FM- Frequency Modulation
PM - Phase Modulation

EELE445-14
Lecture 30

DSB-SC, AM, FM and PM

DSB-SC Complex Envelope: $g(t) = A_c\, m(t)$

AM Complex Envelope: $g(t) = A_c\,(1 + m(t))$

SSB-SC Complex Envelope: $g(t) = A_c[m(t) \pm j\hat{m}(t)]$

PM Complex Envelope: $g(t) = A_c\, e^{jD_p m(t)}$

FM Complex Envelope: $g(t) = A_c\, e^{jD_f \int_{-\infty}^{t} m(\sigma) d\sigma}$

FM and PM

$g(t) = R(t)e^{j\theta(t)} = A_c e^{j\theta(t)}$ *Complex Envelope*

$R(t) = |g(t)| = A_c$ *the real envelope is a constant*

$\rightarrow$ *power is constant*

Transmitted angle - modulated signal :

$$s(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right] = A_c \cos[\omega_c t + \theta(t)]$$

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FM and PM

Transmitted angle - modulated signal :

$$s(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right] = A_c \cos[\omega_c t + \theta(t)]$$

for PM : $\theta(t) = D_p m(t)$

$D_p \equiv$ *phase sensitivity or modulation constant* $\frac{rad}{volt}$

for FM : $\theta(t) = D_f \int_{-\infty}^{t} m(\sigma)d\sigma$

$D_f \equiv$ *frequency deviation or modulation constant* $\frac{rad}{volt - \sec}$

$D_f = 2\pi \frac{Hz}{volt}$

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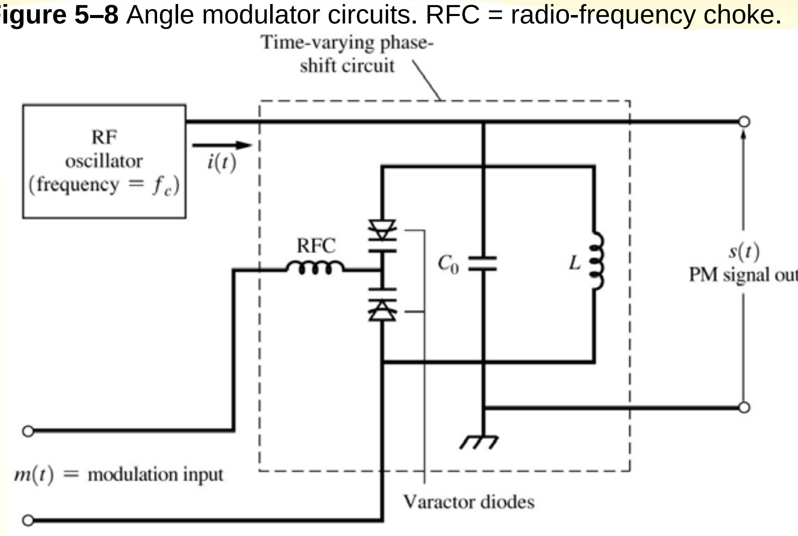
FM and PM

Relationship between $m_f(t)$ and $m_p(t)$:

$$m_f(t) = \frac{D_p}{D_f}\left[\frac{dm_p(t)}{dt}\right]$$

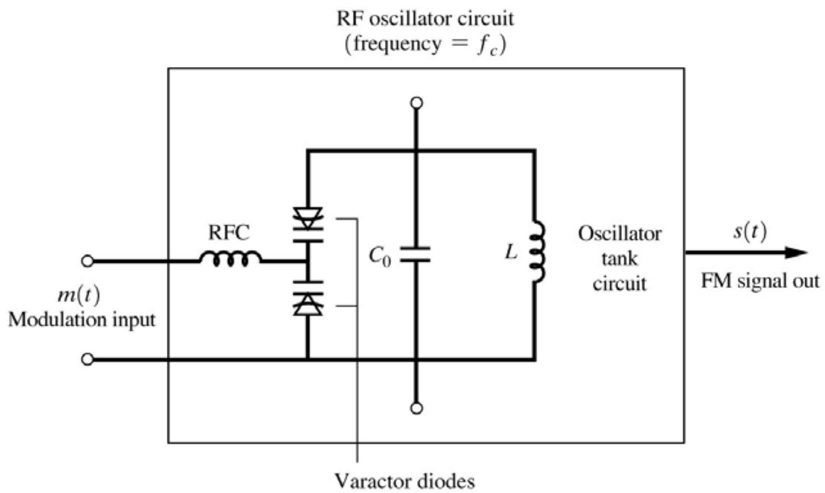
$$m_p(t) = \frac{D_f}{D_p}\int_{-\infty}^{t} m_f(\sigma)\,d\sigma$$

Figure 5–8 Angle modulator circuits. RFC = radio-frequency choke.



(a) A Phase Modulator Circuit

Figure 5–8 Angle modulator circuits. RFC = radio-frequency choke.



(b) A Frequency Modulator Circuit

Couch, Digital and Analog Communication Systems, Seventh Edition

Instantaneous Frequency

$$s(t) = R(t)\cos\psi(t)$$

$$= A_c \cos\psi(t) \quad FM \ or \ PM$$

$$= A_c \cos(\omega_c t + \theta(t))$$

The instantaneous frequency in Hz is :

$$f_i(t) = \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\left[\frac{d\psi(t)}{dt}\right] = f_c + \frac{\dot{\theta}(t)}{2\pi}$$

FM and PM differences

PM: instantaneous <u>phase</u> deviation of the carrier phase is proportional to the amplitude of m(t)

$$\theta(t) = D_p m(t) \;\; radians$$

$$D_p \;\; in \;\; \frac{radians}{volt}$$

- **Modulation Constant**
- **Modulation sensitivity**
- **Phase sensitivity**

Instantaneous phase in radians : $\psi(t) = \omega_c t + \theta(t) = \omega_c t + D_p m(t)$

Instantaneous frequency in Hz : $f_i(t) = \frac{d\psi(t)}{2\pi dt} = f_c + \frac{\dot{\theta}(t)}{2\pi} = f_c + \frac{D_p \dot{m}(t)}{2\pi}$

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FM and PM differences

FM: instantaneous frequency <u>deviation</u> from the carrier frequency is proportional to m(t)

$$\theta(t) = D_f \int_{-\infty}^{t} m(\alpha) d\alpha \;\; radians$$

$$D_f \;\; in \;\; \frac{radians}{volt - \sec}$$

The instantaneous phase in radians : $\psi(t) = \omega_c t + \theta(t) = \omega_c t + D_f \int_{-\infty}^{t} m(\alpha) d\alpha$

The instantaneous frequency in Hz : $f_i(t) = \frac{d\psi(t)}{2\pi dt} = f_c + \frac{\dot{\theta}(t)}{2\pi} = f_c + \frac{D_f m(t)}{2\pi}$

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FM and PM differences

FM: instantaneous **frequency deviation** from the carrier frequency is proportional to m(t)

$$f_d(t) \equiv f_i(t) - f_c = \frac{1}{2\pi}\dot{\theta}(t) = \frac{1}{2\pi} D_f m(t)$$

Modulation Constants

$$D_p = K_p \Rightarrow \frac{radians}{volt}$$

$$D_f = K_f \Rightarrow \frac{rad}{volt - \sec} = 2\pi \frac{Hz}{volt}$$

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FM

frequency deviation $\equiv f_d(t) = f_i(t) - f_c = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$

peak frequency deviation $\equiv \Delta F = \max\left\{\frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]\right\} = \frac{1}{2\pi} D_f V_p$

$V_p = \max[m(t)]$

frequency modulation index $\equiv \beta_f = \frac{\Delta F}{B}$ $\quad$ B is the bandwidth of m(t)

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PM and digital modulation

phase deviation $\equiv \theta(t)$

peak phase deviation $\equiv \Delta\theta = \max[\theta(t)] = D_p V_p$

$V_p = \max[m(t)]$

phase modulation index $\equiv \beta_p = \Delta\theta$

note:

when m(t) is a sinusoidal signal set such that the PM and FM signals have the same peak frequency deviation, then $\beta_p = \beta_f$

For Digital signals the modulation index:

$$h \equiv \frac{2\Delta\theta}{\pi}$$

where $2\Delta\theta$ is the pk-pk phase change in one symbol duration, T_s

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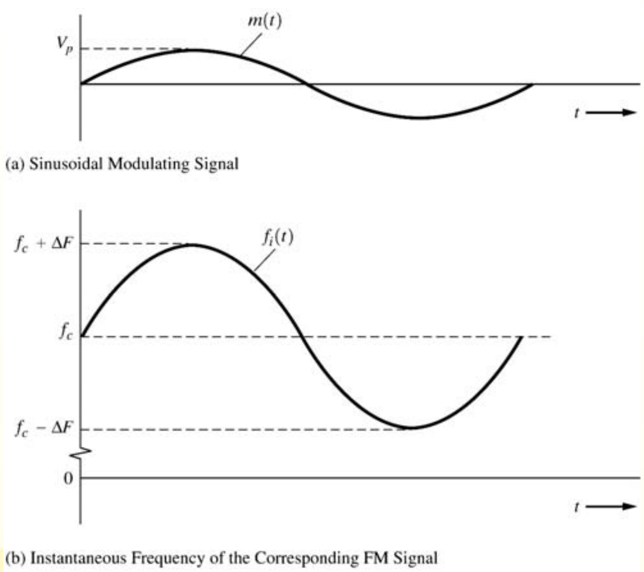
Figure 5–9 FM with a sinusoidal baseband modulating signal.



Couch, *Digital and Analog Communication Systems*, Seventh Edition

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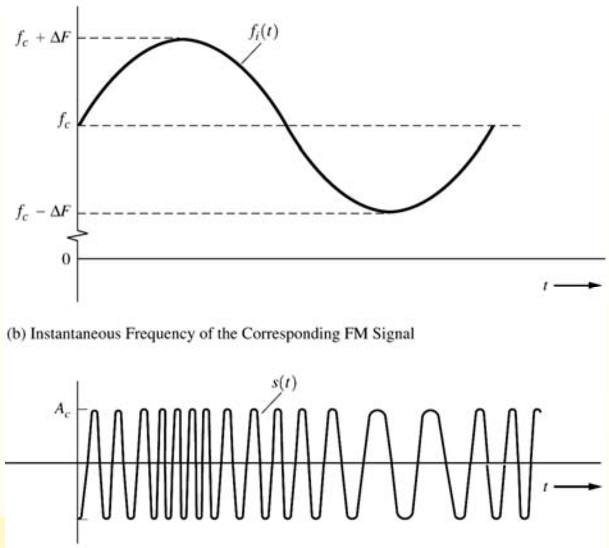
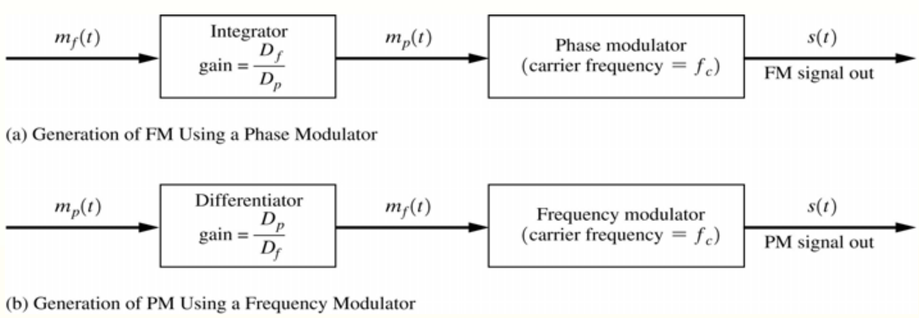
Figure 5–9 FM with a sinusoidal baseband modulating signal.

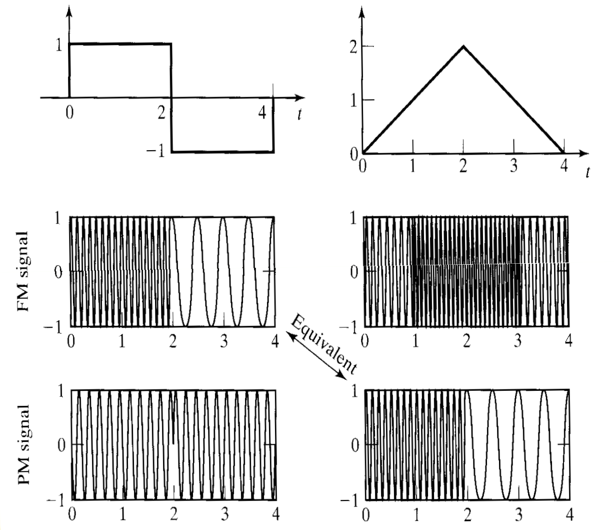


(c) Corresponding FM Signal

FM from PM and PM from FM



FM/PM s(t) waveforms



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FM and PM with m(t)=cos(2πf$_m$

Let $m(t) = a\cos(2\pi f_m t)$

For PM $\phi(t) = k_p m(t) = k_p a \cos(2\pi f_m t),$

For FM $$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau = \frac{k_f a}{f_m} \sin(2\pi f_m t).$$

$$u(t) = \begin{cases} A_c \cos\left(2\pi f_c t + k_p a \cos(2\pi f_m t)\right), & \text{PM} \\ A_c \cos\left(2\pi f_c t + \frac{k_f a}{f_m} \sin(2\pi f_m t)\right), & \text{FM} \end{cases}.$$

Define the modulation indices: $\beta_p = k_p a \qquad \beta_f = \frac{k_f a}{f_m},$

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FM and PM Signals

Define the modulation indices:

$$\beta_p = k_p a \qquad \beta_f = \frac{k_f a}{f_m},$$

$$\beta_p = k_p \max[|m(t)|] \qquad \beta_f = \frac{k_f \max[|m(t)|]}{W}$$

$$\beta_p = \Delta\phi_{\max}; \qquad \beta_f = \frac{\Delta f_{\max}}{W}.$$

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FM and PM Signals

Then

$$u(t) = \begin{cases} A_c \cos\left(2\pi f_c t + \beta_p \cos(2\pi f_m t)\right), & \text{PM} \\ A_c \cos\left(2\pi f_c t + \beta_f \sin(2\pi f_m t)\right), & \text{FM} \end{cases}.$$

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Spectrum Characteristics of FM

- FM/PM is exponential modulation

Let $\phi(t) = \beta \sin(2\pi f_m t)$

$$u(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \\ = \text{Re}\left(A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}\right)$$

u(t) is periodic in f_m

we may therefore use the Fourier series

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Spectrum Characteristics of FM

- FM/PM is exponential modulation

$$u(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \\ = \text{Re}\left(A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}\right)$$

u(t) is periodic in f_m

we may therefore use the Fourier series

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Spectrum with Sinusoidal Modulation

$$g(t) = e^{j\beta \sin(2\pi f_m t)}$$

u(t) is periodic in f_m
we may therefore use the Fourier series

$$c_n = f_m \int_0^{\frac{1}{f_m}} e^{j\beta \sin 2\pi f_m t} e^{-jn2\pi f_m t}\, dt$$
$$\stackrel{u=2\pi f_m t}{=} \frac{1}{2\pi} \int_0^{2\pi} e^{j\beta(\sin u - nu)}\, du.$$

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J_n Bessel Function

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}.$$

$$u(t) = \mathrm{Re}\left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right)$$
$$= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos\left(2\pi (f_c + n f_m) t\right).$$

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J_n Bessel Function

$$J_n(\beta) \approx \frac{\beta^n}{2^n n!}.$$

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}.$$

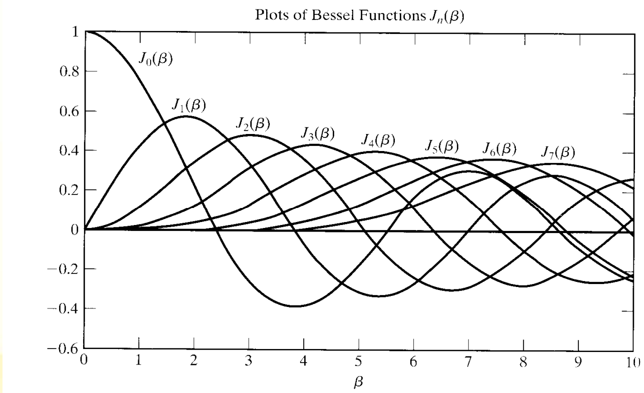


Figure 4.4 Bessel functions for various values of n.

TABLE 5–2 FOUR-PLACE VALUES OF THE BESSEL FUNCTIONS $J_n(\beta)$

TABLE 5–2 FOUR-PLACE VALUES OF THE BESSEL FUNCTIONS $J_n(\beta)$

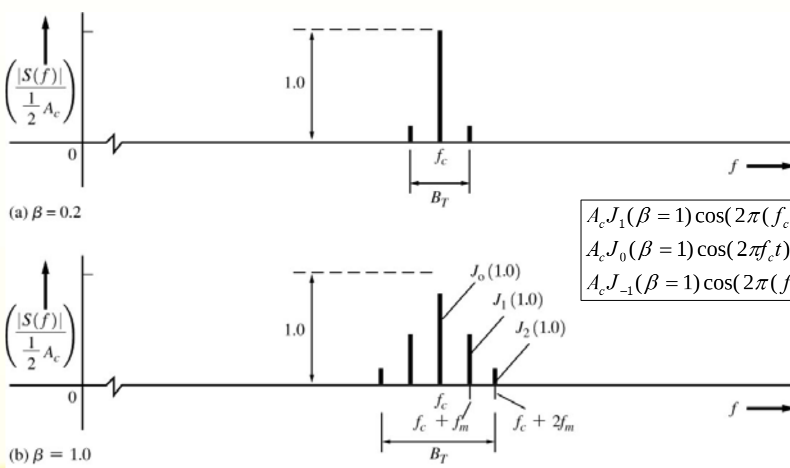
n \ β:	0.5	1	2	3	4	5	6	7	8	9	10
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.09033	−0.2459
1	0.2423	0.4401	0.5767	0.3391	−0.06604	−0.3276	−0.2767	−0.004683	0.2346	0.2453	0.04347
2	0.03060	0.1149	0.3528	0.4861	0.3641	0.04657	−0.2429	−0.3014	−0.1130	0.1448	0.2546
3	0.002564	0.01956	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809	0.05838
4		0.002477	0.03400	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655	−0.2196
5			0.007040	0.04303	0.1321	0.2611	0.3621	0.3479	0.1858	−0.05504	−0.2341
6			0.001202	0.01139	0.04909	0.1310	0.2458	0.3392	0.3376	0.2043	−0.01446
7				0.002547	0.01518	0.05338	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.004029	0.01841	0.05653	0.1280	0.2235	0.3051	0.3179
9						0.005520	0.02117	0.05892	0.1263	0.2149	0.2919
10						0.001468	0.006964	0.02354	0.06077	0.1247	0.2075
11							0.002048	0.008335	0.02560	0.06222	0.1231
12								0.002656	0.009624	0.02739	0.06337
13									0.003275	0.01083	0.02897
14									0.001019	0.003895	0.01196
15										0.001286	0.004508
16											0.001567

TABLE 5–3 ZEROS OF BESSEL FUNCTIONS: VALUES FOR β WHEN $J_n(\beta) = 0$

TABLE 5–3 ZEROS OF BESSEL FUNCTIONS: VALUES FOR β WHEN $J_n(\beta) = 0$

	Order of Bessel Function, n						
	0	1	2	3	4	5	6
β for 1st zero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β for 2nd zero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β for 3rd zero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β for 4th zero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β for 5th zero	14.93	16.47	17.96	19.41	20.83	22.21	23.59
β for 6th zero	18.07	19.61	21.12	22.58	24.02	25.43	26.82
β for 7th zero	21.21	22.76	24.27	25.75	27.20	28.63	30.03
β for 8th zero	24.35	25.90	27.42	28.91	30.37	31.81	33.23

Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.



$A_c J_1(\beta = 1)\cos(2\pi(f_c + 1f_1)t)$

$A_c J_0(\beta = 1)\cos(2\pi f_c t)$

$A_c J_{-1}(\beta = 1)\cos(2\pi(f_c - 1f_1)t)$

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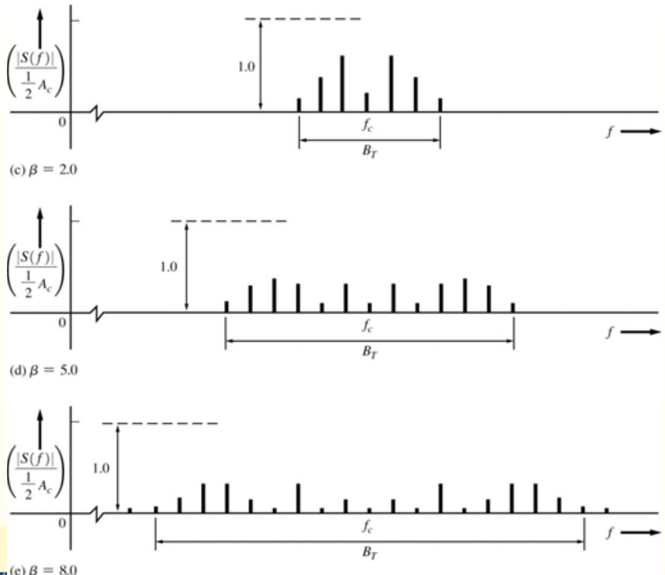

NBFM- Narrowband Frequency Modulation
WBFM - Wideband Frequency
Modulation Carson's Bandwidth Rule

EELE445-14

Lecture 31

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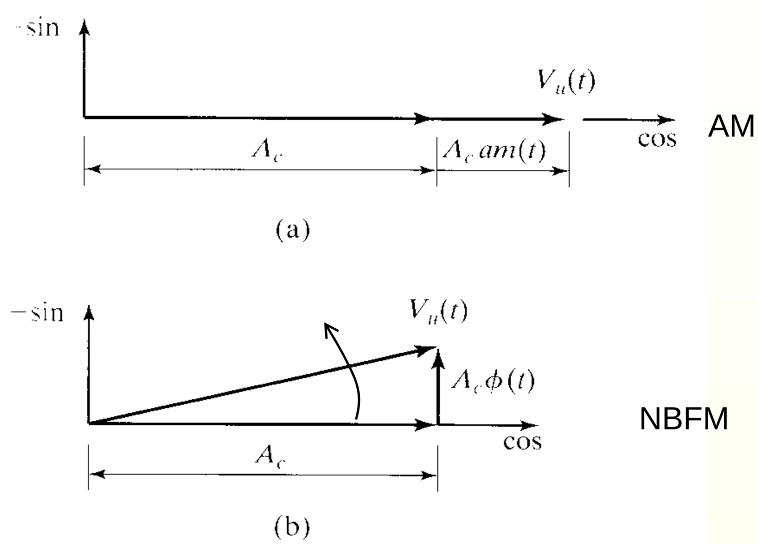
Narrowband FM

•Only the J_o and J_1 terms are significant
•Same Bandwidth as AM
•Using Eulers identity, and $\phi(t) << 1$:

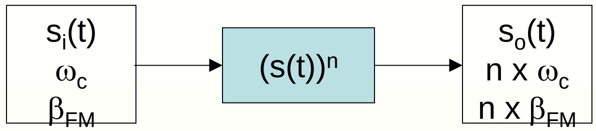
$$u(t) = A_c \cos 2\pi f_c t \cos\phi(t) - A_c \sin 2\pi f_c t \sin\phi(t)$$
$$\approx A_c \cos 2\pi f_c t - A_c \phi(t) \sin 2\pi f_c t,$$

Notice the sidebands are “sin”, not “cos” as in AM

Narrowband FM as a Phaser



Frequency Multiplication: Wideband FM from Narrowband FM



$$s_o(t) = \mathrm{Re}(e^{j\phi(t)}e^{j2\pi f_c t})^n = \mathrm{Re}(e^{jn\phi(t)}e^{j2\pi n f_c t})$$

$$n\phi(t) = nD_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

$$\beta_{fmout} = n\beta_{f\min}$$

- The Output Carrier frequency = n x f_c
- The output modulation index = n x β_{fm}
- The output bandwidth increases according to Carson's Rule

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Effective Bandwidth- Carson's Rule for Sine Wave Modulation

$$B_c = 2(\beta + 1) f_m,$$

Where β is the modulation index f_m is the sinusoidal modulation frequency

$$m(t) = a\cos(2\pi f_m t).$$

$$B_c = 2(\beta + 1) f_m = \begin{cases} 2(k_p a + 1) f_m, & \text{PM} \\ 2\left(\frac{k_f a}{f_m} + 1\right) f_m, & \text{FM} \end{cases}$$

- Notice for FM, if $k_f a \gg f_m$, increasing fm does not increase B_c much
- B_c is linear with f_m for PM

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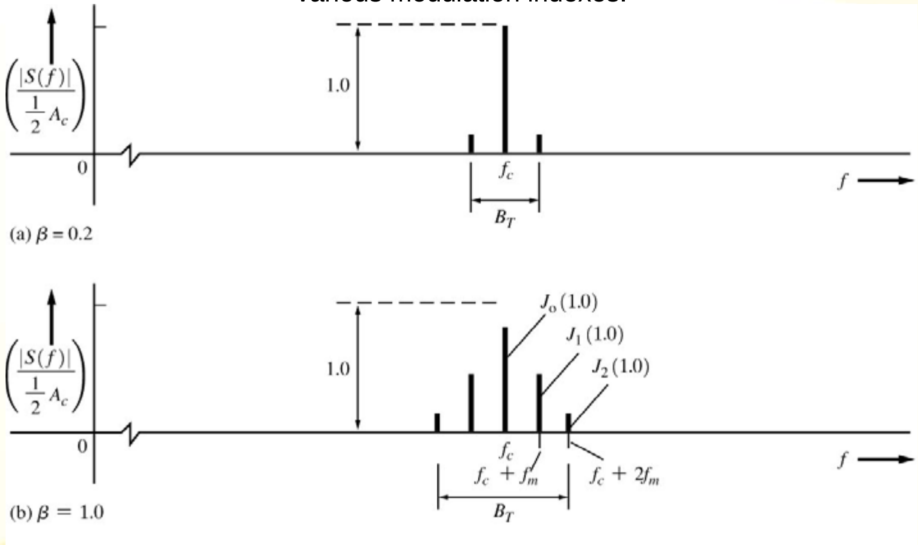
Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.



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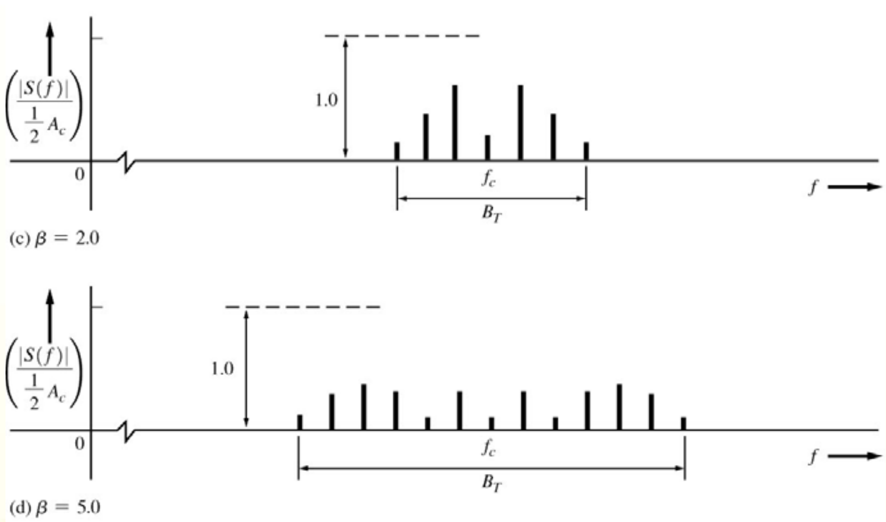

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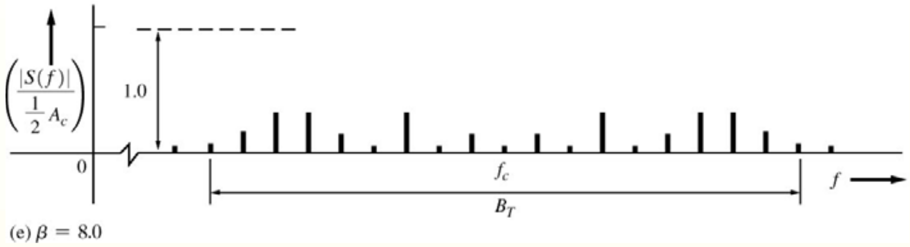
Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for Various modulation indexes.



When m(t) is a sum of sine waves

Consider now the case where $m(t)$ is the sum of K separate sine waves. That is, let

$$m(t) = \sum_{i=1}^{K} C_i \cos(\omega_i t + \theta_i) \tag{2.4.14}$$

where C_i, ω_i, and θ_i are the corresponding deviations, frequency, and phase angles.

When m(t) is a sum of sine waves

$$c(t) = A\cos\left[\omega_c t + \sum_{i=1}^{K}\beta_i \sin(\omega_i t + \theta_i) + \psi\right]$$

$$c(t) = A\sum_{k_K=-\infty}^{\infty}\cdots\sum_{k_1=-\infty}^{\infty}\left[\prod_{i=1}^{K} J_{k_i}(\beta)\right]\cos\left[\omega_c t + \sum_{i=1}^{K} k_i(\omega_i t + \theta_i) + \psi\right] \quad (2.4.16)$$

The preceding equation represents the general expression for the FM carrier modulated by K sinusoids. Note that it corresponds to a collection of harmonic frequencies at all the sidebands $\Sigma_{i=1}^{K} k_i\omega_i$, where all combinations of integers for the $\{k_i\}$ must be considered. Each such combination $\{k_1, k_2, \ldots, k_K\}$ yields a different sinusoid, each with its own phase, $\Sigma_{i=1}^{K}(k_i\theta_i + \psi)$ and its own amplitude $\{A\,\Pi_i^K J_{k_i}(\beta_i)\}$. In particular, we note that the carrier component at ω_c corresponds to $k_1 = k_2 = \cdots = k_K = 0$ and has amplitude $\{A\,\Pi_i^K J_0(\beta_i)\}$, whereas the frequency component at frequency $(\omega_c + \omega_j)$ corresponds to $k_1 = 0, k_2 = 0, \ldots, k_j = 1, k_{j+1} = 0, \ldots, k_K = 0$ and has amplitude $\{AJ_1(\beta_j)\,\Pi_{i=1,i\neq j}^{K} J_0(\beta_i)\}$. We also note that the component at $(\omega + \omega_j)$ contains the exact phase angle of the jth sine wave in (2.4.14) added to that of the carrier.

Minds

Sideband Power

Signal Amplitude: $A_c := 1V$

Modulating frequency: $f_m := 1KHz$

Carrier peak deveation: $\Delta f := 2.4KHz$

Modulation index: $\beta := \frac{\Delta f}{f_m}$ $\quad \beta = 2.4$

Reference equation: $x(t) = \sum_{n=-\infty}^{\infty}\left[A_c \cdot Jn(n,\beta)\cdot\cos\left[\left(\omega_c + n\cdot\omega_m\right)\cdot t\right]\right]$

Power in the signal: $P_c := \frac{A_c^{\,2}}{2\cdot 1\Omega}$ $\quad P_c = 0.5\ W$

Carsons rule bandwidth: $BW := 2\cdot(\beta + 1)\cdot f_m$ $\quad BW = 6.8\times 10^3\ \frac{1}{s}$

Order of significant sidbands predicted by Carsons rule: $n := \text{round}(\beta + 1)$ $\quad n = 3$

Power as a function of number of sidebands: $P_{sum}(k) := \sum_{n=-k}^{k}\frac{\left(A_c\cdot Jn(|n|,\beta)\right)^2}{2\cdot 1\Omega}$

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Percent of power predicted by Carsons rule: $\frac{P_{sum}(n)}{P_c}\cdot 100 = 99.118$

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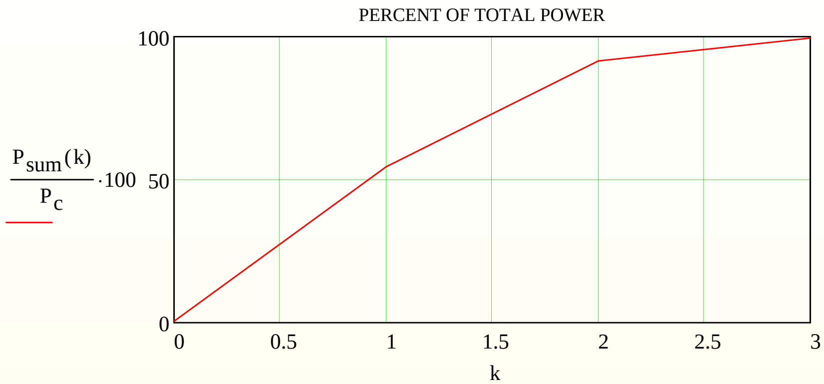
Power vs Bandwidth



Sideband Power β=2.4

$k := 0 .. 10$

$J_k := Jn(k, \beta)$

$P_k := (J_k)^2$

$\beta = 2.4$

$n = 3$

J =

	0
0	$2.508 \cdot 10^{-3}$
1	0.52
2	0.431
3	0.198
4	0.064
5	0.016
6	$3.367 \cdot 10^{-3}$
7	$5.927 \cdot 10^{-4}$
8	$9.076 \cdot 10^{-5}$
9	$1.23 \cdot 10^{-5}$
10	$1.496 \cdot 10^{-6}$

P =

	0
0	$6.288 \cdot 10^{-6}$
1	0.271
2	0.186
3	0.039
4	$4.135 \cdot 10^{-3}$
5	$2.638 \cdot 10^{-4}$
6	$1.134 \cdot 10^{-5}$
7	$3.513 \cdot 10^{-7}$
8	$8.237 \cdot 10^{-9}$
9	$1.513 \cdot 10^{-10}$
10	$2.238 \cdot 10^{-12}$

$$P_0 + 2 \cdot \sum_{j=1}^{n} P_j = 0.991$$

Sideband Power β=0.1

$j := 0..5$ $\beta := 0.1$ $n := 1$

$V_j := Jn(j, \beta)$ $U_j := (V_j)^2$

$$V = \begin{pmatrix} 0.998 \\ 0.05 \\ 1.249 \times 10^{-3} \\ 2.082 \times 10^{-5} \\ 2.603 \times 10^{-7} \\ 2.603 \times 10^{-9} \end{pmatrix} \quad U = \begin{pmatrix} 0.995 \\ 2.494 \times 10^{-3} \\ 1.56 \times 10^{-6} \\ 4.335 \times 10^{-10} \\ 6.775 \times 10^{-14} \\ 0 \end{pmatrix} \quad U_0 + 2 \cdot \sum_{j=1}^{n} U_j = 1$$

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Sideband Power β=0.6

$\beta := 0.6$ $n := 1$

$W_j := Jn(j, \beta)$ $X_j := (W_j)^2$

$$W = \begin{pmatrix} 0.912 \\ 0.287 \\ 0.044 \\ 4.4 \times 10^{-3} \\ 3.315 \times 10^{-4} \\ 1.995 \times 10^{-5} \end{pmatrix} \quad X = \begin{pmatrix} 0.832 \\ 0.082 \\ 1.907 \times 10^{-3} \\ 1.936 \times 10^{-5} \\ 1.099 \times 10^{-7} \\ 3.979 \times 10^{-10} \end{pmatrix} \quad X_0 + 2 \cdot \sum_{j=1}^{n} X_j = 0.996$$

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filename: fmsidebands.mcd
avo 09/21/04
last edit date:2/27/07

FM/PM modulation index set to π/2 for peak phase dev of π/2
set to Δf/fm for frequency modulation. spectrum is the same for sinewave modulation.

$A_c := 1$

$f_c := 0 \cdot 10^4$

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$Fm := 10^0$ Modulating frequency- single sinewave

$M := \frac{x}{10}$

$n := \text{round}(M + 1)$ 2 * n is the number of significant sidebands per Carsons rule

$n = 9$ Bandwidth:= $2 \cdot n \cdot Fm$ Modulation_index= M

$$Si(f) := A_c \cdot \left[(J0(M)) \cdot \delta(f, f_c) + \sum_{k=1}^{n} \left[Jn(k, M) \cdot \delta[f, (f_c + k \cdot Fm)] + (-1)^k \cdot Jn(k, M) \cdot \delta[f, (f_c - k \cdot Fm)] \right] \right]$$

$$B(f) := \delta[f, f_c + (n+0) \cdot Fm] + \delta(f, f_c - n \cdot Fm) \qquad f := f_c - (n+1) \cdot Fm, (f_c - n \cdot Fm) .. [f_c + (n+1) \cdot Fm]$$

Bandwidth= 18 Modulation_index= 7.9

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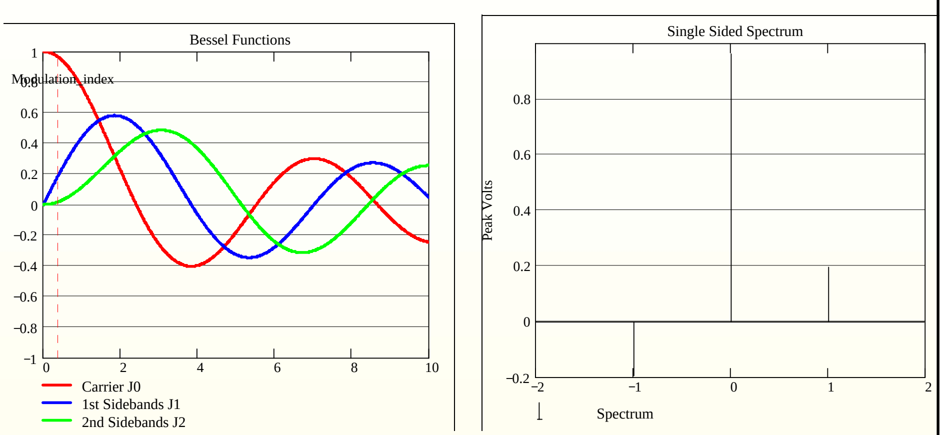
β=.4, Sideband Level =β/2 for Narrowband FM



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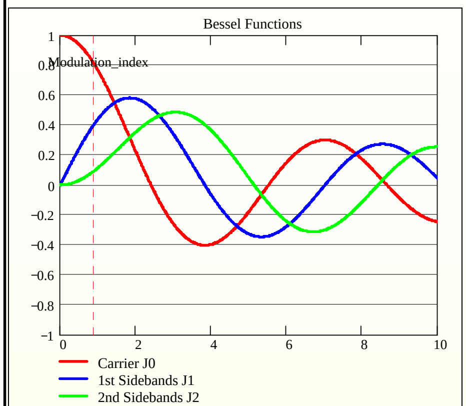
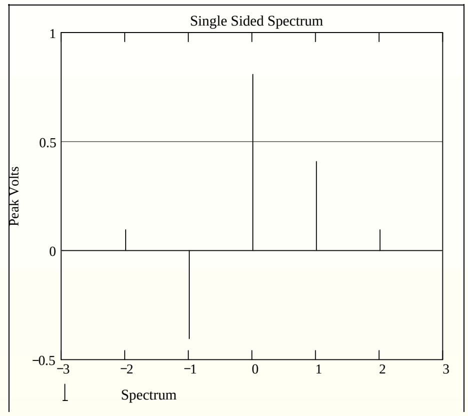
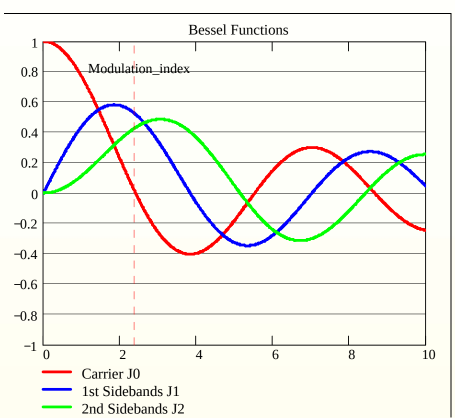
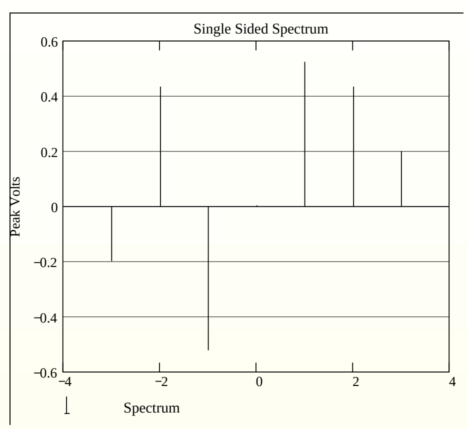
β=.9, Sideband Level =β/2 for Narrowband FM



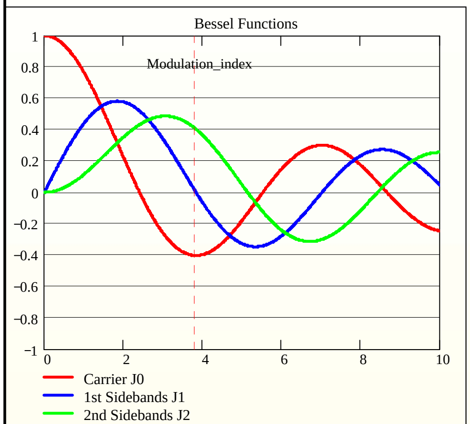


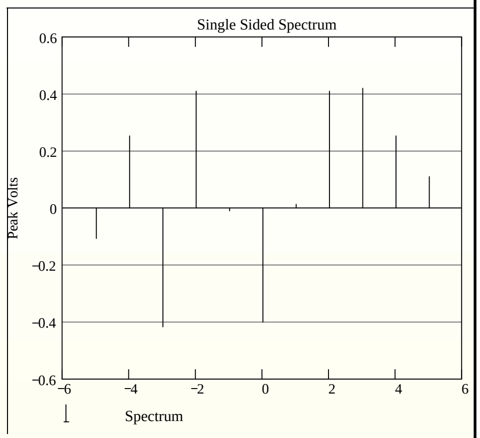
β=2.4, Carrier Null



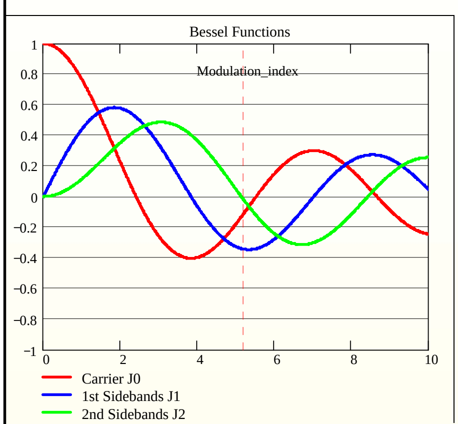


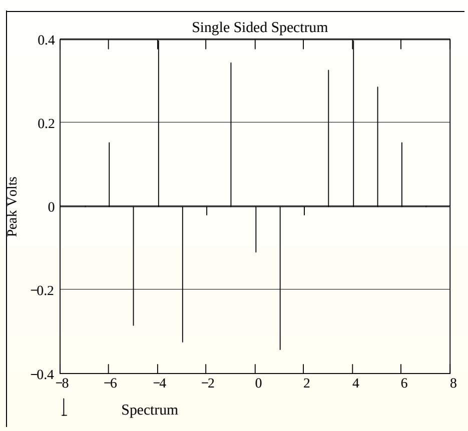
β=3.8, first sideband null





β=5.1, second sideband null





Power vs BW, β=0.1

$$P(M,n) := \left(\frac{J0(M)^2}{2} + \sum_{k=1}^{n} Jn(k,M)^2 \right)$$

second term includes power in +Jn and power in -Jn, i.e the upper and lower sideband pairs

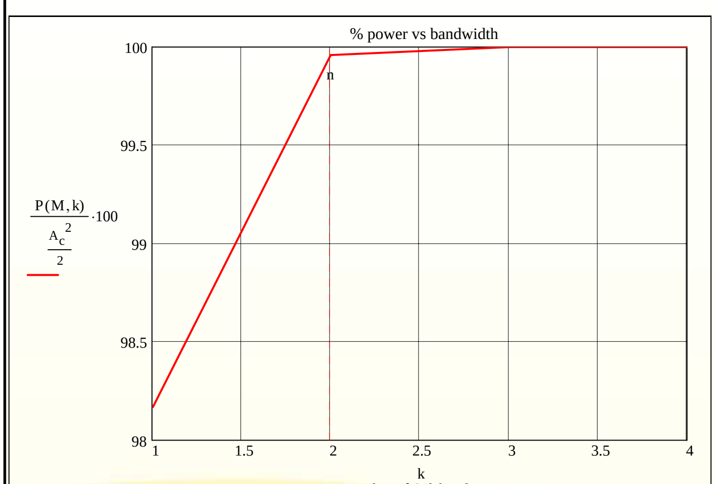
M = 0.1

Fm = 1 Hz

Bandwidth= 2 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}} \cdot 100 = 100$$

Power vs BW, β=0.9



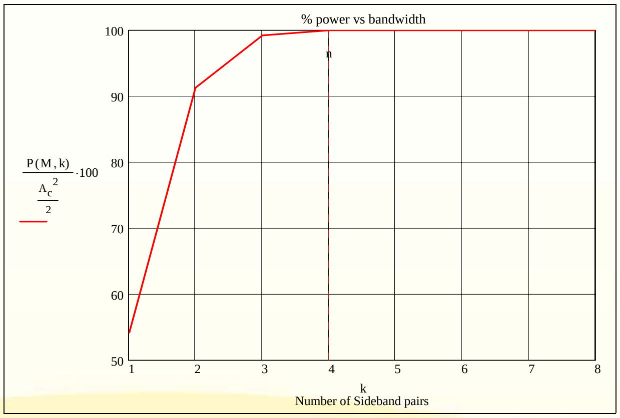
M = 0.9

Fm = 1 Hz

Bandwidth= 4 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}} \cdot 100 = 99.958$$

Power vs BW, β=2.4



M = 2.4

Fm = 1 Hz

Bandwidth = 8 Hz

$$\frac{P(M,n)}{\frac{A_c^2}{2}}\cdot 100 = 99.945$$