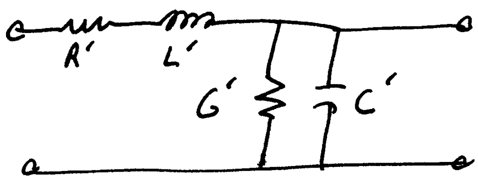
9/23 ee433 1

Transmission Line Circuits vs Lumped element circuits.

Rember $Z_o = \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$



for electrically short wavelengths at high frequencies: $R \ll \omega L$, $G \ll \omega C \Rightarrow \alpha \approx 0$

then $Z_o \cong \sqrt{\dfrac{L'}{C'}} = \dfrac{1}{VC}$, $\beta = \omega\sqrt{L'C'}$, $V = \dfrac{1}{\sqrt{L'C'}}$

consider the admittance of an open circuit line:

$Y = jVC' \tan\beta l$

at Low frequencies $\beta l \ll 1$ so $\tan\beta l \simeq \beta l$

then $Y = jvC'\beta\ell = jvC'\frac{\omega\ell}{v} = j\omega C'\ell = j\underbrace{\omega C}_{\text{Lumped Capacitance}}$

at high frequencies

$$Y_{oc} = \frac{j}{Z_o}\tan\beta\ell = \frac{j}{Z_o}\tan\left(\frac{\pi}{2}\frac{\omega}{\omega_o}\right)$$

where $\omega_o \triangleq$ the frequency where $\ell = \frac{\lambda}{4}$

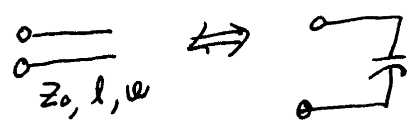
now Define $S = j\Omega \triangleq j\tan\left(\frac{\pi}{2}\frac{\omega}{\omega_o}\right)$

then $Y_{oc} = \frac{j\Omega}{Z_o} = j\Omega C$ so $\frac{1}{Z_o} = \underline{\underline{C}}$

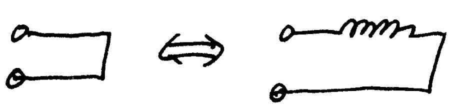
this is Richards transform for an open circuit Line.

Similarly, for a shorted Line:

$$Z_{SC} = jZ_o\tan\left(\frac{\pi}{2}\frac{\omega}{\omega_o}\right) = j\Omega Z_o = j\Omega \underline{\underline{L}}$$

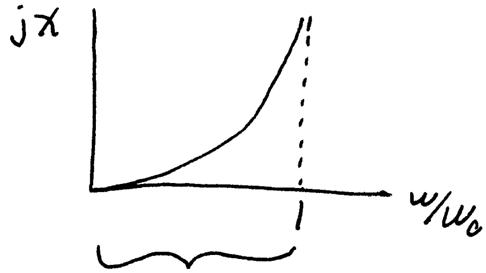


$C = 1/Z_o$
$\Omega = \tan\left(\frac{\pi}{2}\frac{\omega}{\omega_o}\right)$



$L = Z_o$
$\Omega = \tan\left(\frac{\pi}{2}\frac{\omega}{\omega_o}\right)$

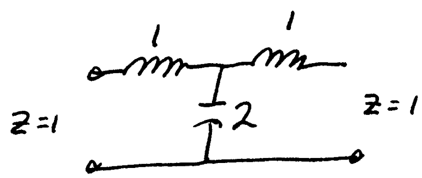
for a shorted Line:

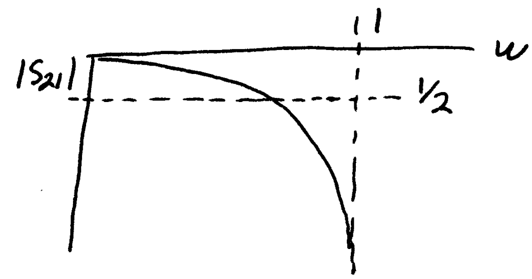


all values $0 < X < \infty$ are realized for $0 \leq w \leq Z_o$

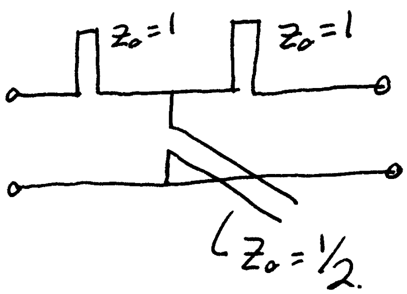
Circuit example:

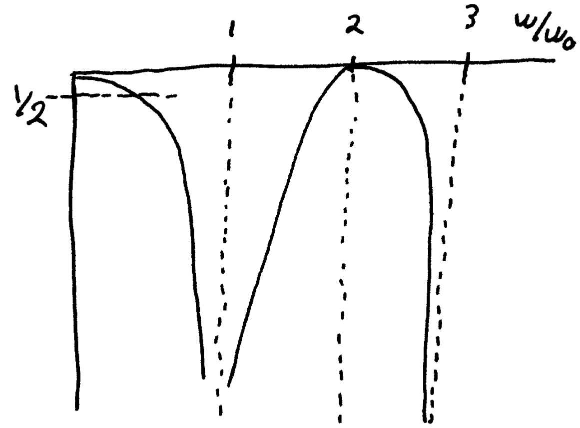
Lumped.





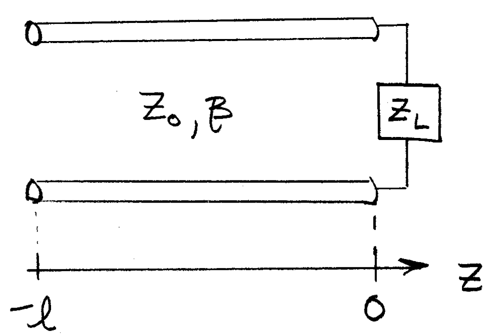
distributed lines





The Smith Chart

A variety of transmission line problems may be solved rather easily with the aid of a graphical tool developed by Phillip H. Smith in the 1930's. His so-called "Smith Chart" may be constructed by considering the reflection coefficient of a terminated transmission line.



Transmission line is characterized by Z_0 and β

Characteristic impedance

phase constant

Recall that the reflection coefficient at the load is given by $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$. Now however we seek the reflection coefficient at an arbitrary point on the transmission line.

The voltage on the TL may be expressed as

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

Thus, $\Gamma(z) = \dfrac{V_o^- e^{j\beta z}}{V_o^+ e^{-j\beta z}} = \dfrac{V_o^-}{V_o^+} e^{j2\beta z}$

$$\Gamma(z=0) \equiv \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{V_o^-}{V_o^+}$$

$$\therefore \quad \Gamma(z) = \Gamma_L e^{j2\beta z}$$

Γ_L may be complex $\rightarrow$ $\Gamma_L = |\Gamma_L| e^{j\theta}$

$$\Rightarrow \quad \Gamma(z) = |\Gamma_L| e^{j(2\beta z + \theta)}$$

$$= |\Gamma_L| \left[\cos(2\beta z + \theta) + j\sin(2\beta z + \theta)\right]$$

$$= |\Gamma_L| \angle \phi$$

From the above we see that as we move along the line, the magnitude of $\Gamma(z)$ is constant and its phase repeats every $z = \frac{\lambda}{2}$.

($2\beta z + \theta = 2\pi + \theta \rightarrow \beta z = \pi$, but $\beta = 2\pi/\lambda$ and thus $z = \lambda/2$)

Recall that the input impedance is given by:

$$Z_{in}(z) = \frac{V(z)}{I(z)} = \frac{V_0^+\left[e^{-j\beta z} + \Gamma_L e^{j\beta z}\right]}{\frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma_L e^{j\beta z}\right]}$$

$$\Rightarrow \quad Z_{in}(z) = Z_0 \frac{1 + \Gamma_L e^{j2\beta z}}{1 - \Gamma_L e^{j2\beta z}}$$

$$\boxed{Z_{in}(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)}}$$

In the construction of the Smith Chart, we work in normalized impedance. Thus let's normalize $Z_{in}(z)$ by Z_0.

$$\Rightarrow \quad \hat{z}_{in}(z) = \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = \hat{r}_{in} + j\hat{x}_{in}$$

normalized wrt Z_0

Recall that $\Gamma(z) = |\Gamma_L| \angle \phi = \Gamma_r + j\Gamma_i$

real part imaginary part

Thus,
$$\frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} = \tilde{r}_{in} + j\tilde{x}_{in}$$

The above equation may be separated to form equations for the real and imaginary components. Solving these equations we find:

$$r_{in} = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x_{in} = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Let's see where the r_{in} equation may lead...

$$r_{in}\left(1-2\Gamma_r + \Gamma_r^2\right) + r_{in}\Gamma_i^2 = 1-\Gamma_r^2 - \Gamma_i^2$$

$$\Gamma_r^2\left(1+r_{in}\right) + \Gamma_i^2\left(1+r_{in}\right) - 2\Gamma_r r_{in} + r_{in} - 1 = 0$$

Dividing by $(1+r_n)$

$$\Gamma_r^2 + \Gamma_i^2 - 2\Gamma_r \frac{r_{in}}{1+r_{in}} + \frac{r_{in}}{1+r_{in}} - \frac{1}{1+r_{in}} = 0$$

$$\Gamma_r^2 - 2\Gamma_r \frac{r_{in}}{1+r_{in}} + \Gamma_i^2 = \frac{1-r_{in}}{1+r_{in}}$$

Completing the square by adding $\left(\frac{r_{in}}{1+r_{in}}\right)^2$ to both sides...

$$\Gamma_r^2 - 2\Gamma_r \frac{r_{in}}{1+r_{in}} + \left(\frac{r_{in}}{1+r_{in}}\right)^2 + \Gamma_i^2 = \frac{1-r_{in}}{1+r_{in}} + \left(\frac{r_{in}}{1+r_{in}}\right)^2$$

$$\left[\Gamma_r - \left(\frac{r_{in}}{1+r_{in}}\right)\right]^2 + \Gamma_i^2 = \frac{(1-r_{in})(1+r_{in}) + r_{in}^2}{(1+r_{in})^2} = \frac{1-r_{in}^2+r_{in}^2}{(1+r_{in})^2}$$

$$\left[\Gamma_r - \left(\frac{r_{in}}{1+r_{in}}\right)\right]^2 + \Gamma_i^2 = \left(\frac{1}{1+r_{in}}\right)^2 \qquad (1)$$

And similarly,

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_{in}}\right)^2 = \left(\frac{1}{x_{in}}\right)^2 \qquad (2)$$

<end lecture 11>

Equations ① and ② are in the form of circles.

$$(x-a)^2+(y-b)^2=r^2$$

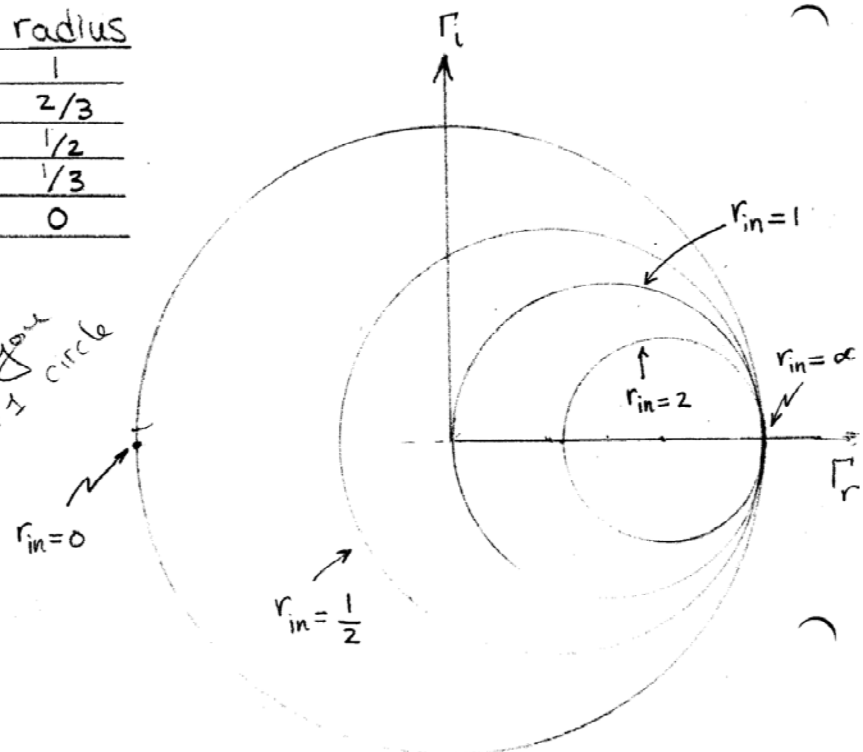
$\Rightarrow$ center at (a, b), radius $= r$

▸ Let's plot the constant r_{in} circles.

$$\text{center} = \left(\frac{r_{in}}{1+r_{in}}, 0\right) \quad \text{radius} = \frac{1}{1+r_{in}}$$

r_{in}	center	radius
0	(0,0)	1
1/2	(1/3,0)	2/3
1	(1/2,0)	1/2
2	(2/3,0)	1/3
∞	(1,0)	0

* notice that if you are inside $r_{in}=1$ circle $R_{in} > Z_o$

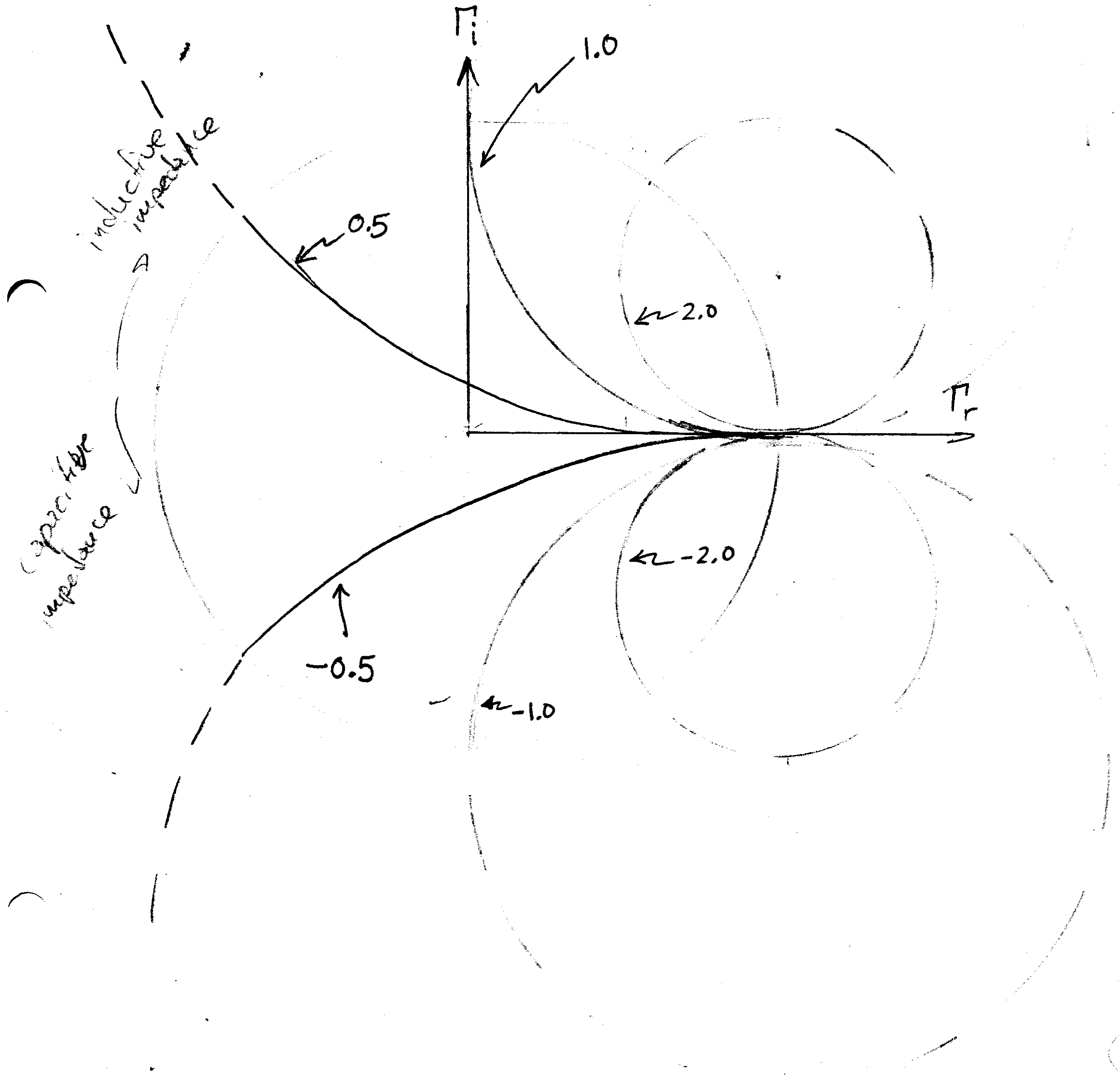


► Lets plot the constant x_{in} circles

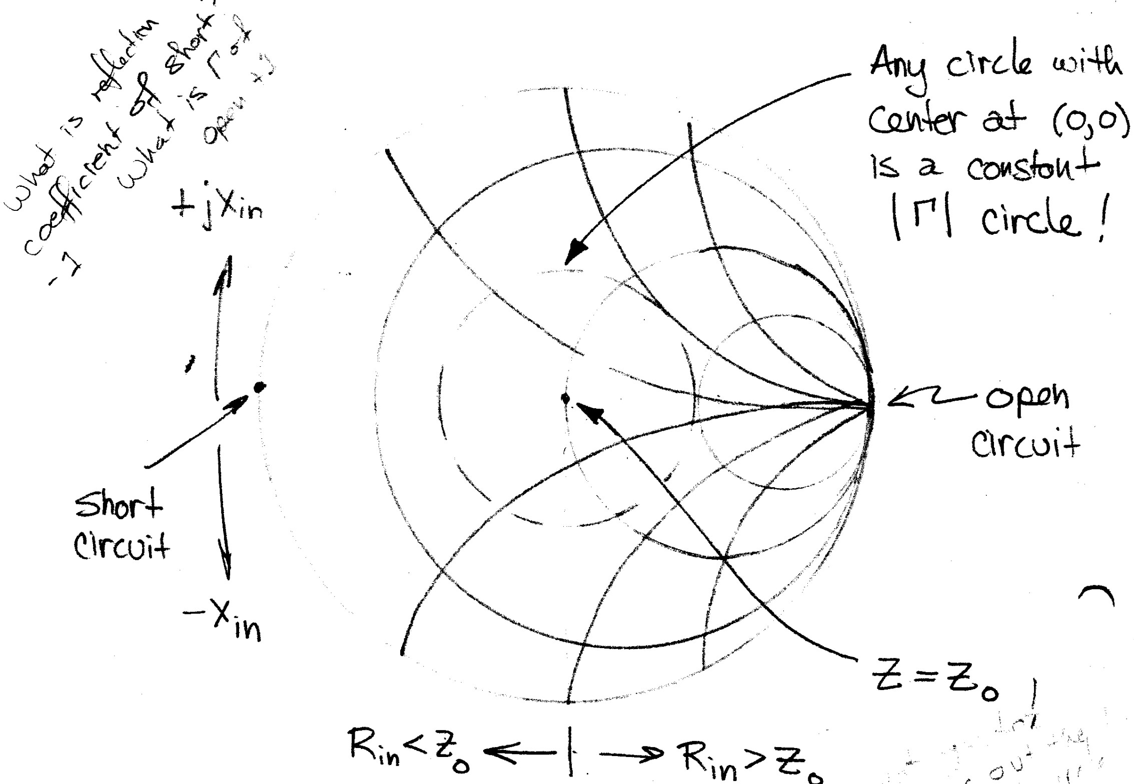
x_{in}	center	radius
1	(1,1)	1
-1	(1,-1)	1
2	(1,½)	½
½	(1,2)	2

center $= \left(1, \frac{1}{x_{in}}\right)$

radius $= \frac{1}{x_{in}}$



The Smith Chart is a superposition of the constant r_{in} and x_{in} circles for $|\Gamma| \le 1$.



We have plotted the impedance, how might we read admittance from the Smith Chart?

Recall the TL equation (normalized to Z_0 in this case)

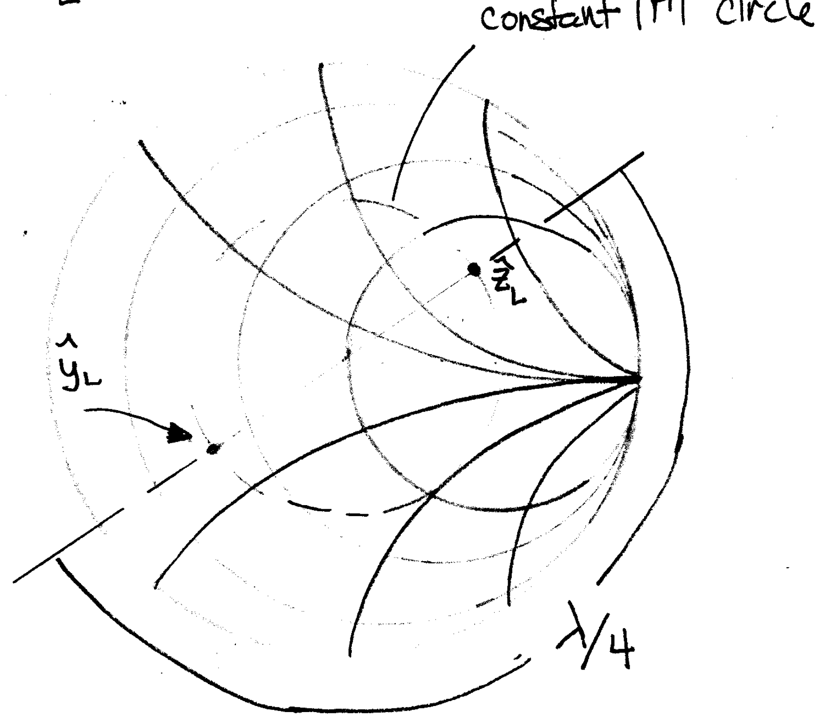
$$\tilde{Z}_{in}(\ell) = \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} \qquad \text{Let } \ell = \frac{\lambda}{4}$$

$$\rightarrow \beta\ell = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad \tan\left(\frac{\pi}{2}\right) = \infty$$

(44)

$$\hat{z}_{in}\left(\ell=\frac{\lambda}{4}\right)=\frac{z_L+jz_0(\infty)}{z_0+jz_L(\infty)}=\frac{z_0}{z_L}\rightarrow\hat{y}_L$$

Thus we can determine $\hat{y}_L$ by rotating $\frac{\lambda}{4}$ from z_L.



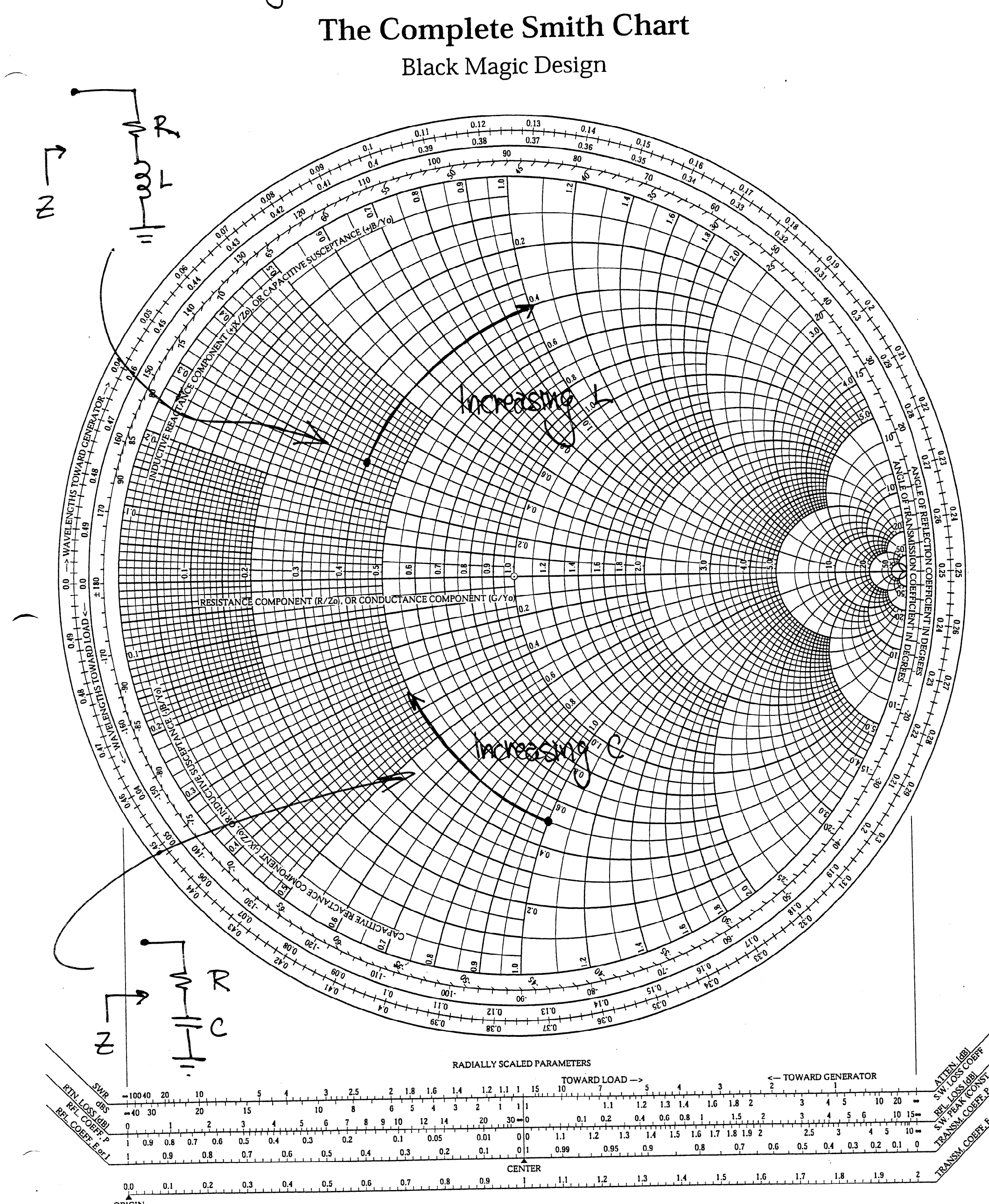
Recall that Γ and z_{in} repeat every $\lambda/2$ which is one complete revolution on the Smith Chart.

See the following plots for these questions

(1) Explain the trends observed in the "Effect of Adding Series L/C On Impedance At A Given Frequency" Smith Chart.

(2) Explain the trends observed in the "Impedance of Series RL and RC With Frequency" Smith Chart. Where will each curve end at infinite frequency?

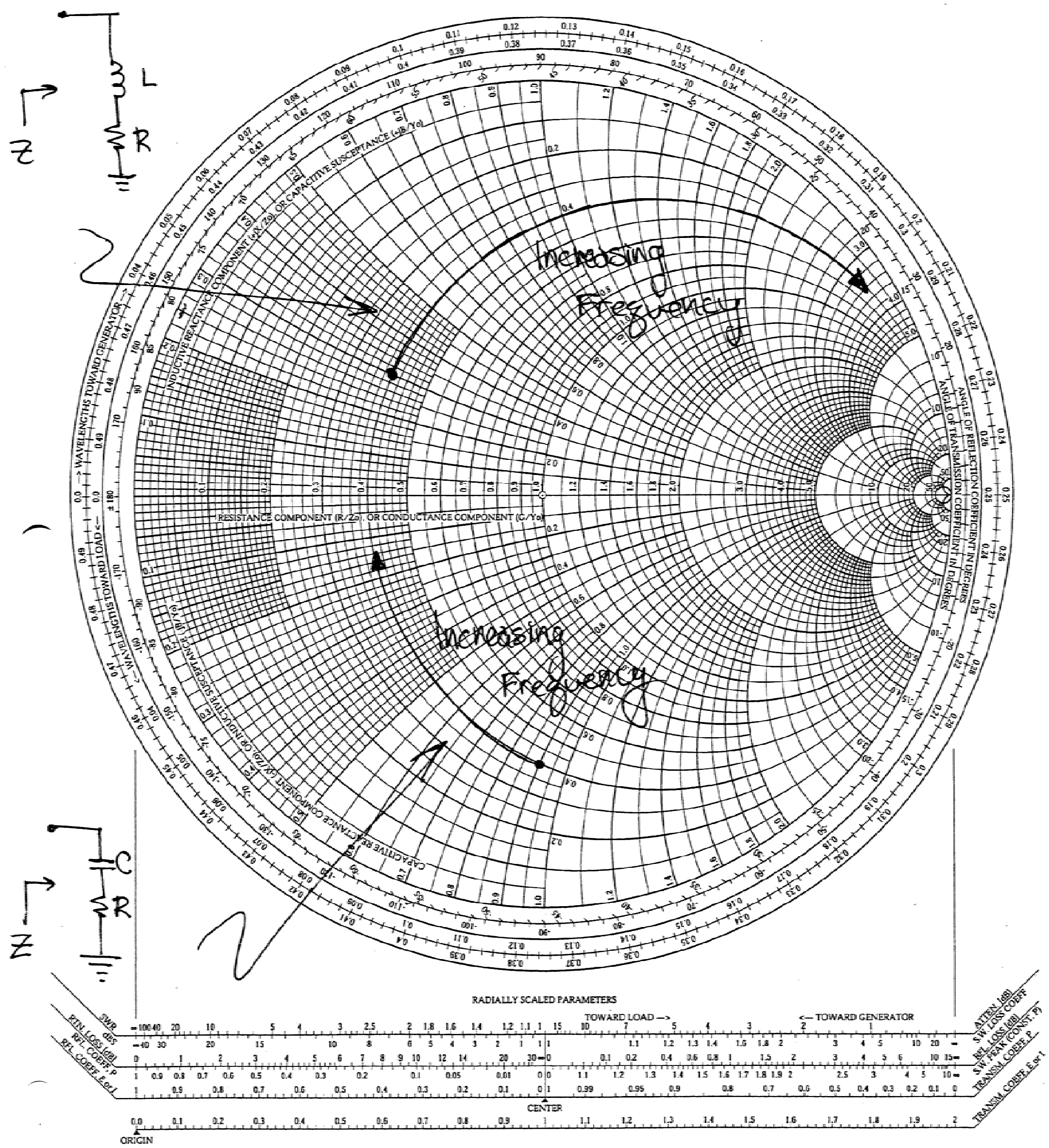
Effect of Adding Series L/C on Impedance At A Given Frequency



Impedance of Series RL & RC With Frequency

The Complete Smith Chart

Black Magic Design

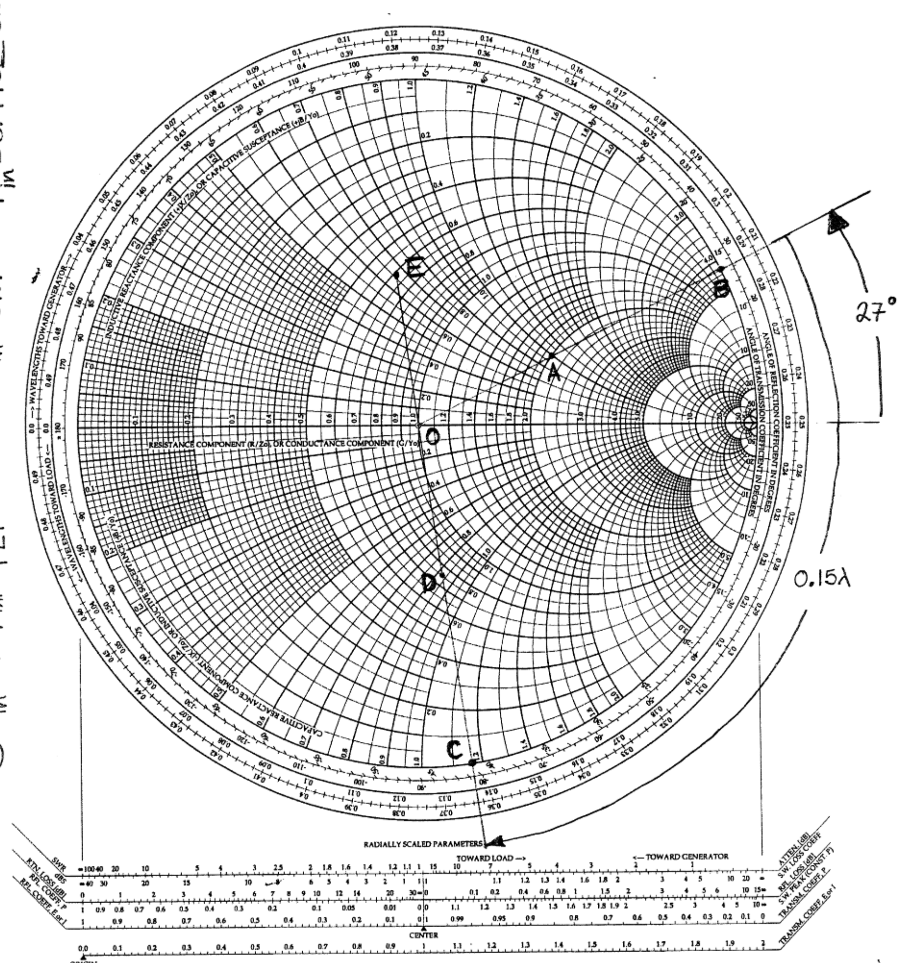


Example: A load of $Z_L = 100 + j50\,\Omega$ terminates a $50\,\Omega$ line. What is the input impedance, input admittance, and reflection coefficient $0.15\,\lambda$ from the load? What is the reflection coefficient at the load?

⑤ Rotate $\lambda/4$ on constant SWR circle (radius = 28.5mm) to locate $\hat{y}_{in} \doteq$ pt. E $\quad \hat{y}_{in} = 0.62 + j0.67$

$y_{in} = \frac{1}{Z_0}\hat{y}_{in} = \frac{1}{50}(0.62 + j0.67) \quad y_{in} = 0.012 + j0.013\ S$

⑥ $\Gamma_{in} = ?\ \ |\Gamma_{in}| = |\Gamma_L| = 0.445 \quad \angle\Gamma_{in} \sim -81.7^\circ \qquad \Gamma_{in} = 0.445\angle -81.7^\circ$



Solution

① $\hat{z}_L = \frac{Z_L}{Z_0} = 2 + j\ \Omega \rightarrow$ pt. A

② $\Gamma_L = ?\ |\Gamma_L| = \frac{OA}{OB} = \frac{28.5\text{mm}}{64\text{ mm}} = 0.445$

$\angle\Gamma_L \sim 27^\circ$

③ Rotate $0.15\,\lambda$ toward generator

$\rightarrow$ rotate CW $\rightarrow 0.213\lambda + 0.15\lambda = 0.363\,\lambda$

④ Either draw constant SWR circle through A and take intersection of circle with OC or measure 28.5mm from O along OC

$\hookrightarrow$ pt D $\rightarrow \hat{z}_{in} = 0.75 - j0.82$

$Z_{in} = 50\,(0.75 - j0.82)$

$Z_{in} = 37.5 - j41\,\Omega$

Solution of the previous example via analytical expression

▷ $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{100 + j50 - 50}{100 + j50 + 50} = \dfrac{50 + j50}{150 + j50}$

$$\boxed{\Gamma_L = 0.447 \angle 26.6^\circ}$$

▷ $\Gamma_{in} \triangleq \Gamma(l = 0.15\lambda) = \Gamma_L e^{-j2\beta l} = 0.447\, e^{j26.6}\, e^{-j2\frac{2\pi}{\lambda}(0.15\lambda)}$

$$\Gamma_{in} = 0.447\, e^{j26.6}\, e^{-j108}$$

$$\boxed{\Gamma_{in} = 0.447\, e^{-j81.4^\circ}}$$

▷ $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$ or $Z_{in} = Z_0 \dfrac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$

$$Z_{in} = 50\, \frac{1 + 0.447e^{-j81.4}}{1 - 0.447e^{-j81.4}} = \boxed{\begin{array}{l} 37.53 - j41.46\ \Omega \\ = Z_{in} \end{array}}$$

$$\boxed{Y_{in} = \frac{1}{Z_{in}} = 0.012 + j0.013\ S}$$