common equivalent circuits are the T- and π-networks.

▷ T Equivalent

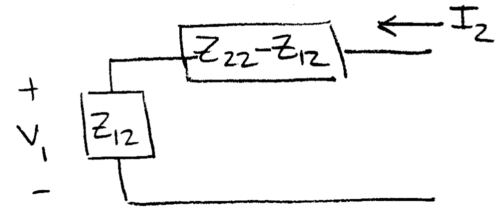


How is this derived?

- $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$



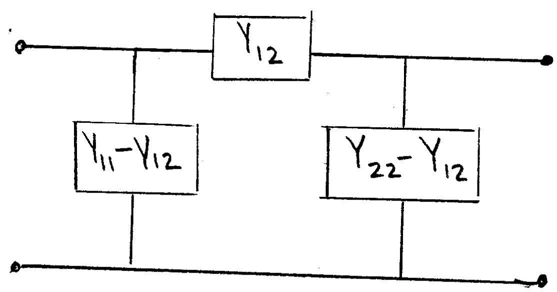
• $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $\rightarrow$



check ✓

And similarly with Z_{21} & Z_{22}.

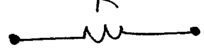
▷ π Equivalent



<end lecture 16 >

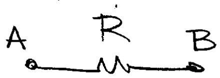
Passive Components at Microwave Frequencies

▷ Resistors

At DC 

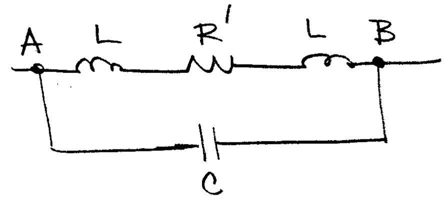
$V = IR$

At RF/Microwave frequencies a simple resistor is no longer such. Rather, a variety of parasitics emerge.



DC

becomes



RF/Microwave

$L \rightarrow$ inductance from lead wire
$C \rightarrow$ parasitic capacitance
$R' \rightarrow$ skin effect $(R' > R)$

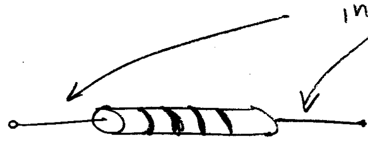
} often frequency dependent

Overall the impedance tends to decrease with increasing frequency due to parasitics

Resistor Types

- Carbon

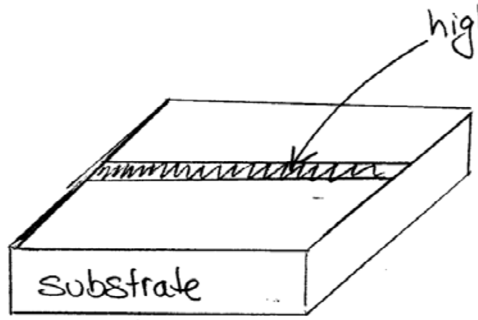
inductance



spiral cut in film
good to perhaps 100 MHz or so

carbon granuales induce parasitic capacitance

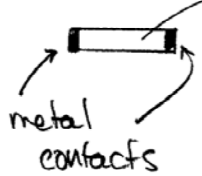
- Planar thin or thick film resistor



ex. NiCr ("Nichrome")
$Ta_x N_y$ (Tantalum nitride)
Doped Semiconductor

↳ used extensively in Monolithic Microwave Integrated Circuits (MMICs)

- Chip Resistor



high resistance film on a ceramic substrate

CHIP SIZE CODE	WIDTH / LENGTH (MILS)	
1218	120/180	
1206	120/60	
0805	80/50	1.52
0603	60/30	0.76
0504	50/40	
0402	40/20	

↳ used on surface mount boards

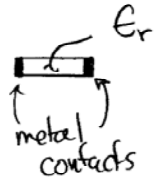
↳ smaller component not only reduce real estate; parasitics are smaller → good to higher freq.

▷ Capacitors

↳ Again, parasitics a problem. Behavior strongly influenced by quality of the dielectric.

Capacitor Types

- Chip capacitor



C up to a couple 100pF → depends on frequency range

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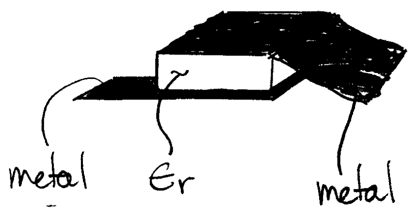
- Metal-Insulator-metal (MIM)



MIM's can provide C up to ~ 25pF

(Metals are not shorted)

example dielectrics:

SiO_2 ($\epsilon_r \sim 3.9$)

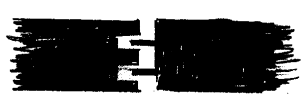
Si_3N_4 ($\epsilon_r \sim 6-7$)

Al_2O_3 ($\epsilon_r \sim 6-9$)

Ta_2O_5 ($\epsilon_r \sim 20-25$)

- Transmission line

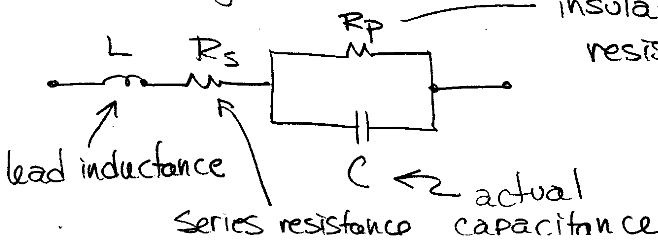
Shunt stub $C < 0.1pF$
(more on this later)



Interdigital $C < 0.5pF$

The values quoted are approximate,

One potential equivalent circuit of a chip capacitor is as follows:



clearly non-ideal

(64)

1

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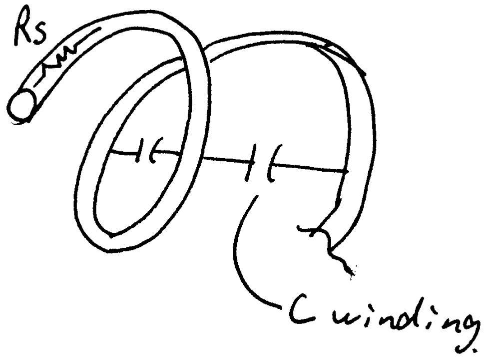
The Lumped inductor.

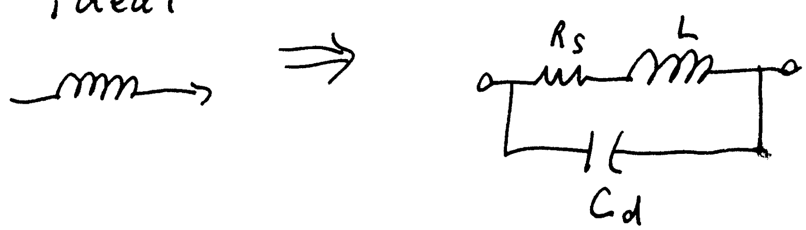


a practical coil inductor ~~h~~ has a series resistance R_s and interwinding capacitance. C_d

The R_s is the dc resistance of the wire - <u>plus</u> additional frequency dependent R due to skin effect and radiation loss.

ideal

Equivalent mw circuit.



2

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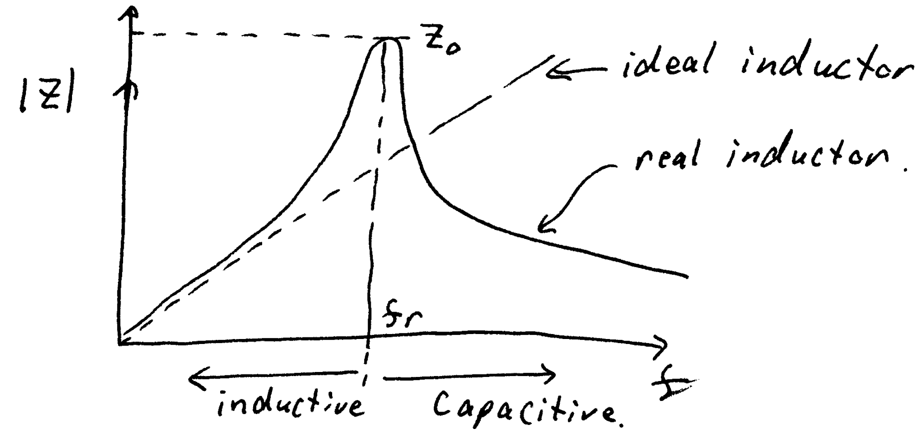
the impedance of the real inductor is:



← inductor properties → (see Lab 2)

SRF ⇒ self resonant Frequency f_r

$$f_r = \frac{1}{2\pi\sqrt{L\,C_d}}$$ L ⇒ value of inductor. Low freq.

R_{dc} → Low frequency R_s of the inductor.

$R_s = \frac{\omega_Q L}{Q_w}$ ← high frequency R_s.

Q_w = Q of inductor at a frequency ω_Q

$\omega_Q = 2\pi f_Q$

(see Lab 2)

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example.

Coilcraft LL2012-F series.

$L = 10\,nH \rightarrow \quad Q = 16 \quad @ \; f_Q = 100\ mHz.$

$\qquad\qquad\qquad\quad Q = 46 \quad @ \; f_Q = 800\ mHz$

$SRF = 2500\ mHz.$

$RDC = 0.3\,\Omega.$

to find the model at 800 mHz.

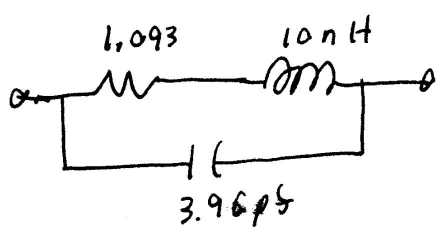
$$SRF = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{(2\pi\, 8\times10^{8})^2\ 10^{-8}}$$

$\underline{C = 3.96\,pf}$

$$Q = \frac{X_L}{R_s} = \frac{\omega L}{R_s} \qquad R_s = \frac{(2\pi\ 8\times10^{8})\ 10^{-8}}{46}$$

$\underline{R_s = 1.093\,\Omega.}$

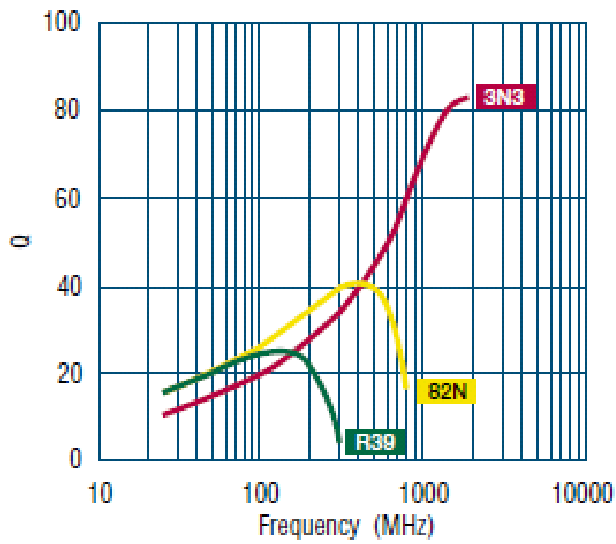
So the model ~~m~~ at 800 mHz is

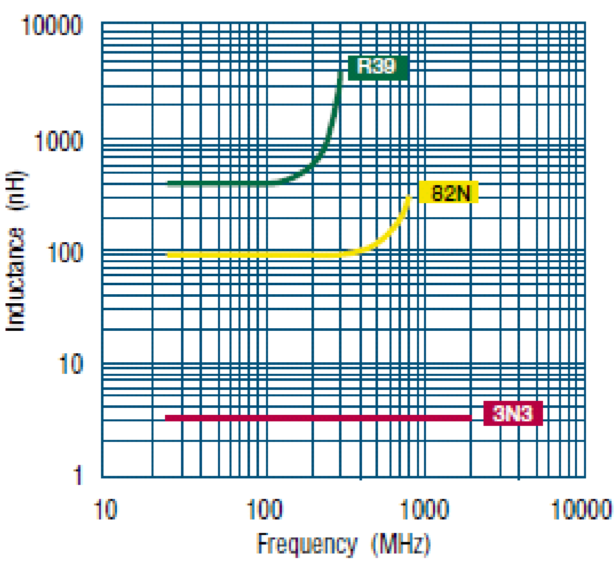


An actual data sheet:

TYPE LL2012-FHL

TOKO Part Number	Inductance & Tolerance					Q min.	Q (Typ.)						SRF (MHz) min.	RDC (Ω) max.	IDC (mA) max.
	at 100MHz		at 800 (500, 300, 200) MHz												
	Lo (nH)	L Tol.*	Lo (nH)	L Tol.*	Freq. (MHz)	100 MHz	100 MHz	300 MHz	500 MHz	800 MHz	1000 MHz	1800 MHz			
LL2012-FHL1N5S	1.5	S	1.5	± 0.5nH	800	11	15.3	27.5	37.5	52.0	61.5	79.3	4000	0.10	300
LL2012-FHL1N8S	1.8	S	1.7	± 0.5nH	800	12	14.0	25.0	33.9	46.6	54.0	78.4	4000	0.10	300
LL2012-FHL2N2S	2.2	S	2.1	± 0.5nH	800	12	16.7	29.5	39.9	55.0	62.6	96.4	3800	0.10	300
LL2012-FHL2N7S	2.7	S	2.42	± 0.5nH	800	12	15.5	27.5	36.8	50.8	57.8	89.0	3600	0.10	300
LL2012-FHL3N3S	3.3	S	3.0	± 0.5nH	800	12	15.4	29.0	39.2	52.6	59.2	96.4	3400	0.10	300
LL2012-FHL3N9S	3.9	S	3.7	± 0.5nH	800	12	16.0	29.7	39.7	53.4	59.7	76.8	3200	0.10	300
LL2012-FHL4N7S	4.7	S	4.6	± 0.5nH	800	12	16.5	30.4	40.9	54.3	61.0	81.0	2800	0.12	300
LL2012-FHL5N6S	5.6	S	5.7	± 0.5nH	800	12	17.0	31.3	42.1	55.2	61.0	76.9	2800	0.15	300
LL2012-FHL6N8J	6.8	J	6.7	± 10%	800	12	18.7	33.3	44.6	58.1	63.9	89.7	2100	0.15	300
LL2012-FHL8N2J	8.2	J	8.2	± 10%	800	15	18.5	32.2	42.4	54.8	59.5	73.2	2000	0.18	300
LL2012-FHL10NJ	10	J	10.2	± 10%	800	15	18.9	33.7	44.4	56.9	61.5	75.7	1600	0.20	300
LL2012-FHL12NJ	12	J	12.7	± 10%	800	16	20.5	36.5	47.5	60.8	65.9	79.8	1350	0.22	300





Comments on inductor Q & effective value:

for an inductor $Q = \frac{X_L}{R_S}$

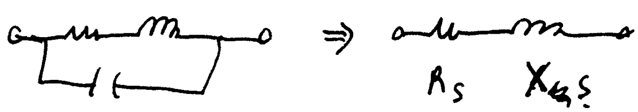
at low frequencies X_L is small & Q is small. for our example inductor:

$Q = 16$ at 100mHz & $Q = 46$ at 800mHz.

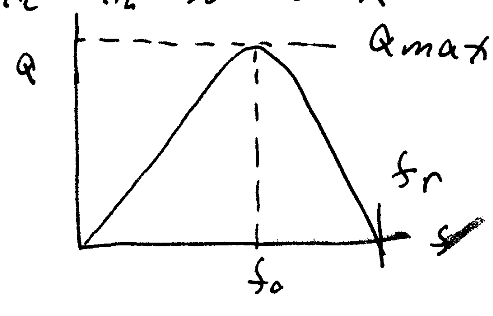
the Q will increase to some maximum value at a frequency f_o (not SRF)

Below Resonance:

equiv model



note that at SRF $\Rightarrow X_S = 0$ because $X_C = X_L$ so $Q = 0$



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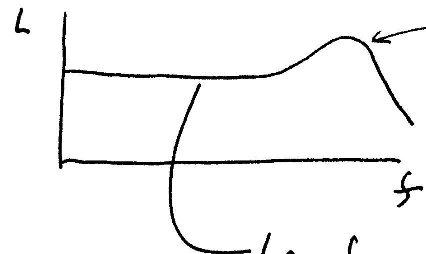
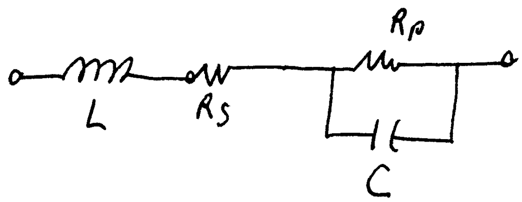
data sheets often display an "effective" L vs frequency



inductor Looks larger as $f \rightarrow$ SRF

low frequency L ie the "marked" inductance.

Real Capacitor model



C = actual capacitor

$R_s \rightarrow$ Lead or termination resistance.

$L \rightarrow$ Lead or package parasitic inductance

$R_p \rightarrow$ insulation resistance & dielectric loss

R_s and R_p are functions of frequency

R_p is usually Large & is ignored.

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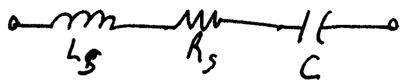
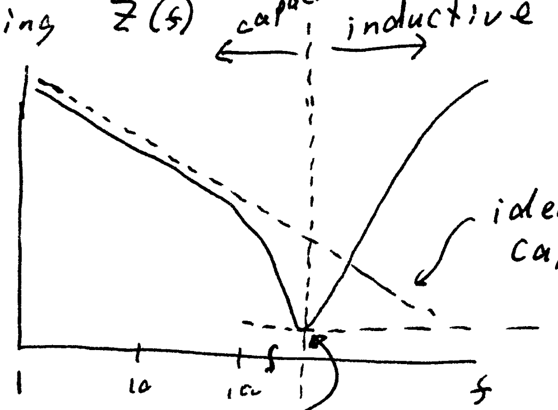
Real capacitor model



plotting $Z(f)$



$f_r \Rightarrow$ SRF

$Z(f_r) = R_s \pm j\,0$

$\lfloor$ series resistance.

note that near resonance the actual $-jX_c$ is greater than an ideal capacitor

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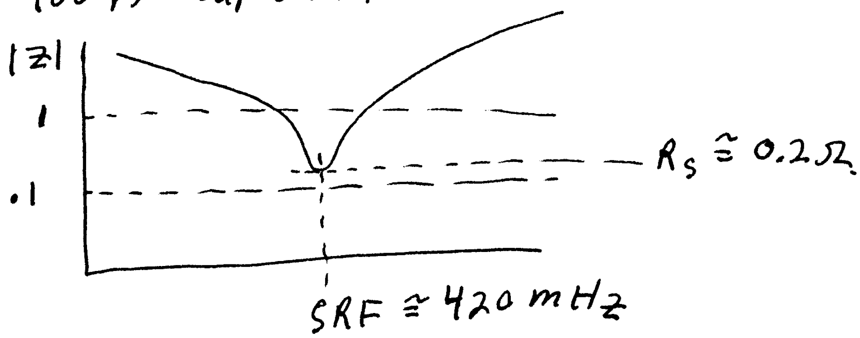
Example: from Lab 2 curves for a

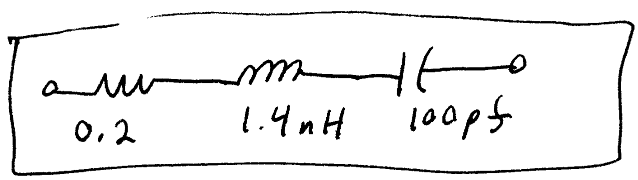
100 pf capacitor:



$$L_S = \frac{1}{(2\pi\, 420 \times 10^6)^2\, 10^{-10}}$$

$$L_S = 1.44 \text{ nH}$$

So the model is



For the same value capacitor.

The smaller the physical part –

the higher the SRF ⇒ the smaller L_S is.

<end lecture 17>