Lecture 28-29
FM- Frequency Modulation
PM - Phase Modulation

EE445-10

FM and PM

Representation of PM and FM Signals

Phase modulation (PM) and *frequency modulation* (FM) are special cases of angle-modulated signaling. In this kind of signaling the complex envelope is

$$g(t) = A_c e^{j\theta(t)} \quad (5\text{–}33)$$

Here the real envelope, $R(t) = |g(t)| = A_c$, is a constant, and the phase $\theta(t)$ is a linear function of the modulating signal $m(t)$. However, $g(t)$ is a *nonlinear* function of the modulation. Using Eq. (5–33), we find the resulting *angle-modulated signal* to be

$$s(t) = A_c \cos[\omega_c t + \theta(t)] \quad (5\text{–}34)$$

$$\theta(t) = D_p m(t) \quad (5\text{–}35)$$

D_p is the phase sensitivity or phase modulation constant

FM and PM

for FM:

$$\theta(t) = D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma \quad (5\text{–}36)$$

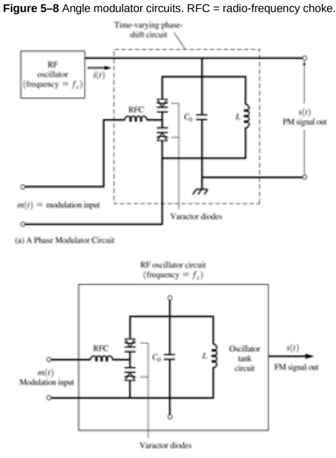
Relationship between $m_f(t)$ and $m_p(t)$:

$$m_f(t) = \frac{D_p}{D_f}\left[\frac{dm_p(t)}{dt}\right] \quad (5\text{–}37)$$

where the subscripts f and p denote frequency and phase, respectively. Similarly, if we have an FM signal modulated by $m_f(t)$, the corresponding phase modulation on this signal is

$$m_p(t) = \frac{D_f}{D_p}\int_{-\infty}^{t} m_f(\sigma)\, d\sigma \quad (5\text{–}38)$$

Figure 5–8 Angle modulator circuits. RFC = radio-frequency choke.



FM and PM

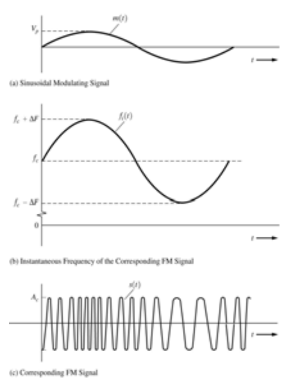
DEFINITION. If a bandpass signal is represented by

$$s(t) = R(t)\cos\psi(t)$$

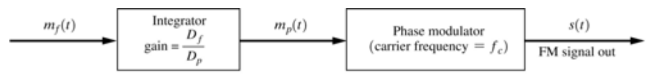
where $\psi(t) = \omega_c t + \theta(t)$, then the *instantaneous* frequency (hertz) of $s(t)$ is [Boashash, 1992]

$$f_i(t) = \frac{1}{2\pi}\,\omega_i(t) = \frac{1}{2\pi}\left[\frac{d\psi(t)}{dt}\right]$$

Figure 5–9 FM with a sinusoidal baseband modulating signal.



FM and PM



(a) Generation of FM Using a Phase Modulator

(b) Generation of PM Using a Frequency Modulator

FM and PM differences

PM:

$$\theta(t) = D_p m(t) \Rightarrow$$ phase is proportional to m(t)

FM:

$$\theta(t) = D_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$$

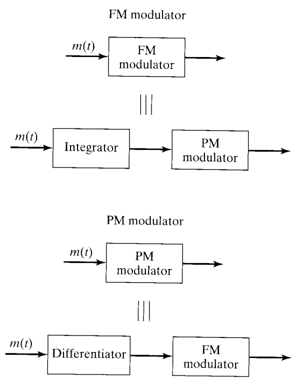
$$f_i(t) - f_c = D_f m(t) \Rightarrow$$ instantaneous frequency <u>deviation</u> from the carrier is proportional to m(t)

Modulation Constants

$$D_p = K_p \Rightarrow \frac{radians}{volt}$$

$$D_f = K_f \Rightarrow \frac{Hz}{volt}$$

FM from PM
PM from FM



$$\phi(t) = \begin{cases} k_p m(t), & \text{PM} \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau, & \text{FM} \end{cases}.$$

$$\frac{d}{dt}\phi(t) = \begin{cases} k_p \frac{d}{dt} m(t), & \text{PM,} \\ 2\pi k_f m(t), & \text{FM} \end{cases}$$

Figure 4.1 A comparison of frequency and phase modulators.

FM from PM
PM from FM

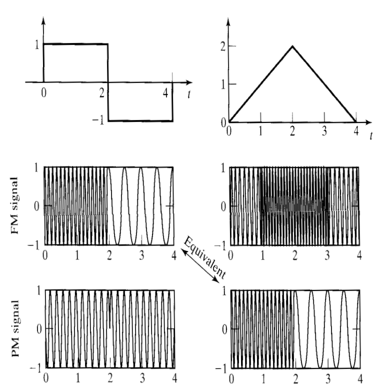


Figure 4.2 Frequency and phase modulation of square and sawtooth waves.

FM and PM Signals

Maximum phase deviation in PM:

$$\Delta\phi_{\max} = k_p \max[|m(t)|],$$

Maximum frequency deviation in FM:

$$\Delta f_{\max} = k_f \max[|m(t)|].$$

Example

Let $m(t) = a\cos(2\pi f_m t)$

For PM $\phi(t) = k_p m(t) = k_p a \cos(2\pi f_m t),$

For FM $\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau = \frac{k_f a}{f_m}\sin(2\pi f_m t).$

$$u(t) = \begin{cases} A_c \cos\left(2\pi f_c t + k_p a \cos(2\pi f_m t)\right), & \text{PM} \\ A_c \cos\left(2\pi f_c t + \frac{k_f a}{f_m}\sin(2\pi f_m t)\right), & \text{FM} \end{cases}.$$

Define the modulation indices: $\beta_p = k_p a \qquad \beta_f = \frac{k_f a}{f_m},$

Example

Define the modulation indices:

$$\beta_p = k_p a \qquad \beta_f = \frac{k_f a}{f_m},$$

$$\beta_p = k_p \max[|m(t)|] \qquad \beta_f = \frac{k_f \max[|m(t)|]}{W}$$

$$\beta_p = \Delta\phi_{\max}; \qquad \beta_f = \frac{\Delta f_{\max}}{W}.$$

Sine Wave Example

Then

$$u(t) = \begin{cases} A_c \cos\left(2\pi f_c t + \beta_p \cos(2\pi f_m t)\right), & \text{PM} \\ A_c \cos\left(2\pi f_c t + \beta_f \sin(2\pi f_m t)\right), & \text{FM} \end{cases}.$$

Spectrum Characteristics of FM

- FM/PM is exponential modulation

Let $\phi(t) = \beta \sin(2\pi f_m t)$

$$u(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$
$$= \mathrm{Re}\left(A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}\right)$$

u(t) is periodic in f_m

we may therefore use the Fourier series

Spectrum Characteristics of FM

- FM/PM is exponential modulation

$$c(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$
$$= \mathrm{Re}\left(A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}\right)$$

c(t) is periodic in f_m

we may therefore use the Fourier series

Spectrum Characteristics with Sinusoidal Modulation

$$e^{j\beta \sin 2\pi f_m t}.$$

u(t) is periodic in f_m
we may therefore use the Fourier series

$$c_n = f_m \int_0^{\frac{1}{f_m}} e^{j\beta \sin 2\pi f_m t} e^{-jn2\pi f_m t}\, dt$$

$$\overset{u=2\pi f_m t}{=} \frac{1}{2\pi}\int_0^{2\pi} e^{j\beta(\sin u - nu)}\, du.$$

J_n Bessel Function

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}.$$

$$u(t) = \mathrm{Re}\left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}\right)$$

$$= \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos\left(2\pi(f_c + n f_m)t\right).$$

J_n Bessel Function

$$J_n(\beta) \approx \frac{\beta^n}{2^n n!}. \qquad J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}.$$

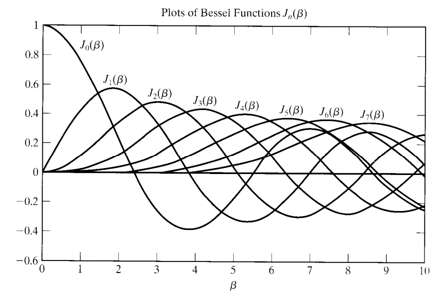
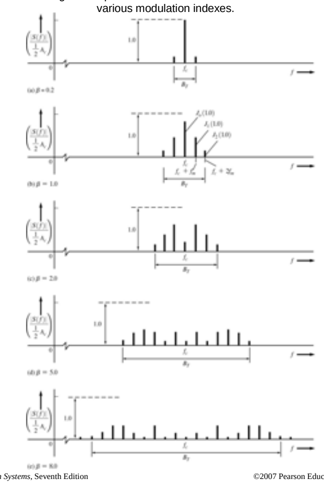


Figure 4.4 Bessel functions for various values of n.

Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.



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Lecture 29
FM- Frequency Modulation
PM - Phase Modulation
(continued)

EE445-10

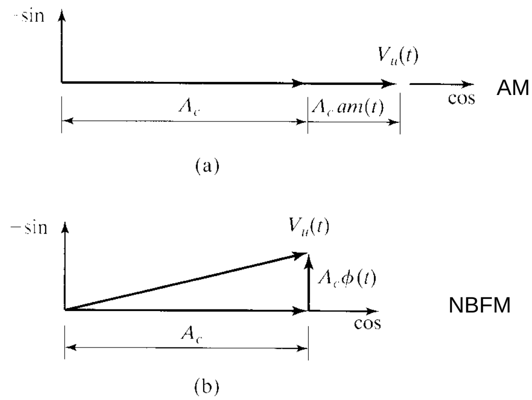
Narrowband FM

- Only the J_o and J_1 terms are significant
- Same Bandwidth as AM
- Using Eulers identity, and $\phi(t)<<1$:

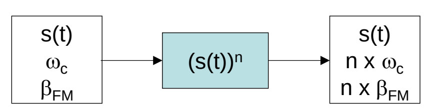
$$u(t) = A_c \cos 2\pi f_c t \cos\phi(t) - A_c \sin 2\pi f_c t \sin\phi(t)$$
$$\approx A_c \cos 2\pi f_c t - A_c\phi(t)\sin 2\pi f_c t,$$

Notice the sidebands are “sin”, not “cos” as in AM

Narrowband FM as a Phaser



Wideband FM from Narrowband FM



- The Output Carrier frequency = n x f_c
- The output modulation index = n x β_c
- The output bandwidth increases according to Carson’s Rule

Effective Bandwidth- Carson's Rule for Sine Wave Modulation

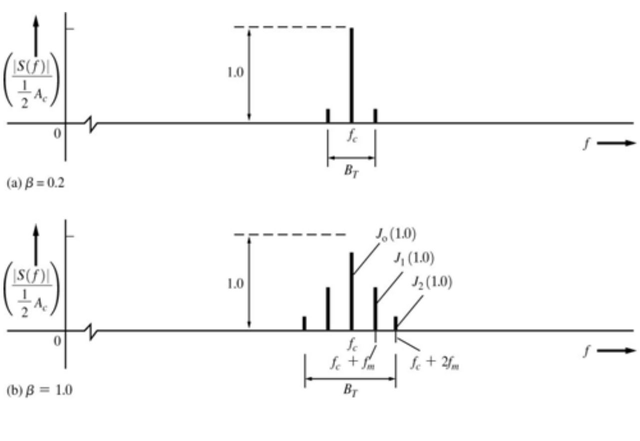
$$B_c = 2(\beta + 1) f_m,$$ Where β is the modulation index
f_m is the sinusoidal modulation frequency

$$m(t) = a\cos(2\pi f_m t).$$

$$B_c = 2(\beta+1) f_m = \begin{cases} 2(k_p a + 1) f_m, & \text{PM} \\ 2\left(\frac{k_f a}{f_m} + 1\right) f_m, & \text{FM} \end{cases}$$

•Notice for FM, if $k_f a >> f_m$, increasing fm does not increase B_c much
•B_c is linear with f_m for PM

Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.



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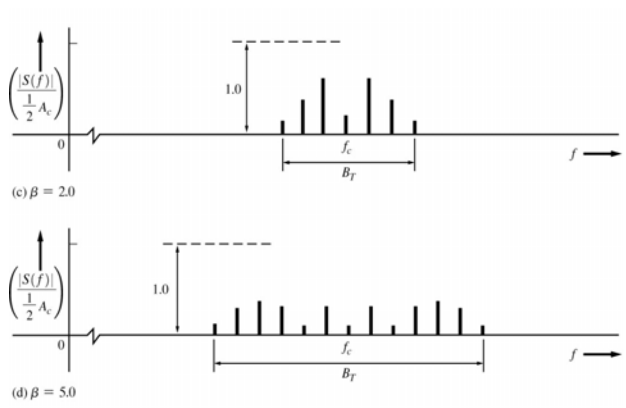

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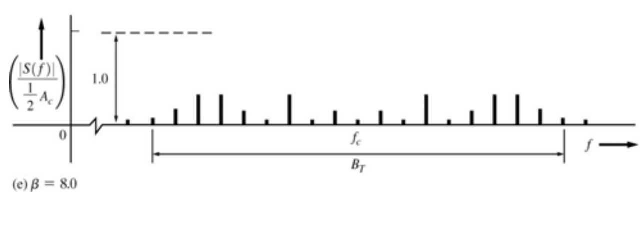


When m(t) is a sum of sine waves

Consider now the case where $m(t)$ is the sum of K separate sine waves. That is, let

$$m(t) = \sum_{i=1}^{K} C_i \cos(\omega_i t + \theta_i) \qquad (2.4.14)$$

where C_i, ω_i, and θ_i are the corresponding deviations, frequency, and phase angles. We then have

When m(t) is a sum of sine waves

$$c(t) = A\cos\left[\omega_c t + \sum_{i=1}^{K} \beta_i \sin(\omega_i t + \theta_i) + \psi\right]$$

$$c(t) = A \sum_{k_K=-\infty}^{\infty} \cdots \sum_{k_1=-\infty}^{\infty} \left[\prod_{i=1}^{K} J_{k_i}(\beta)\right] \cos\left[\omega_c t + \sum_{i=1}^{K} k_i(\omega_i t + \theta_i) + \psi\right] \qquad (2.4.16)$$

The preceding equation represents the general expression for the FM carrier modulated by K sinusoids. Note that it corresponds to a collection of harmonic frequencies at all the sidebands $\Sigma_{i=1}^{K} k_i\omega_i$, where all combinations of integers for the $\{k_i\}$ must be considered. Each such combination $\{k_1, k_2, \ldots, k_K\}$ yields a different sinusoid, each with its own phase, $\Sigma_{i=1}^{K}(k_i\theta_i + \psi)$ and its own amplitude $\{A\,\Pi_i^K J_{k_i}(\beta_i)\}$. In particular, we note that the carrier component at ω_c corresponds to $k_1 = k_2 = \cdots = k_K = 0$ and has amplitude $\{A\,\Pi_i^K J_0(\beta_i)\}$, whereas the frequency component at frequency $(\omega_c + \omega_j)$ corresponds to $k_1 = 0$, $k_2 = 0, \ldots, k_j = 1$, $k_{j+1} = 0, \ldots, k_K = 0$ and has amplitude $\{AJ_1(\beta_j)\,\Pi_{i=1,i\neq j}^{K} J_0(\beta_i)\}$. We also note that the component at $(\omega + \omega_j)$ contains the exact phase angle of the jth sine wave in (2.4.14) added to that of the carrier.

Sideband Power

Signal Amplitude: $A_c := 1V$

Modulating frequency: $f_m := 1KHz$

Carrier peak deveation: $\Delta f := 2.4KHz$

Modulation index: $\beta := \frac{\Delta f}{f_m}$ $\quad \beta = 2.4$

Reference equation: $x(t) = \sum_{n=-\infty}^{\infty} \left[A_c \cdot Jn(n,\beta)\cdot\cos\left[(\omega_c + n\cdot\omega_m)\cdot t\right]\right]$

Power in the signal: $P_c := \frac{A_c^{\,2}}{2\cdot 1\Omega}$ $\quad P_c = 0.5\text{ W}$

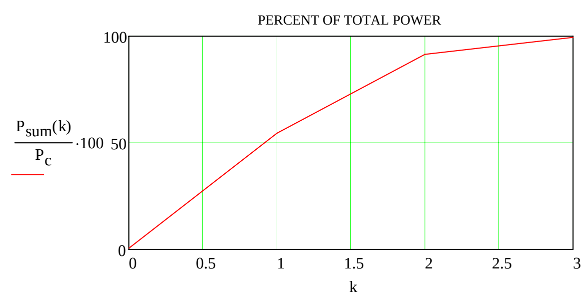
Carsons rule bandwidth: $BW := 2\cdot(\beta + 1)\cdot f_m$ $\quad BW = 6.8\times 10^3\ \frac{1}{s}$

Order of significant sidbands predicted by Carsons rule: $n := round(\beta + 1)$ $\quad n = 3$

Power as a function of number of sidebands: $P_{sum}(k) := \sum_{n=-k}^{k} \frac{(A_c\cdot Jn(|n|,\beta))^2}{2\cdot 1\Omega}$

Percent of power predicted by Carsons rule: $\frac{P_{sum}(n)}{P_c}\cdot 100 = 99.118$

Sideband Power



Sideband Power

$k := 0 .. 10$ $J_k := Jn(k, \beta)$ $P_k := (J_k)^2$ $\beta = 2.4$ $n = 3$

J =

	0
0	$2.508 \cdot 10^{-3}$
1	0.52
2	0.431
3	0.198
4	0.064
5	0.016
6	$3.367 \cdot 10^{-3}$
7	$5.927 \cdot 10^{-4}$
8	$9.076 \cdot 10^{-5}$
9	$1.23 \cdot 10^{-5}$
10	$1.496 \cdot 10^{-6}$

P =

	0
0	$6.288 \cdot 10^{-6}$
1	0.271
2	0.186
3	0.039
4	$4.135 \cdot 10^{-3}$
5	$2.638 \cdot 10^{-4}$
6	$1.134 \cdot 10^{-5}$
7	$3.513 \cdot 10^{-7}$
8	$8.237 \cdot 10^{-9}$
9	$1.513 \cdot 10^{-10}$
10	$2.238 \cdot 10^{-12}$

$$P_0 + 2 \cdot \sum_{j=1}^{n} P_j = 0.991$$

Sideband Power

$j := 0 .. 5$ $\beta := 0.1$ $n := 1$

$V_j := Jn(j, \beta)$ $U_j := (V_j)^2$

$$V = \begin{pmatrix} 0.998 \\ 0.05 \\ 1.249 \times 10^{-3} \\ 2.082 \times 10^{-5} \\ 2.603 \times 10^{-7} \\ 2.603 \times 10^{-9} \end{pmatrix} \quad U = \begin{pmatrix} 0.995 \\ 2.494 \times 10^{-3} \\ 1.56 \times 10^{-6} \\ 4.335 \times 10^{-10} \\ 6.775 \times 10^{-14} \\ 0 \end{pmatrix}$$

$$U_0 + 2 \cdot \sum_{j=1}^{n} U_j = 1$$

Sideband Power

$\beta := 0.6$ $n := 1$

$W_j := Jn(j, \beta)$ $X_j := (W_j)^2$

$$W = \begin{pmatrix} 0.912 \\ 0.287 \\ 0.044 \\ 4.4 \times 10^{-3} \\ 3.315 \times 10^{-4} \\ 1.995 \times 10^{-5} \end{pmatrix} \quad X = \begin{pmatrix} 0.832 \\ 0.082 \\ 1.907 \times 10^{-3} \\ 1.936 \times 10^{-5} \\ 1.099 \times 10^{-7} \\ 3.979 \times 10^{-10} \end{pmatrix}$$

$$X_0 + 2 \cdot \sum_{j=1}^{n} X_j = 0.996$$

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FM/PM modulation index set to $\pi/2$ for peak phase dev of $\pi/2$
set to $\Delta f/fm$ for frequency modulation. spectru
is the same for sinewave
modulation.

$A_c := 1$

$f_c := 0 \cdot 10^4$

79

$Fm := 10^0$ Modulating frequency- single sinewave

$M := \frac{x}{10}$

$n := round(M + 1)$ 2 * n is the number of significant sidebands per Carsons rule

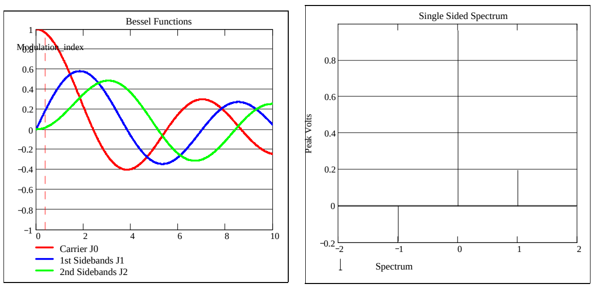
$n = 9$ $Bandwidth = 2 \cdot n \cdot Fm$ $Modulation\_index = M$

$$Si(f) := A_c \cdot \left[(J0(M)) \cdot \delta(f, f_c) + \sum_{k=1}^{n} \left[Jn(k, M) \cdot \delta[f, (f_c + k \cdot Fm)] + (-1)^k \cdot Jn(k, M) \cdot \delta[f, (f_c - k \cdot Fm)] \right] \right]$$

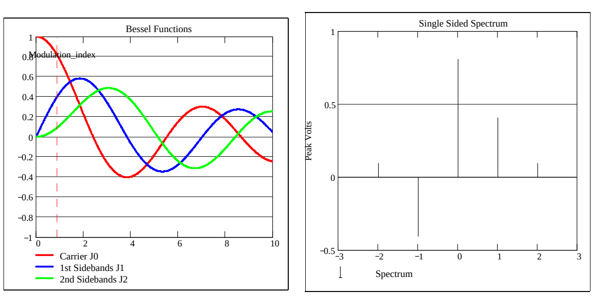
$$B(f) := \delta[f, f_c + (n+0) \cdot Fm] + \delta(f, f_c - n \cdot Fm) \qquad f := f_c - (n+1) \cdot Fm, (f_c - n \cdot Fm) .. [f_c + (n+1) \cdot Fm]$$

$Bandwidth = 18$ $Modulation\_index = 7.9$

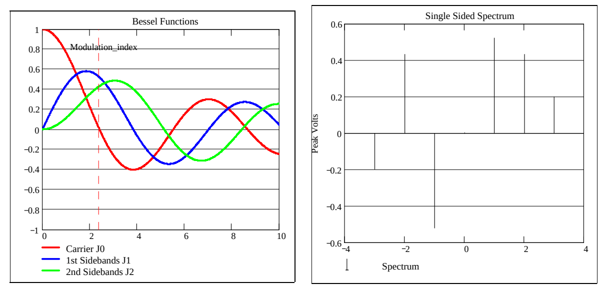
M=.4, Sideband Level =M/2 for Narrowband FM



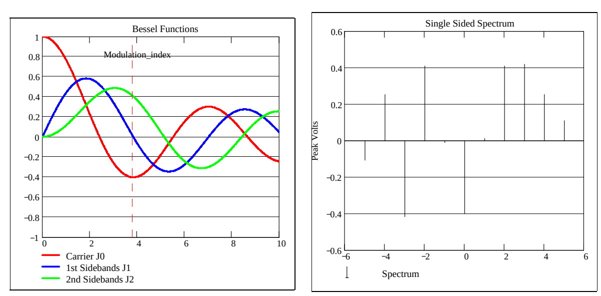
M=.9, Sideband Level =M/2 for Narrowband FM



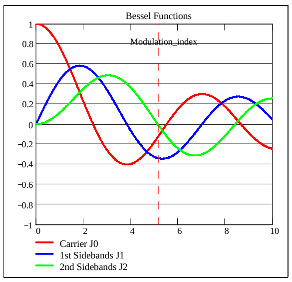
M=2.4, Carrier Null

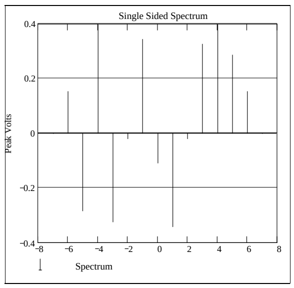


M=3.8, first sideband null



M=5.1, second sideband null

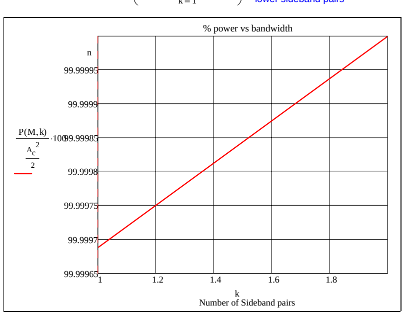




Power vs BW, M=0.1

$$P(M,n) := \left(\frac{J0(M)^2}{2} + \sum_{k=1}^{n} Jn(k,M)^2 \right)$$

second term includes power in +Jn and power in -Jn, i.e the upper and lower sideband pairs



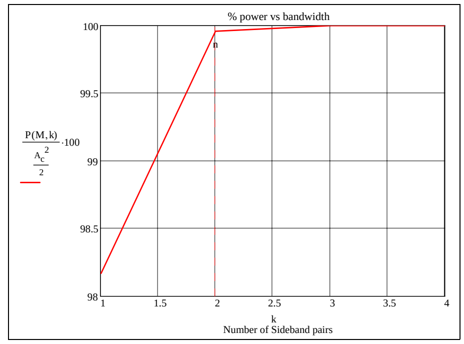
M = 0.1

Fm = 1 Hz

Bandwidth = 2 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}} \cdot 100 = 100$$

Power vs BW, M=0.9



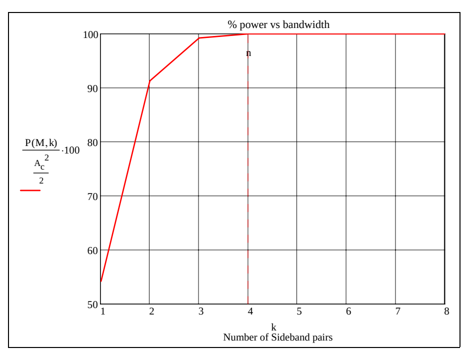
M = 0.9

Fm = 1 Hz

Bandwidth = 4 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}} \cdot 100 = 99.958$$

Power vs BW, M=2.4



M = 2.4

Fm = 1 Hz

Bandwidth = 8 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}} \cdot 100 = 99.945$$

Figure 5–16 Angle-modulated system with preemphasis and deemphasis.

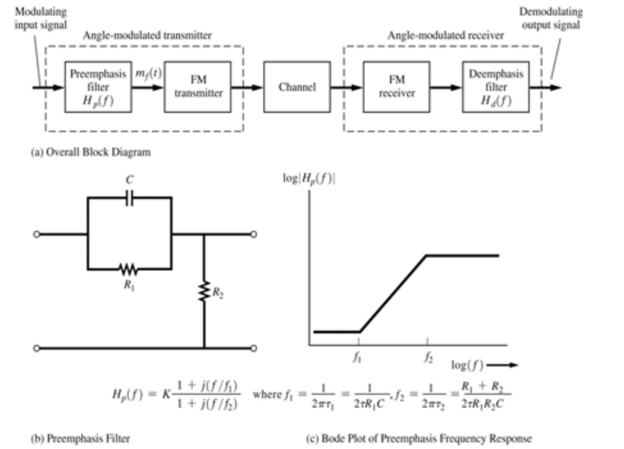


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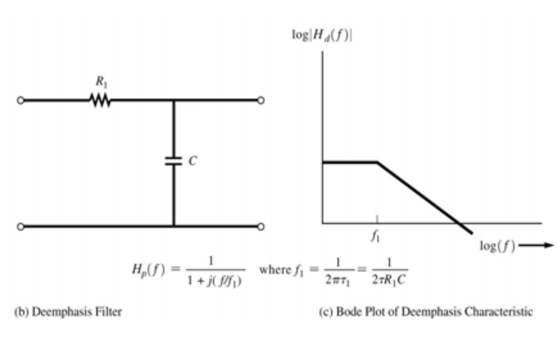
Figure 5–16 Angle-modulated system with preemphasis and deemphasis.



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AM vs FM

- FM capture effect: A phenomenon, associated with FM reception, in which only the stronger of two signals at or near the same frequency will be demodulated.
  - The complete suppression of the weaker signal occurs at the receiver limiter, where it is treated as noise and rejected.
  - When both signals are nearly equal in strength, or are fading independently, the receiver may switch from one to the other.
- Bandwith: B_{AM}=2 x f_m B_{FM} >= 2 x f_m use Carson's Rule
- The Receiver IF amplifier is change to a Limiting Amplifier for FM
  - FM rejects amplitude noise such as lightening and man made noise
- The FM demodulator may be a PLL, Ratio Detector, Foster Sealy Discriminator, or slope detector.