Sampling
PAM- Pulse Amplitude Modulation
(continued)

EELE445-14

Lecture 14

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Sampling

Properties will be looking at for:

- Impulse Sampling
- Natural Sampling
- Rectangular Sampling

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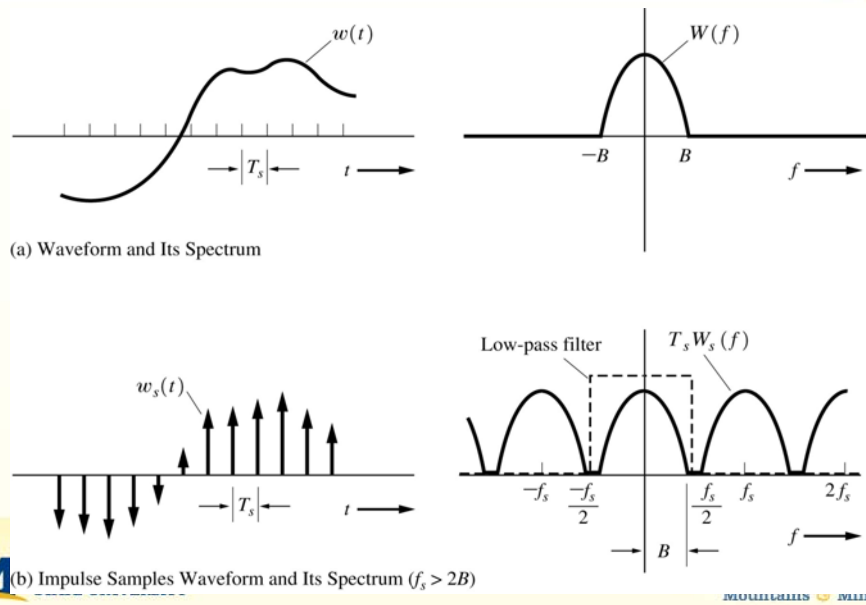
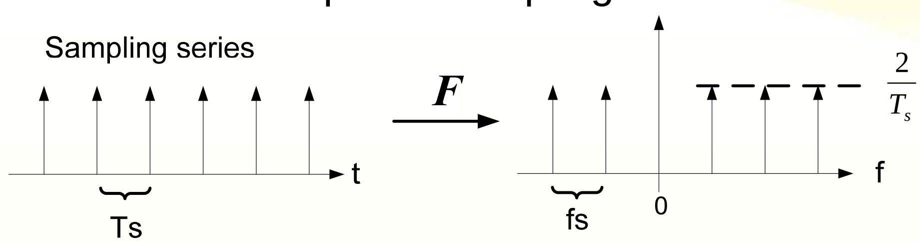
Figure 2–18 Impulse sampling.



Impulse Sampling

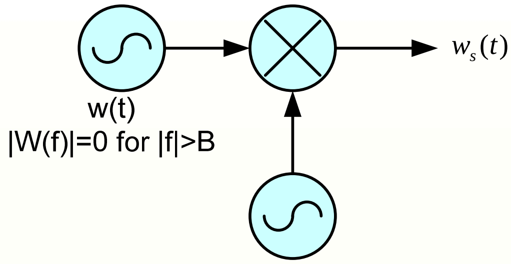


$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \Rightarrow D_0 + \sum_{n=1}^{\infty} D_n \cos(n\omega_s t + \varphi_n)$$

$$\varphi_n = 0 \qquad \omega_s = \frac{2\pi}{T_s} \qquad D_0 = \frac{1}{T_s} \qquad D_n = \frac{2}{T_s}$$

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos(\omega_s t) + 2\cos(2\omega_s t) + 2\cos(3\omega_s t) + \cdots\right]$$

Impulse Sampling



Impulse sampling series $\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$$w_s(t) = w(t)\delta_{T_s}(t) = \frac{w(t)}{T_s}\left[1 + 2\cos(\omega_s t) + 2\cos(2\omega_s t) + 2\cos(3\omega_s t) + \cdots\right]$$

$$W_s(f) = F\left[w_s(t)\right] = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} W(f - nf_s)$$

Impulse Sampling- text

$$w_s(t) = w(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} w(nT_s)\delta(t - nT_s) \qquad eq2-171$$

subsituting ,

$$w_s(t) = w(t)\sum_{n=-\infty}^{\infty} \frac{1}{T_s} e^{jn\omega_s t}$$

$$W_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} W(f - nf_s)$$

Impulse Sampling

The spectrum of the impuse sampled signal is the spectrum of the unsampled signal that is repeated every f_s Hz, where f_s is the sampling frequency or rate (samples/sec). This is one of the basic principles of digital signal processing.

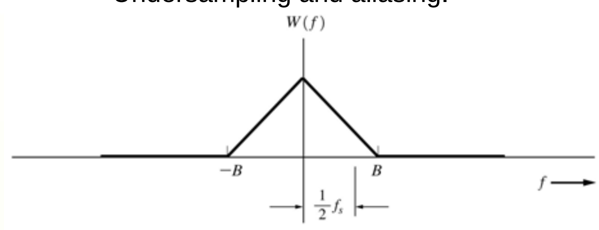
Note:

This technique of impulse sampling is often used to translate the spectrum of a signal to another frequency band that is centered on a harmonic of the sampling frequency, f_s.

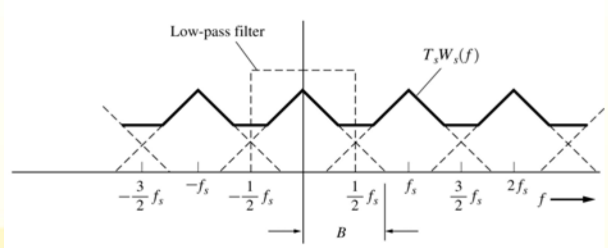
If f_s>=2B, (see fig 2-18), the replicated spectra around each harmonic of f_s do not overlap, and the original spectrum can be regenerated with an ideal LPF with a cutoff of f_s/2.

Impulse Sampling

Undersampling and aliasing.



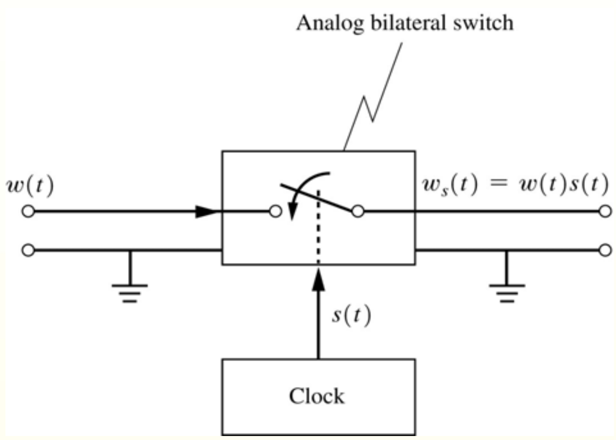
(a) Spectrum of Unsampled Waveform



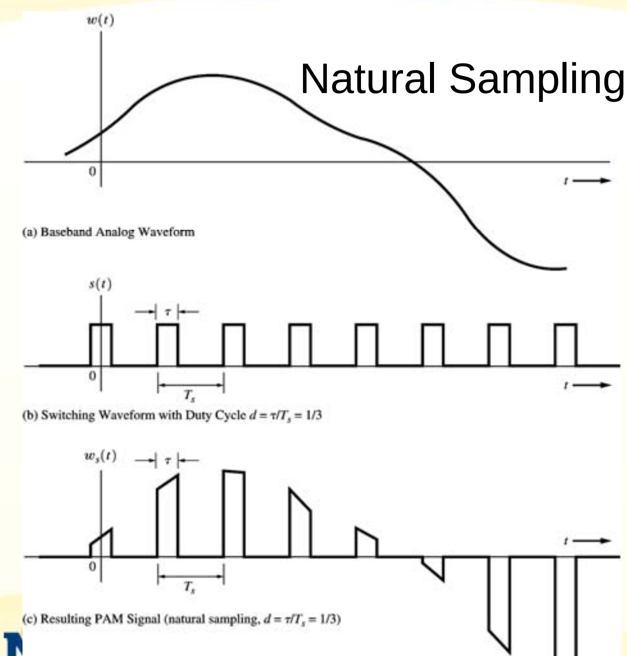
(b) Spectrum of Impulse Sampled Waveform ($f_s < 2B$)

Natural Sampling

Generation of PAM with natural sampling (gating).

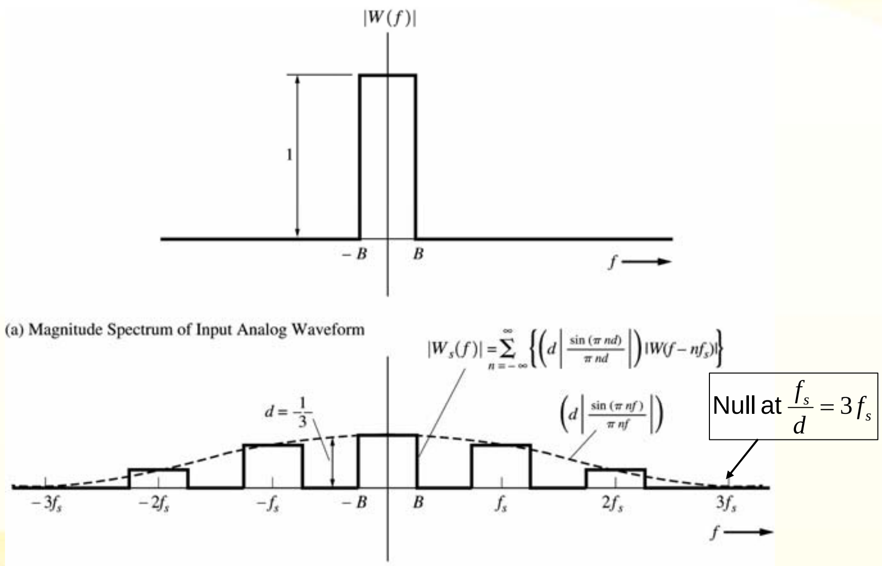


Natural Sampling



Duty cycle =1/3

Natural Sampling



(a) Magnitude Spectrum of Input Analog Waveform

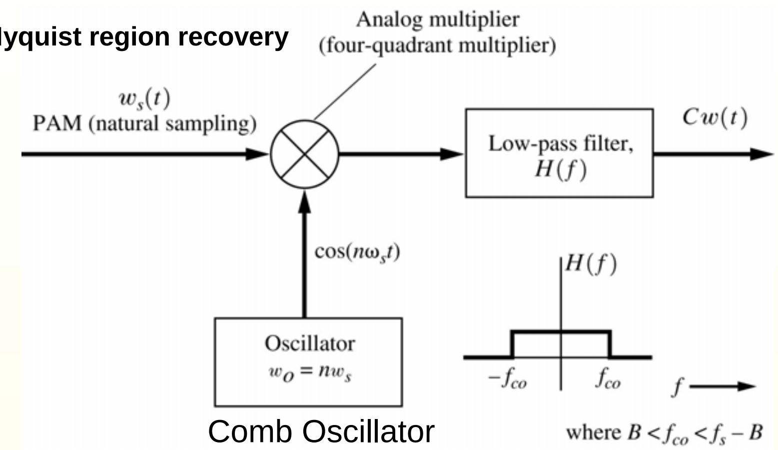
(b) Magnitude Spectrum of PAM (natural sampling) with $d = 1/3$ and $f_s = 4$ B

PAM and PCM

- PAM- Pulse Amplitude Modulation:
  - The pulse may take any real voltage value that is proportional to the value of the original waveform. No information is lost, but the energy is redistributed in the frequency domain.
- PCM- Pulse Code Modulation:
  - The original waveform amplitude is quantized with a resulting loss of information

Figure 3–4 Demodulation of a PAM signal (naturally sampled).

n^{th} Nyquist region recovery



Comb Oscillator
n=1,2,3….

Figure 3–5 PAM signal with flat-top sampling.
Impulse sample and hold

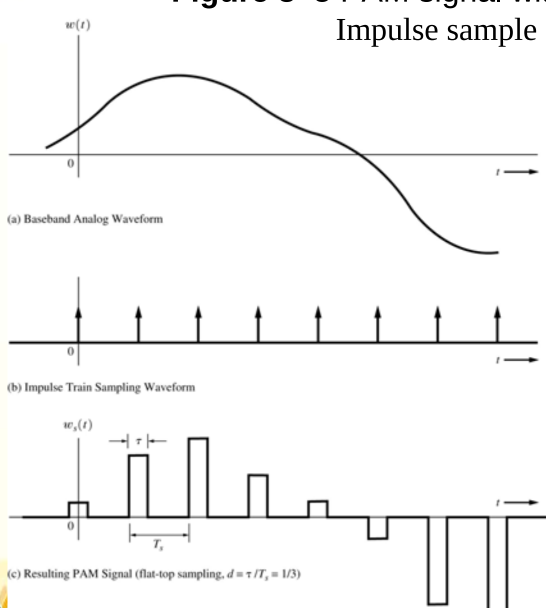


Figure 3–6 Spectrum of a PAM waveform with flat-top sampling.

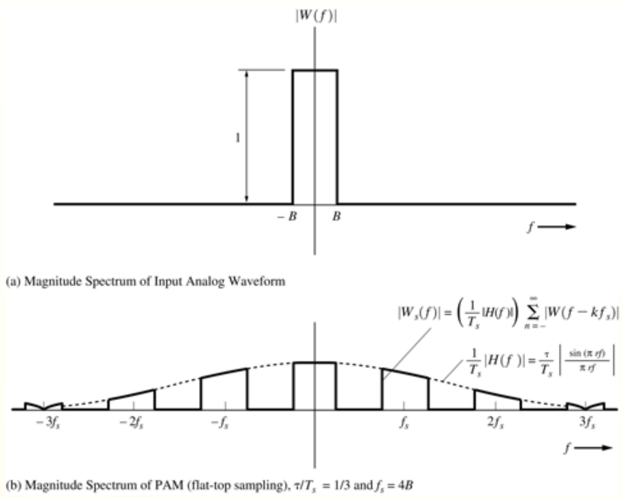
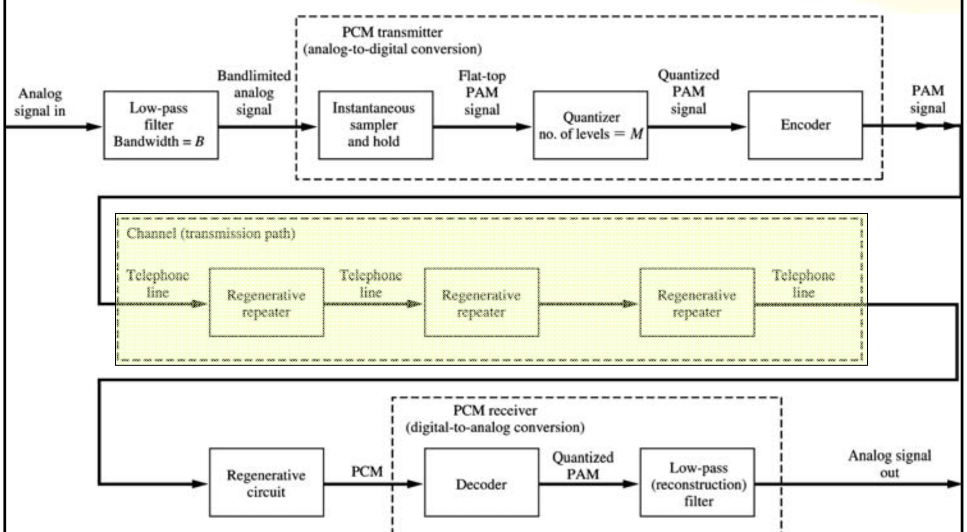


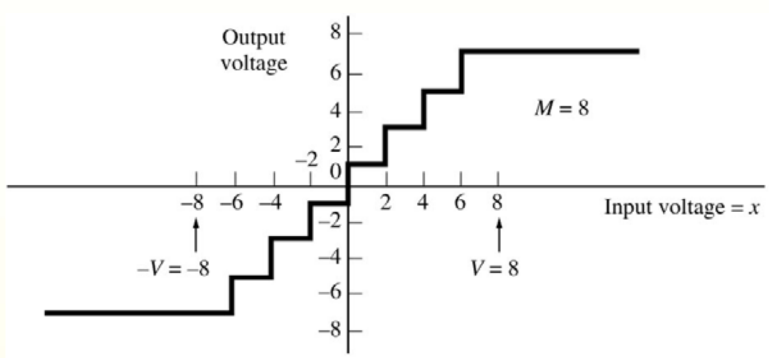
Figure 3–7 PCM trasmission system.



PCM Transmission

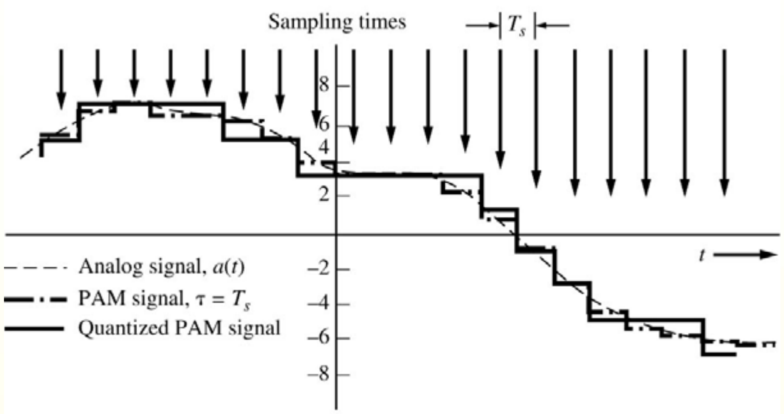
- Negative: The transmission bandwidth of the PCM signal is much larger than the bandwidth of the original signal
- Positive: The transmission range of a PCM signal may be extended with the use of a **<u>regenerative repeater</u>**.

Figure 3–8 Illustration of waveforms in a PCM system.



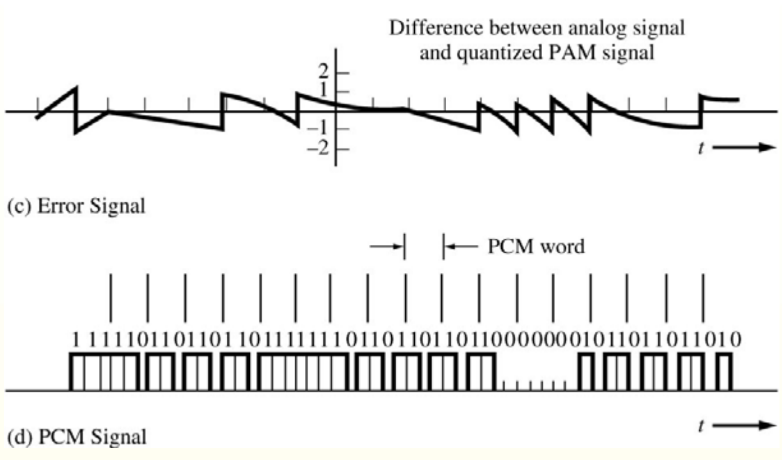
(a) Quantizer Output-Input Characteristics

Figure 3–8 Illustration of waveforms in a PCM system.



(b) Analog Signal, Flat-top PAM Signal, and Quantized PAM Signal

Illustration of waveforms in a PCM system.



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Couch, *Digital and Analog Communication Systems*, Seventh Edition
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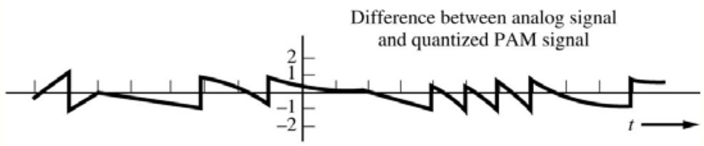
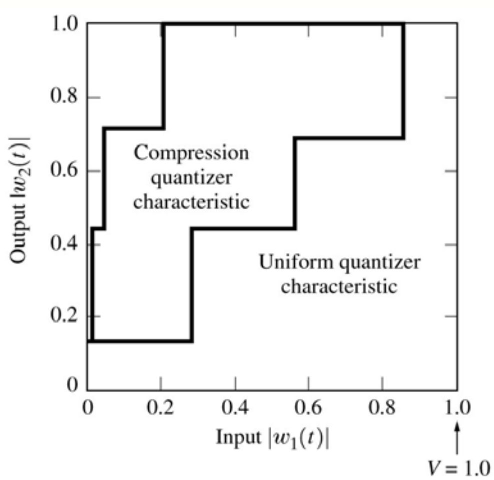
Illustration of waveforms in a PCM system.



(c) Error Signal

- The error signal, (quantization noise), is inversely proportional to the number of quantization levels used to approximate the waveform x(t)
- Companding is used to improve the SQNR (signal to quantization noise ratio) for small amplitude x(T) signals

Figure 3–9 Compression characteristics (first quadrant shown).



(a) $M = 8$ Quantizer Characteristic

Figure 3–9 Compression characteristics (first quadrant shown).

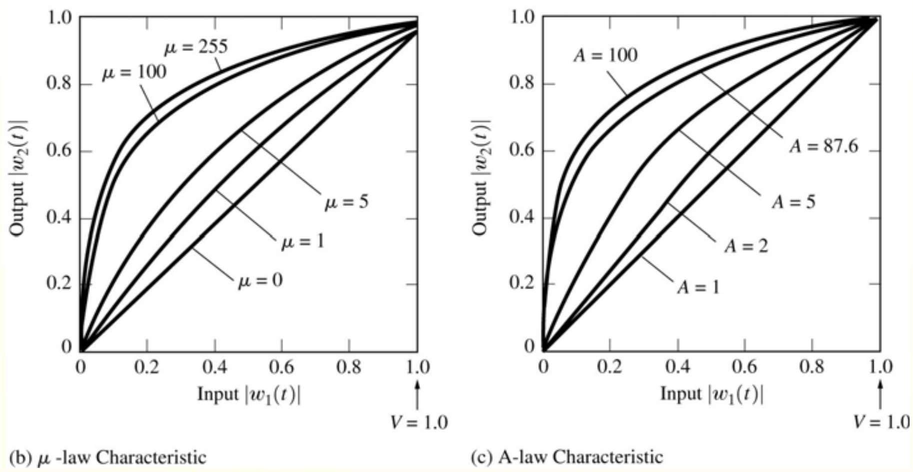
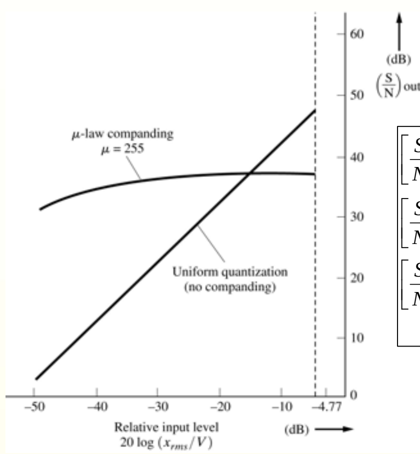


Figure 3–10 Output SNR of 8-bit PCM systems with and without companding.



$$\left[\frac{S}{N}\right]_{dB} = 4.77 + 6.02n - 20\log\left(\frac{V_{peak\ range}}{x_{rms}}\right) \quad \textit{uniform quantion}$$

$$\left[\frac{S}{N}\right]_{dB} = 4.77 + 6.02n - 20\log[\ln(1+\mu)] \quad \mu - \textit{law companding}$$

$$\left[\frac{S}{N}\right]_{dB} = 4.77 + 6.02n - 20\log[1+\ln(A)] \quad A - \textit{law companding}$$

Quantization Noise,
Analog to Digital Converter-A/D

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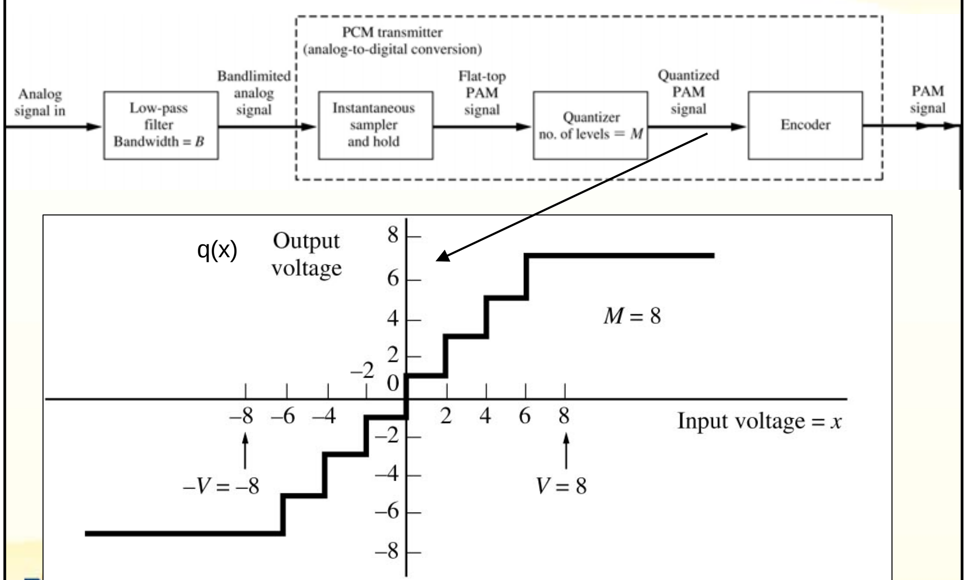
Lecture 16

Figure 3–7 PCM trasmission system.



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Quantization

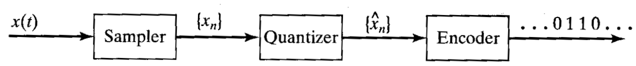
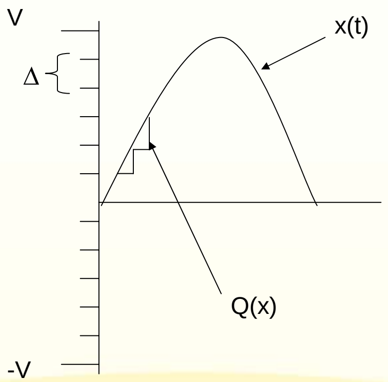


Figure 7.7 Block diagram of a PCM system.

$$\hat{x}_n = Q(x)$$



$$\Delta = \frac{2V}{M} = \frac{V}{2^{n-1}}$$

$$M = 2^n$$

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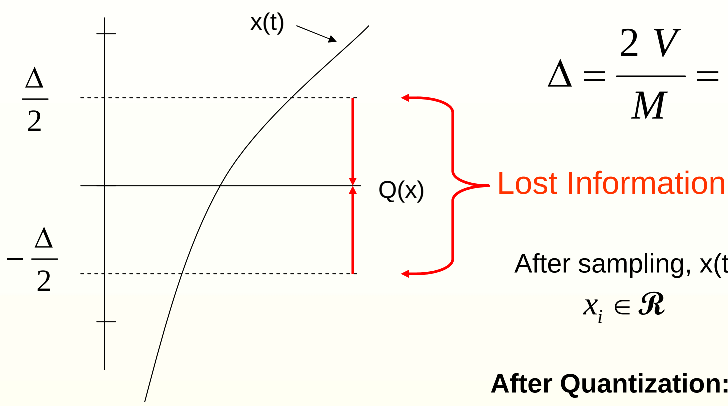
Quantization – Results in a Loss of Information



$$\Delta = \frac{2V}{M} = \frac{V}{2^{n-1}}$$

After sampling, x(t)=x_i

$x_i \in \mathcal{R}$

After Quantization:

$$Q(x) = \hat{x}, \quad x \in \mathcal{R}$$

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Quantization Noise

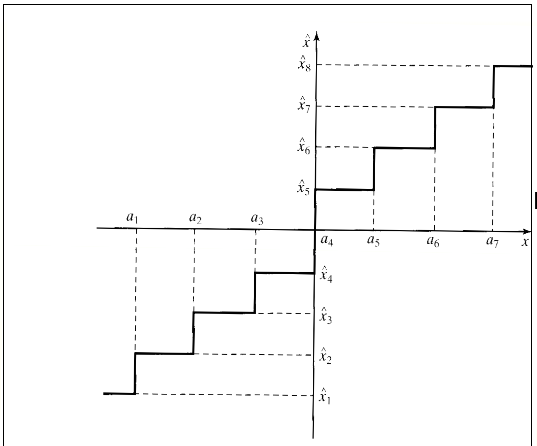


Figure 7.3 Example of an 8-level quantization scheme.

Quantization function:

$$Q(x) = \hat{x}_i \text{ for all } x \in \mathscr{R}_i.$$

Define the mean square <u>distortion</u>:

$$q(x) = (x - Q(x))^2 = \tilde{x}^2$$

and

$$|x - Q(x)| \le \frac{\Delta}{2}$$

Quantization

$$\langle q^2 \rangle = \frac{1}{\Delta} \int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq$$

$$= \frac{\Delta^2}{12}$$

$$= \frac{(V)^2}{3M^2} = P_{nq} \quad \textit{the quantization noise}$$

$$\Delta = \frac{2\,V}{M} = \frac{V}{2^{n-1}}$$

$$M = 2^n$$

where $M=2^n$, V is ½ the A/D input range, and n is the number of bits

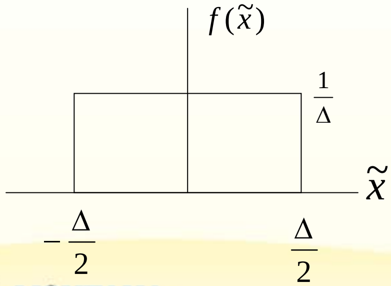
Quantization Noise from the expectation operator:

Since X is a random variable, so are $\hat{X}$ *and* $\tilde{X}$

So we can define the mean squared error (distortion) as:

$$D = E\left[d\left(X, \hat{X}\right)\right] = E\left[(X - Q(X))^2\right]$$

The pdf of the error is uniformly distributed: $\tilde{X} = X - Q(X)$



$$f(\tilde{x}) = \begin{cases} \frac{\Delta}{2} & -\frac{\Delta}{2} \le \tilde{x} \le \frac{\Delta}{2} \\ 0 & otherwise \end{cases}$$

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SQNR – Signal to Quantization Noise

Definition If the random variable X is quantized to $Q(X)$, the *signal-to-quantization noise ratio* (SQNR) is defined by

$$\text{SQNR} = \frac{E[X^2]}{E[X - Q(X)]^2}.$$

When dealing with signals, the quantization noise power is

$$P_{\tilde{X}} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} E\,[X(t) - Q(X(t))]^2\, dt$$

and the signal power is

$$P_X = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} E[X^2(t)]dt.$$

Hence, the signal-to-quantization noise ratio is

$$\text{SQNR} = \frac{P_X}{P_{\tilde{X}}}.$$

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SQNR – Signal to Quantization Noise Ratio

P_x may be found using:

$$P_X = R_X(\tau)|_{\tau=0} = \int_{-\infty}^{\infty} S_X(f)\,df = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx.$$

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SQNR – Signal to Quantization Noise Ratio

The distortion, or "noise", is therefore:

$$E[\tilde{X}^2] = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta}\tilde{x}^2 d\tilde{x} = \frac{\Delta^2}{12} = \frac{x_{\max}^2}{3N^2} = \frac{x_{\max}^2}{3 \times 4^\nu}$$

$$\text{SQNR} = \frac{P_X}{\overline{\tilde{X}^2}} = \frac{3 \times N^2 P_X}{x_{\max}^2} = \frac{3 \times 4^\nu P_X}{x_{\max}^2}$$

Where P_x is the power of the input signal

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SQNR – Linear Quantization

$$E\left[X^2\right] \le x_{\max}^2$$

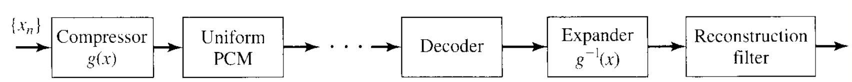
$$\frac{P_x}{x_{\max}^2} < 1$$

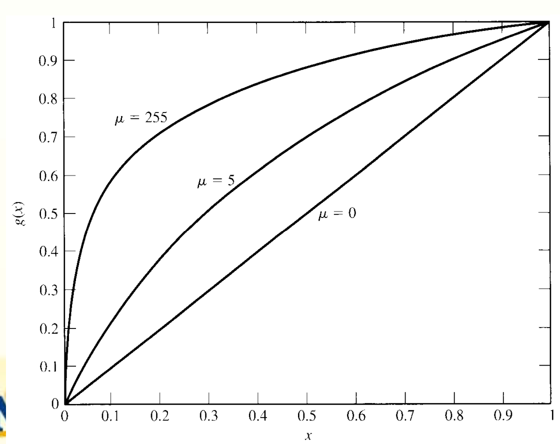
The SQNR decreases as
The input dynamic range
increases

$$\mathrm{SQNR}\big|_{\mathrm{dB}} \approx 10\log_{10}\frac{P_X}{x_{\max}^2} + 6\nu + 4.8.$$

$\mathcal{U}$-Law Nonuniform PCM

used to increase SQNR for given P_x , xmax, and n

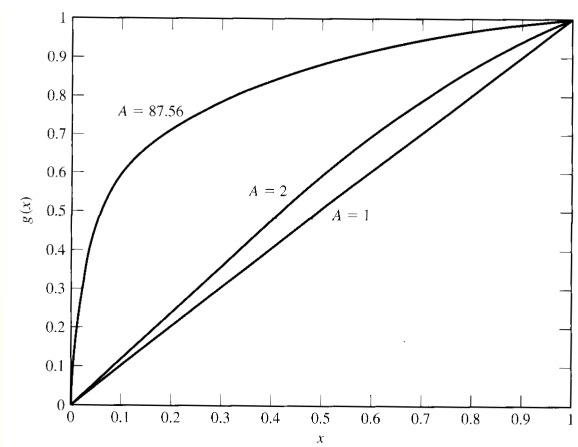




$$g(x) = \frac{\log(1+\mu|x|)}{\log(1+\mu)}\,\mathrm{sgn}(x).$$

$\mathcal{U}$=255 U.S

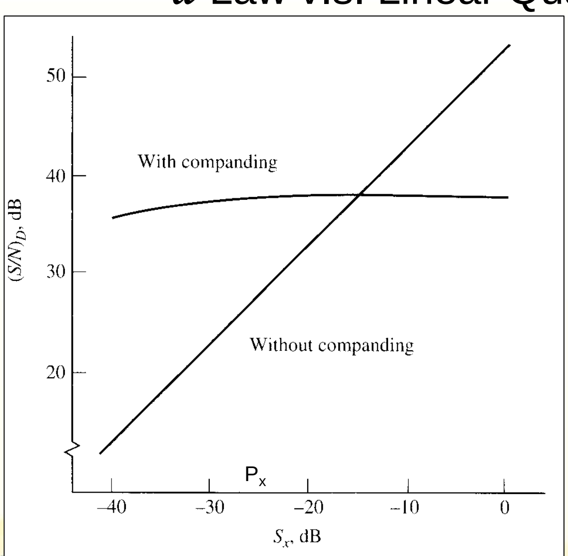
A-Law Nonuniform PCM



$$g(x) = \frac{1 + \log A|x|}{1 + \log A}\,\mathrm{sgn}(x),$$

$\boldsymbol{a}$=87.56 U.S

$\boldsymbol{u}$-Law v.s. Linear Quantization

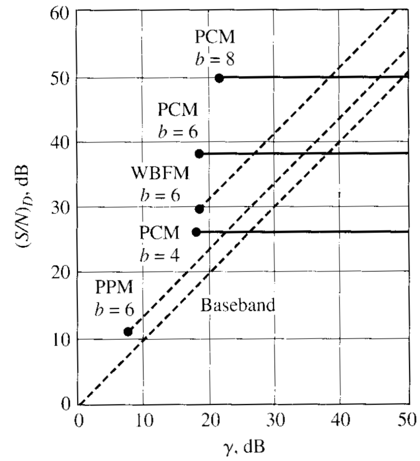


8 bit

P_x is signal power

Relative to full scale

Pulse Code Modulation, PCM, Advantage compared with analog systems



Performance comparison of PCM and analog modulation.

- b is the number of bits
- γ is $(S/N)_{baseband}$ Relative to full scale
- PPM is pulse position modulation

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