Sampling
PAM- Pulse Amplitude Modulation
(continued)

EELE445-14

Lecture 17

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SQNR
with and without
Companding

EELE445-13

Lecture 17

(continuation of lecture 16)

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SQNR - summary

$$P_{nq} = \frac{V_{max}^2}{3M^2} \quad \textit{quantization noise power}$$

$$SQNR = \frac{P_x}{P_{nq}} = \frac{3M^2 P_x}{V_{max}^2} = \frac{3 \times 4^n P_x}{V_{max}^2}$$

- **M is the number of quantization levels**
- **n is the number of bits**
- **V_{max} is ½ the A/D input range**

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$SQNR_{dB}$

$$P_x \le V_{max}^2$$

$$\frac{P_x}{V_{max}^2} \le 1$$

The SQNR decreases as The input dynamic range increases

$$SQNR\,|_{dB} \cong 10\log_{10}\left(\frac{P_x}{V_{max}^2}\right) + 6n + 4.8$$

- V_{max} is $\frac{1}{2}$ the peak to peak range of the quantizer
- n is the number of bits in the full scale quantizer range

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Summary of $SQNR|_{dB}$ for linear, μ–law, A-law

$$SQNR_{dB} = 6.02n + \alpha \quad (3\text{-}25)$$

$$\alpha = 4.77 - 20\log\left(\frac{V_{\max}}{x_{rms}}\right) \quad \text{(uniform quantizing)} \quad (3\text{-}26a)$$

$$\alpha \cong 4.77 - 20\log[\ln(1+\mu)] \quad (\mu\text{-law companding}) \quad (3\text{-}26b)$$

$$\alpha \cong 4.77 - 20\log[1+\ln A] \quad (A\text{-law companding}) \quad (3\text{-}26c)$$

$P_x = x_{rms}^2$ is the input signal power

$V_{\max}$ is the peak design level of the quantizer

Exam 1
Wednesday February 26

Line Codes: Baseband Digital Signals

ELE445-14

Lecture 17

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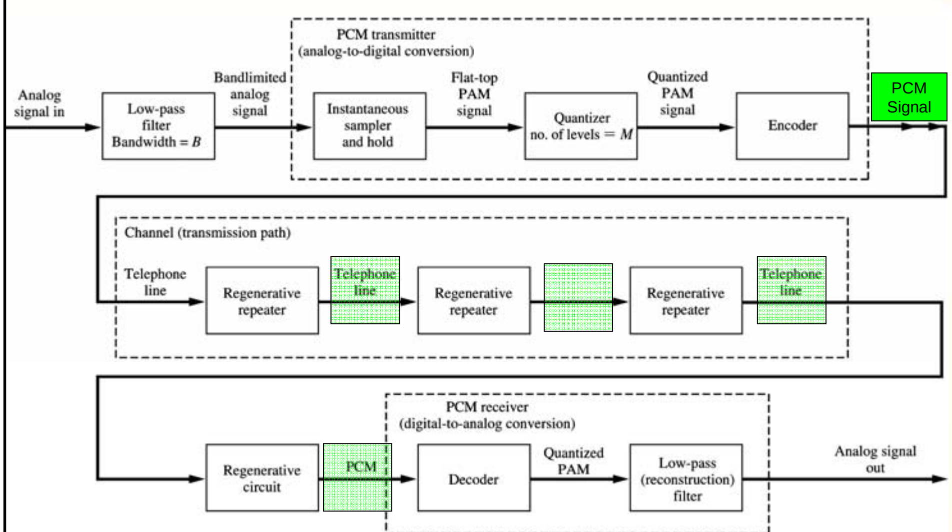
PCM Signal Transmission



Couch, *Digital and Analog Communication Systems*, Seventh Edition

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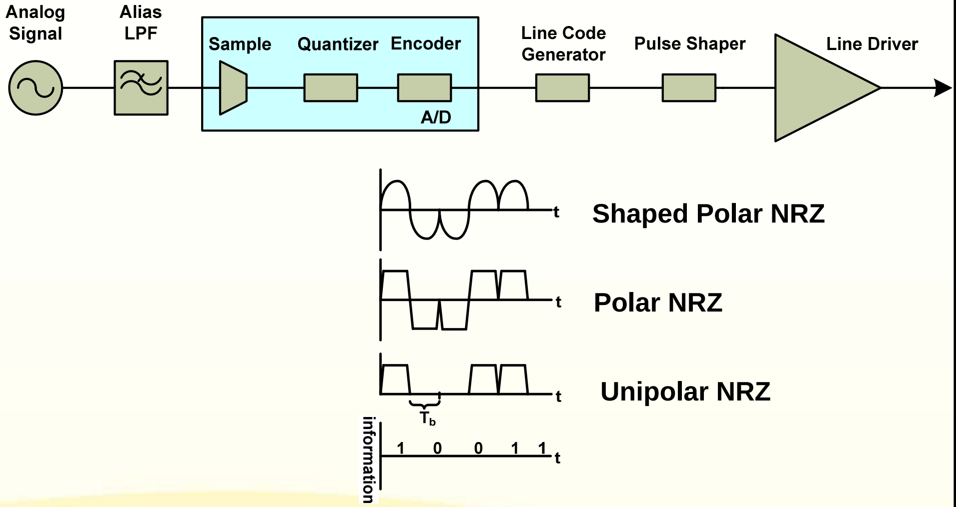
PCM Waveforms



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Digital Signaling – Signal Vectors

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0 < t < T_o$$

Where w_k is the digital data and $\phi_k(t)$ is the set of basis waveforms

Example:

$$\varphi(t) = \Pi\left(\frac{t}{T_o}\right) \quad T_o = \frac{T_s}{2} \quad for \;\; RZ \;\; codes$$

$$T_o = T_s \quad for \;\; NRZ \;\; codes$$

$$w \in (0,1) \quad for\; Unipolar$$

$$w \in (-1,1) \quad for \;\; polar \;\; or \;\; bipolar$$

$$w \in (-.75,-.25,.25,.75) \quad for\; 4\; state\; multilevel$$

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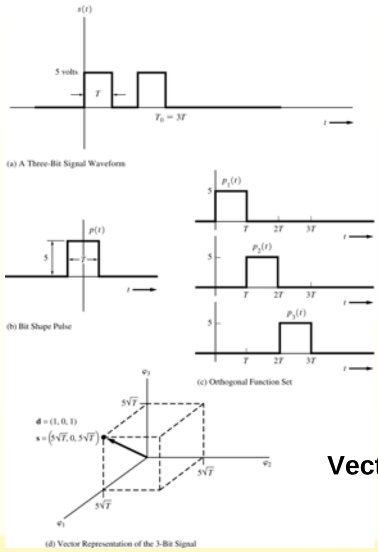
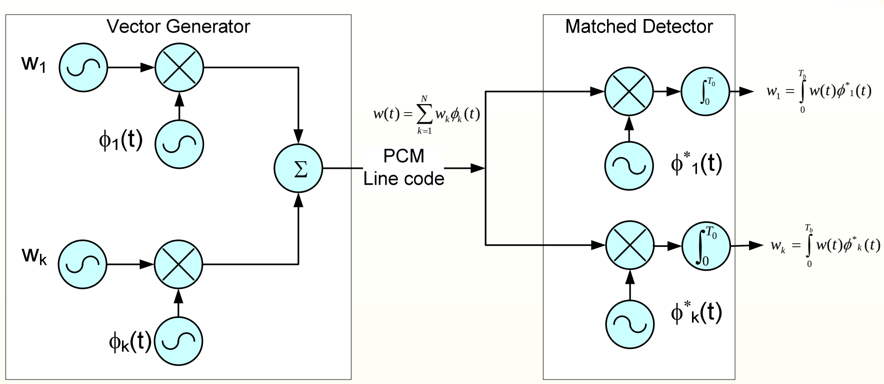
Figure 3–11 Representation for a 3-bit binary digital signal.



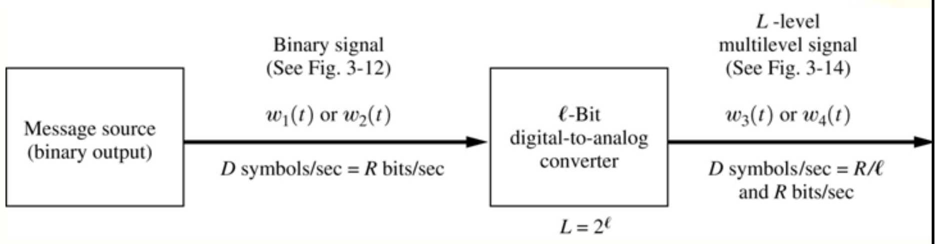
Vector lengths are in units of $\sqrt{E}$

Vector Signaling



The ϕ_k are orthogonal basis functions, w_k are weights that correspond to the digital data.

Figure 3–13 Binary-to-multilevel signal conversion.



$$\varphi(t) = \Pi\left(\frac{t}{T_o}\right) \quad T_o = \frac{T_s}{2} \quad for \;\; RZ \; codes$$

$$T_o = T_s \quad for \;\; NRZ \; codes$$

$$w \in (-3,-1,1,3) \;\; for \; 4 \; state \; multilevel$$

Figure 3–12 Binary signaling (computed).

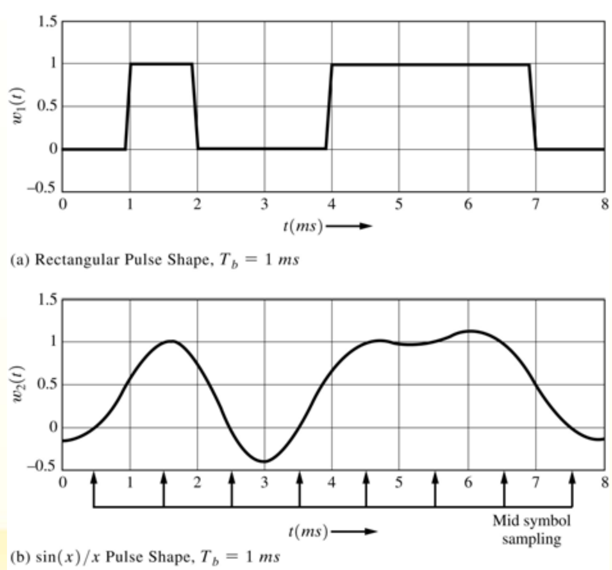
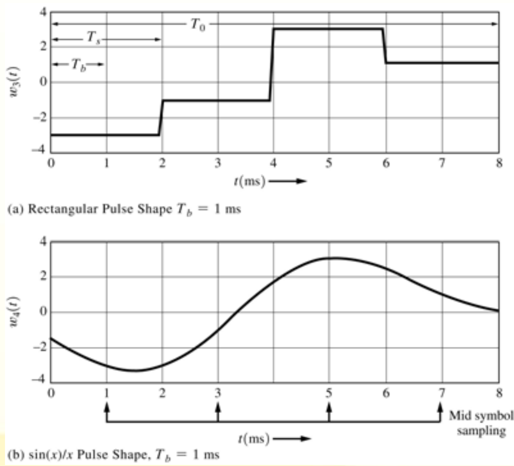


Figure 3–14 L = 4-level signaling (computed).



Couch, *Digital and Analog Communication Systems*, Seventh Edition

Desired Properties of Line Codes

- ***Self-synchronization:***
  There is enough timing information built into the code so that bit synchronizers can be designed to extract the clock signal. A long series of 1's or 0's should not cause a problem.

- ***Low probability of bit error:***
  Receiver can be designed that will recover the binary data with a low probability of bit error when the input data signal is corrupted by noise or ISI (intersymbol interference)

- ***Good Spectral Properties:***
  If the channel is ac coupled, the PSD of the line code signal should be negligible at frequencies near zero. The signal bandwidth needs to be sufficiently small compared to the channel bandwidth, so that ISI will not be a problem.

Desired Properties of Line Codes

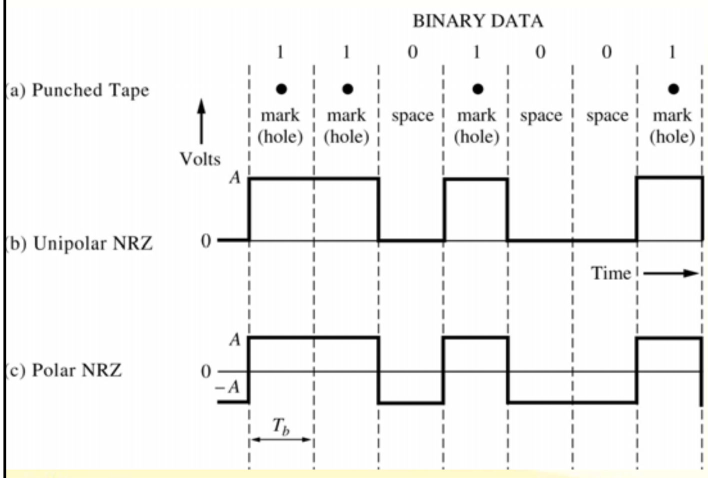
•***Transmission bandwidth:***
As small as possible

•***Error detection capability:***
It should be possible to implement this feature easillyby the addition of channel encoders and decoders, or the feature should be incorporated into the line code.

• ***Transparency:***
The data protocol and line code are designed so that every possible sequence of data is faithfully and transparently received. (The sequence of bits does not matter.)

Figure 3–15 Binary signaling formats.

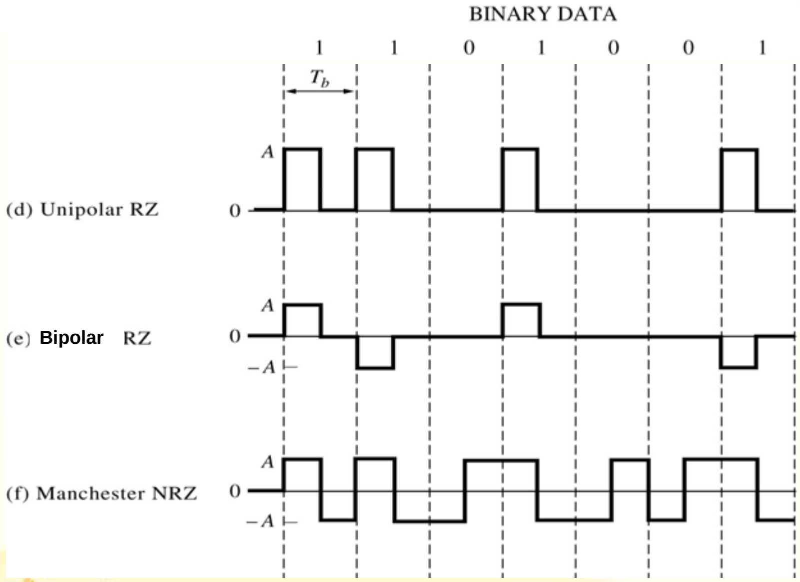
NRZ: non-return to zero has smallest occupied bandwidth but must constrain consecutive 0's or 1's



Good: simple
Bad: DC

Good: Lowest BER

Figure 3–15 Binary signaling formats.



Line Codes:
Power Spectral Density (PSD)

ELE445-14

Lecture 18

PSD of Line Codes

$$P_y(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi k f T_s}$$

$$R(k) = \sum_{i=1}^{l} (a_n a_{n+k})_i P_i$$

for random data

Binary Signals: $T_s = T_b$
Multilevel Signals: $T_s = l\,T_b$

P_i is the probability of $a_n a_{n+k}$ occurring

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R(k) example for a deterministic data file

Data	1	1	0	1	0
Unipolar A= 0,1	1	1	0	1	0
Polar A= -1,1	1	1	-1	1	-1
Bipolar A = 0, (1,-1)	1	-1	0	1	0

$$R(k) = \frac{1}{N} \sum_{k=-N}^{N} (a_n a_{n+k})$$

where N is the number of bits

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R(k) example Polar , k=0

Data	1	1	0	1	0
a_n	1	1	-1	1	-1
a_{n+0}	1	1	-1	1	-1
$(a_n)^2$	1	1	1	1	1
$(1/N)\Sigma(a_n)^2$	1				

$$R(k) = \frac{1}{N}\sum_{k=-N}^{N}(a_n a_{n+k})$$

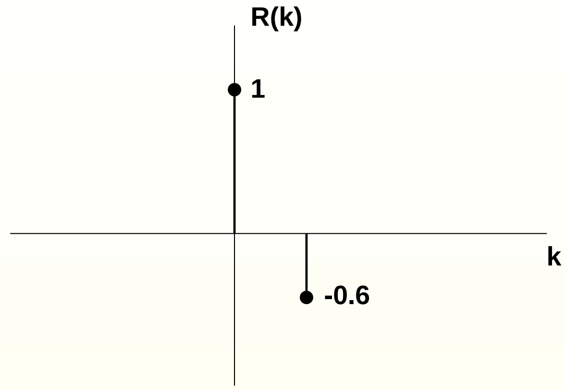
R(k) example Polar , k=1

Data	1	1	0	1	0
a_n	1	1	-1	1	-1
a_{n+1}	-1	1	1	-1	1
$a_n a_{+1}$	-1	1	-1	-1	-1
$(1/N)\Sigma\ a_n a_{+1}$	-0.6				

Shift Right 1 →

See Mathcad file bpsk_psd.xmcd and linecodepsd.xmcd

R(k) example Polar , k=1



PSD of Line Codes

Unipolar NRZ Signaling. For unipolar signaling, the possible levels for the a's are $+A$ and 0 V. Assume that these values are equally likely to occur and that the data are independent. Now, evaluate $R(k)$ as defined by Eq. (3–36b). For $k = 0$, the possible products of $a_n a_n$ are $A \times A = A^2$ and $0 \times 0 = 0$, and consequently, $I = 2$. For random data, the probability of having A^2 is $\frac{1}{2}$ and the probability of having 0 is $\frac{1}{2}$, so that

$$R(0) = \sum_{i=1}^{2} (a_n a_n)_i P_i = A^2 \cdot \tfrac{1}{2} + 0 \cdot \tfrac{1}{2} = \tfrac{1}{2} A^2$$

For $k \neq 0$, there are $I = 4$ possibilities for the product values: $A \times A$, $A \times 0$, and $0 \times A$, 0×0. They all occur with a probability of $\frac{1}{4}$. Thus, for $k \neq 0$,

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k}) P_i = A^2 \cdot \tfrac{1}{4} + 0 \cdot \tfrac{1}{4} + 0 \cdot \tfrac{1}{4} + 0 \cdot \tfrac{1}{4} = \tfrac{1}{4} A^2$$

Hence,

$$R_{\text{unipolar}}(k) = \begin{cases} \dfrac{1}{2} A^2, & k = 0 \\ \dfrac{1}{4} A^2, & k \neq 0 \end{cases} \qquad (3\text{–}37a)$$

PSD of Line Codes

For rectangular NRZ pulse shapes, the Fourier transform pair is

$$f(t) = \Pi\left(\frac{t}{T_b}\right) \leftrightarrow F(f) = T_b \frac{\sin \pi f T_b}{\pi f T_b}. \quad (3\text{–}37b)$$

Using Eq. (3–36a) with $T_s = T_b$, we find that the PSD for the unipolar NRZ line code is

$$\mathcal{P}_{\text{unipolar NRZ}}(f) = \frac{A^2 T_b}{4}\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \left[1 + \sum_{k=-\infty}^{\infty} e^{j2\pi k f T_b}\right]$$

But†

$$\sum_{k=-\infty}^{\infty} e^{jk2\pi f T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \quad (3\text{–}38)$$

Thus,

$$\mathcal{P}_{\text{unipolar NRZ}}(f) = \frac{A^2 T_b}{4}\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \quad (3\text{–}39a)$$

† Equation (3–38) is known as the *Poisson sum formula*, as derived in Eq. (2–115).

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PSD of Line Codes

But because $[\sin(\pi f T_b)/(\pi f T_b)] = 0$ at $f = n/T_b$, for $n \neq 0$, this reduces to

$$\mathcal{P}_{\text{unipolar NRZ}}(f) = \frac{A^2 T_b}{4}\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \left[1 + \frac{1}{T_b}\delta(f)\right] \quad (3\text{–}39b)$$

If A is selected so that the normalized average power of the unipolar NRZ signal is unity, then† $A = \sqrt{2}$. This PSD is plotted in Fig. 3–16a, where $1/T_b = R$, the bit rate of the line code. The disadvantage of unipolar NRZ is the waste of power due to the DC level and the fact that the spectrum is not approaching zero near DC. Consequently, DC coupled circuits are needed. The advantages of unipolar signaling are that it is easy to generate using TTL and CMOS circuits and requires the use of only one power supply.

Check the website for Matlab and mathcad files to plot the psd of line codes

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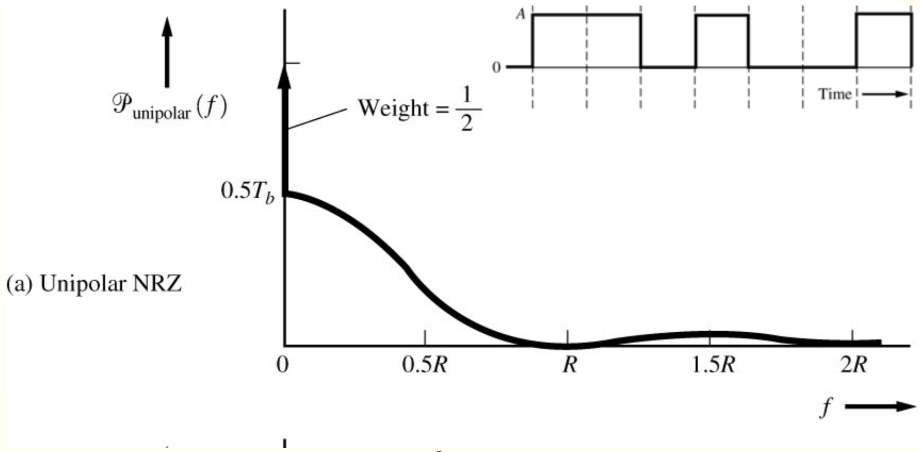
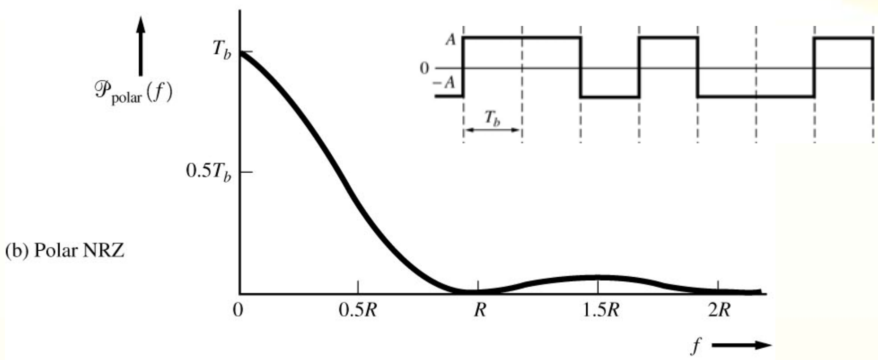
Figure 3–16 PSD for line codes (positive frequencies shown).



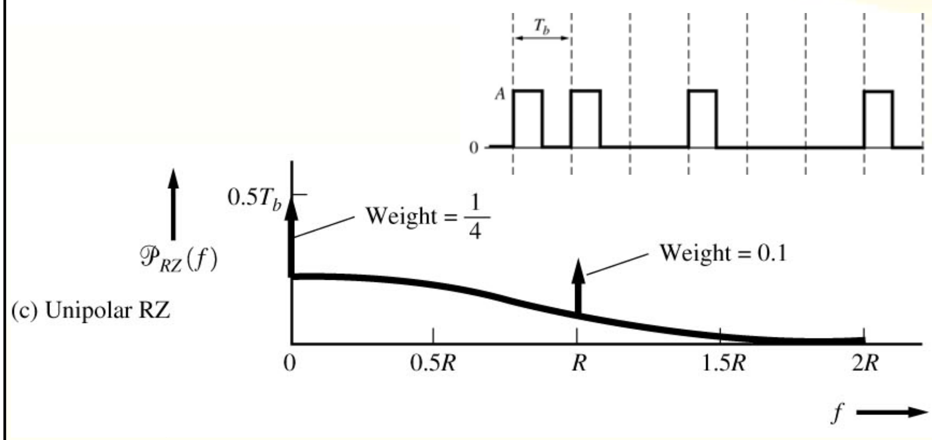
$$P_{unipolar\ NRZ}(f) = \frac{A^2 T_b}{4}\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2\left[1 + \frac{1}{T_b}\delta(f)\right] \quad (3\text{-}39b)$$

Figure 3–16 PSD for line codes (positive frequencies shown).



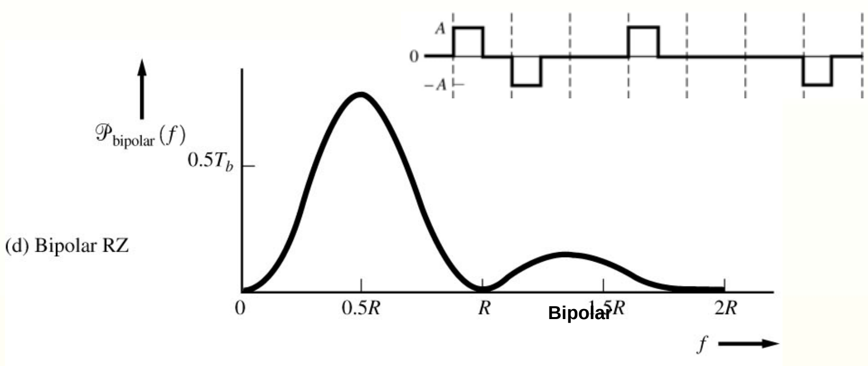
$$P_{polar\ NRZ}(f) = A^2 T_b\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \quad (3\text{-}41)$$

Figure 3–16 PSD for line codes (positive frequencies shown).



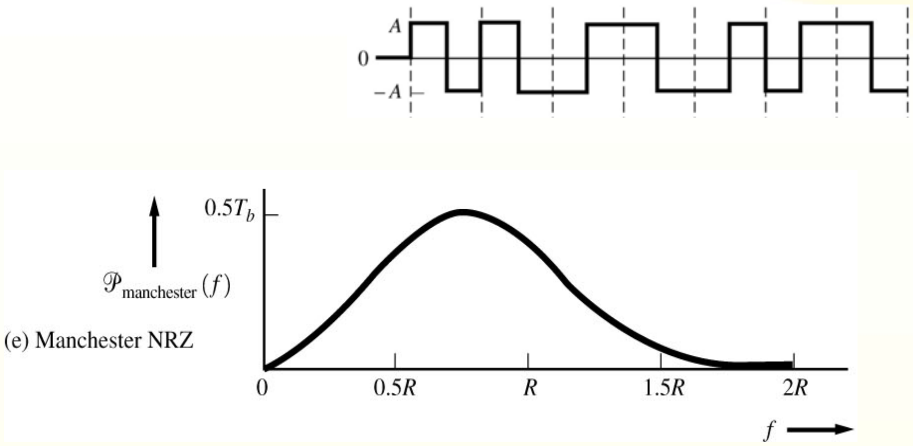
$$P_{unipolar\ RZ}(f) = \frac{A^2 T_b}{16}\left(\frac{\sin \pi f T_b/2}{\pi f T_b/2}\right)^2 \left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \quad (3-43)$$

Figure 3–16 PSD for line codes (positive frequencies shown).



$$P_{bipolar\ RZ}(f) = \frac{A^2 T_b}{8}\left(\frac{\sin \pi f T_b/2}{\pi f T_b/2}\right)^2 \left(1 - \cos(2\pi f T_b)\right) \quad (3-45)$$

Figure 3–16 PSD for line codes (positive frequencies shown).



$$P_{Manchester\ NRZ}(f) = A^2 T_b \left(\frac{\sin \pi f T_b / 2}{\pi f T_b / 2} \right)^2 \sin^2(\pi f T_b / 2) \quad (3\text{-}46c)$$

Eye Diagrams
Clock Recovery
Regenerative Repeater
Spectral Efficiency

EELE445-14

Lecture 19

Exam in Class

EELE445-14
Lecture 20

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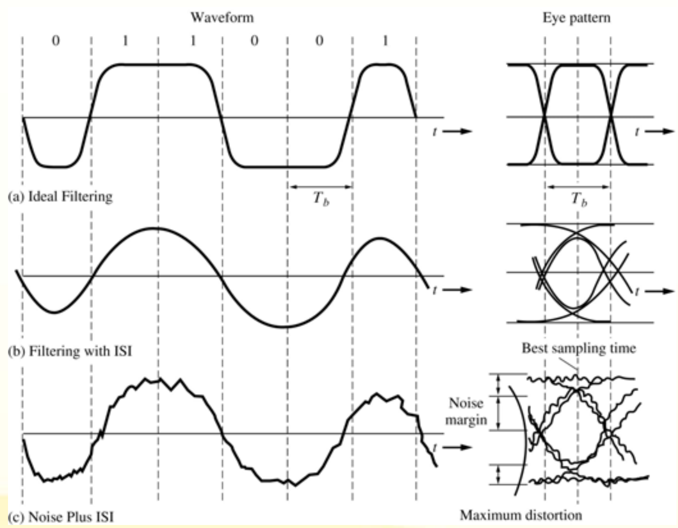
EYE Diagrams

Figure 3–18 Distorted polar NRZ waveform and corresponding eye pattern.



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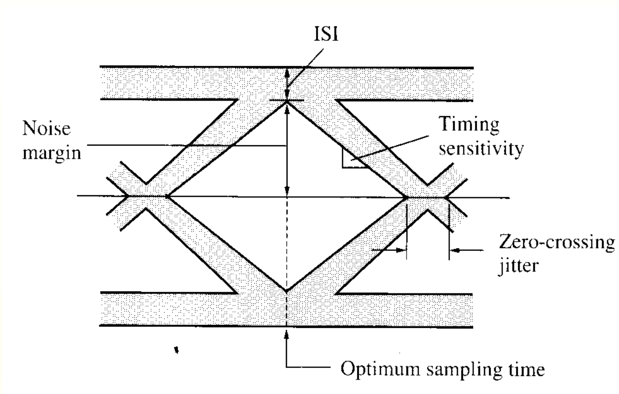
EYE Diagrams



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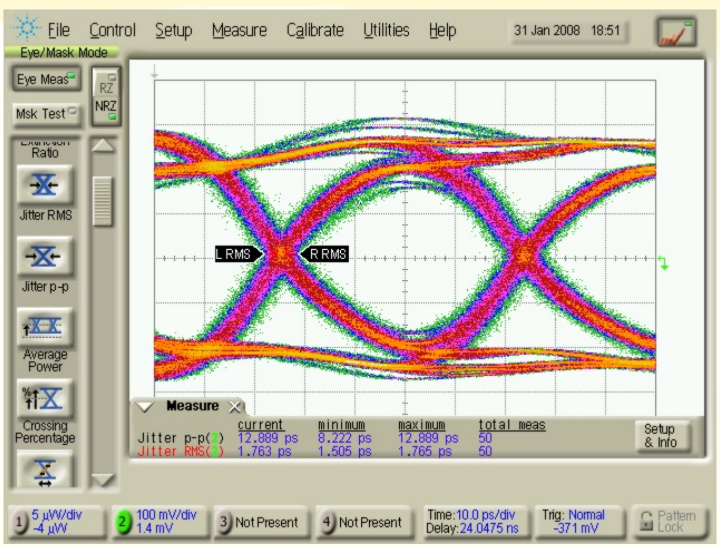

Eye diagram of a 20 Gbps data stream The time scale is set at 10 picoseconds/division.

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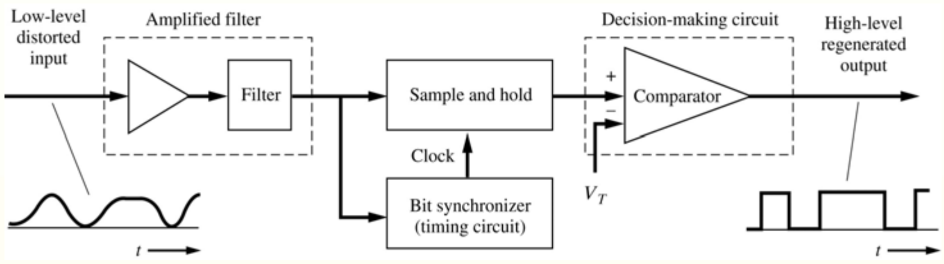
Figure 3–19 Regenerative repeater for unipolar NRZ signaling.



•**The ability to use a regenerative repeater is one of the major advantages of a digital binary system over an analog system**

Figure 3–20 Square-law bit synchronizer for polar NRZ signaling.

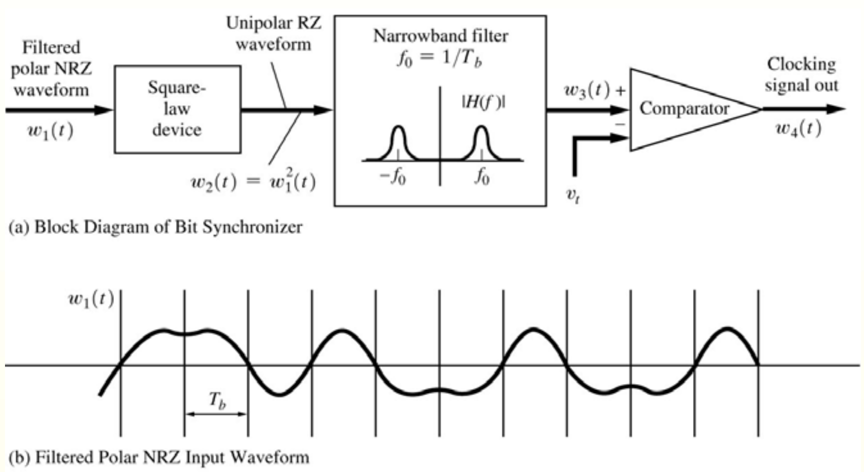


Figure 3–20 Square-law bit synchronizer for polar NRZ signaling.

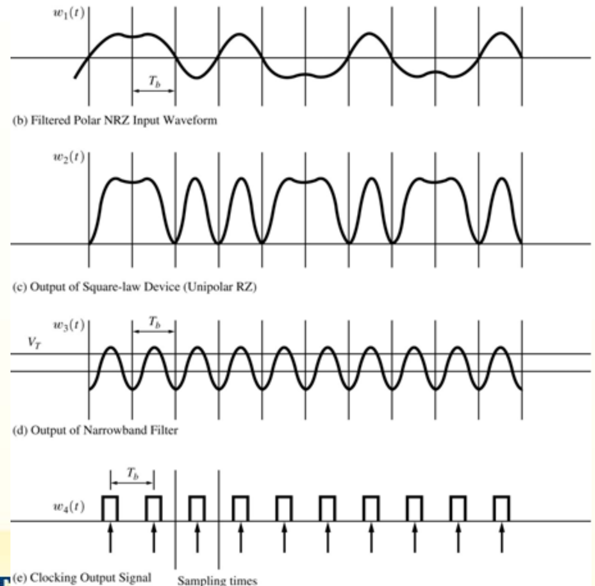


Figure 3–21 Early–late bit synchronizer for polar NRZ signaling.

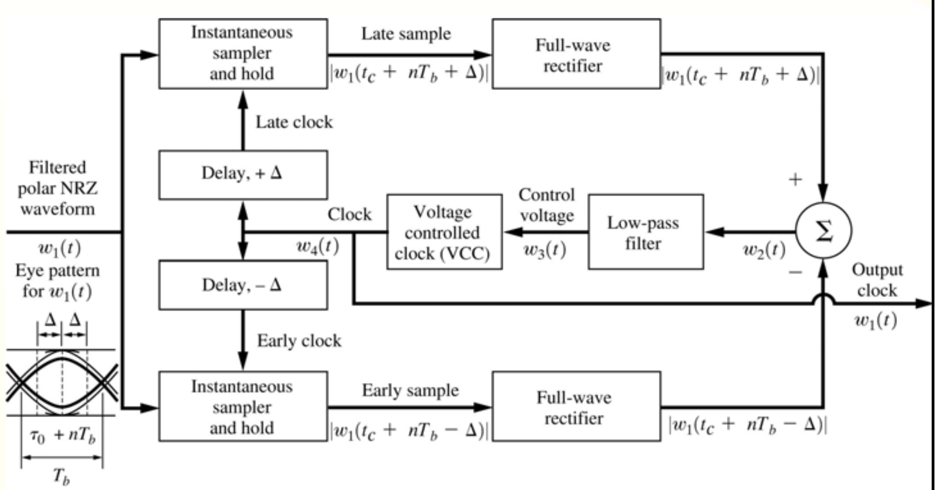


Figure 3–22 Binary-to-multilevel polar NRZ signal conversion.

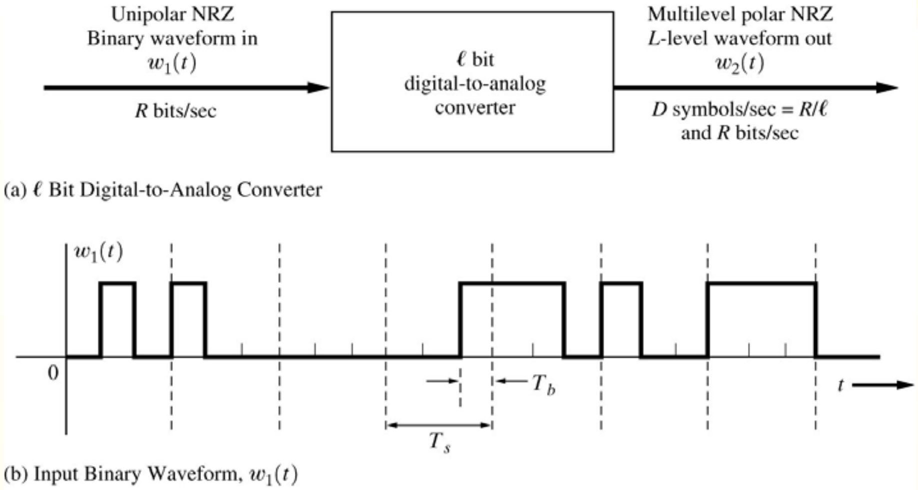
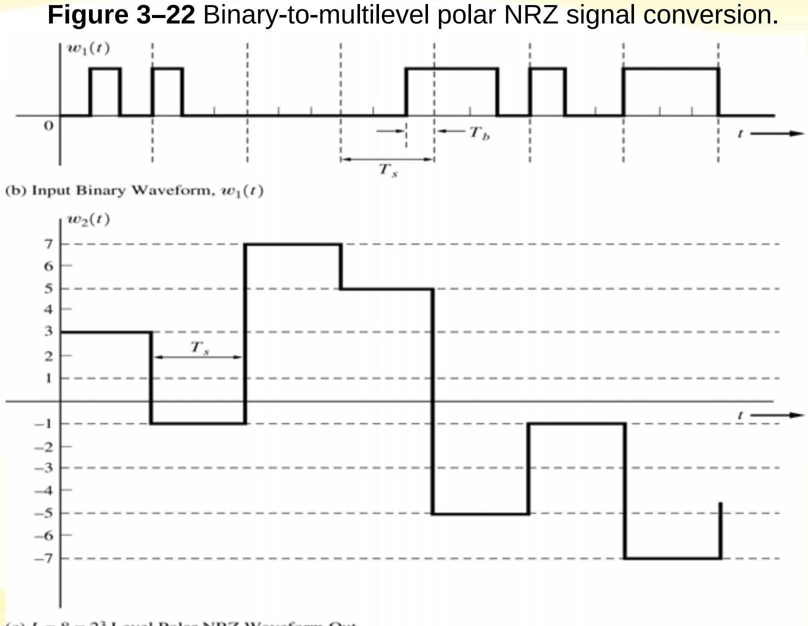


Figure 3–22 Binary-to-multilevel polar NRZ signal conversion.



Spectral Efficiency

Spectral Efficiency

Definition: The **spectral efficiency** of a digital signal is given by the number of bits per second that can be supported by each hertz of bandwidth.

$$\eta = \frac{R}{B} \quad in\ (bits/\sec)/Hz$$

where R is the data rate in bits per second and B is the bandwidth in Hz

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Spectral Efficiency Limits

$$\eta_{\max} = \frac{C}{B} = \log_2\left(1+\frac{S}{N}\right) \qquad (3-56)$$

for multilevel signaling :

$$\eta = \ell (bit/s)/Hz \quad \ell \text{ is the number} \quad (3-57)$$

of bits used in the A/D

- **Shannon's Law is the best we can do**
- We are a long way from this
- Multiple bits/symbol get us closer (***ℓ***- multilevel PCM)
- η is in (bits/sec)/Hz

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Spectral Efficiency

Table 3-6 SPECTRAL EFFICIENCIES OF LINE CODES

Code Type	First null Bandwidth (Hz)	Spectral Efficiency η=R/B
Unipolar NRZ	*R*	1
Polar NRZ	R	1
Unipolar RZ	2R	1/2
Bipolar RZ	R	1
Manchester NRZ	2R	1/2
Multilevel polar NRZ	R/l	l