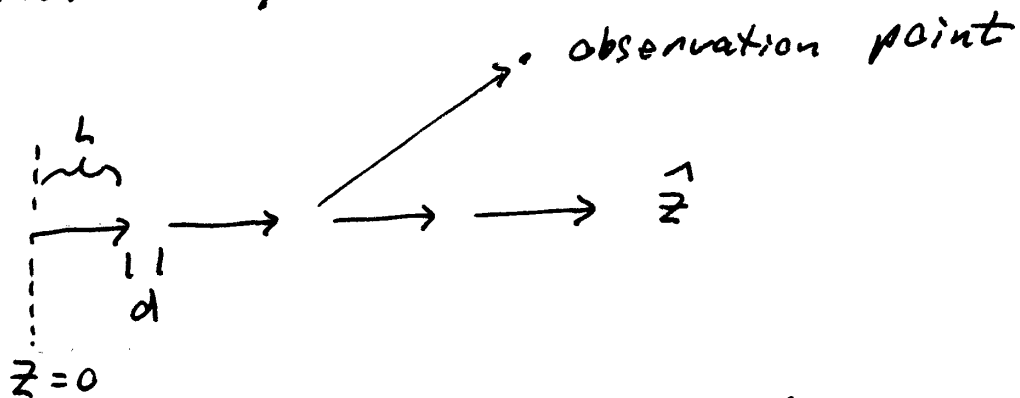


EE533-15

Array antennas: (chapter 6.1-6.3 intro)

Consider a co-linear arrangement or "linear" array of dipoles. (special case)

 $\Rightarrow$ find the vector potential for the array

$$\text{1st element: } A_z = \frac{e^{-jkr}}{4\pi r} L I_0$$

using superposition.

$$A_z = \frac{e^{-jkr}}{4\pi r} L \left[I_0 + I_1 e^{jk d \cos \theta} + I_2 e^{j2kd \cos \theta} + \dots \right]$$

$$A_z = \frac{e^{-jkr}}{4\pi r} L \sum_{n=0}^{N-1} I_n e^{jKnd \cos \theta}$$

N is the total number of Array elements.

EE533-15/16.

$$A_r \approx 0, A_\phi = 0, A_\theta = -\sin\theta A_z$$

$$E_\theta = -j k \eta A_\theta = \frac{j k \eta}{4\pi r} e^{-jkr} \sum_{n=0}^{N-1} I_n e^{j k n d \cos\theta}$$

So the normalized pattern can be written as:

$$F(\theta, \phi) = \underbrace{g_a(\theta, \phi)}_{\text{element factor}} \underbrace{f(\theta, \phi)}_{\text{Array Factor.}}$$

$$F(\theta, \phi) = \sin\theta \sum_{n=0}^{N-1} I_n e^{j k n d \cos\theta}$$

(with proper normalization)

Lecture 16 10/9/08

we can feed each element with different phase and magnitude.

$$I_n = I_0 e^{j\alpha n} \quad \alpha \text{ is the phase progression.}$$

$$\begin{array}{ccc} \xrightarrow{0} & \xrightarrow{\alpha} & \xrightarrow{2\alpha} \\ n=0 & n=1 & n=2 \end{array} \quad \hat{z}$$

EE533-16

$$\sum_{n=0}^{N-1} I_n e^{jkn d \cos \theta} = I_0 \sum_{n=0}^{N-1} e^{jn\psi}$$

where $\psi = kd \cos \theta + \alpha$ (note: our text uses $\psi = kd \cos \theta + \beta$)

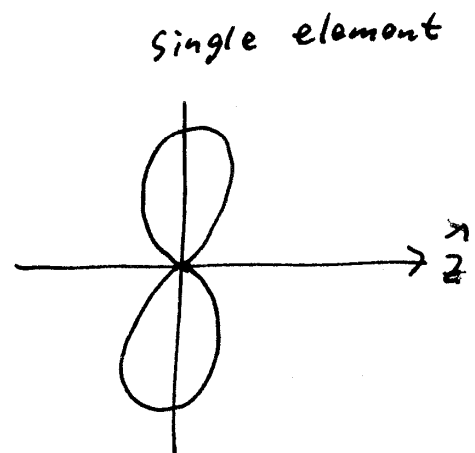
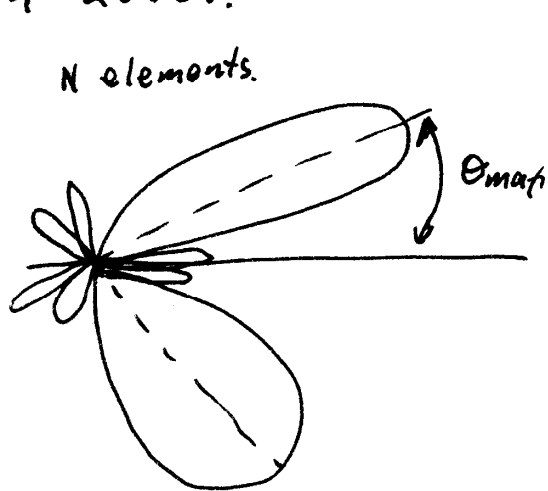
(see eq 6-7 text)

$$F(\theta, \phi) = g_a(\theta, \phi) f(\theta, \phi)$$

$$f(\theta, \phi) = f(\theta) = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \rightarrow \text{uniform spacing on } z \text{ axis.}$$

d is typically $\frac{\lambda}{2}$

For Large N side lobes appear with the main lobes:



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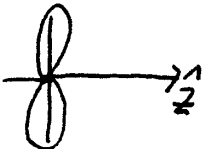
Direction of the peak value occurs

when $\psi = 0 \Rightarrow kd \cos \theta_{max} + \alpha = 0$

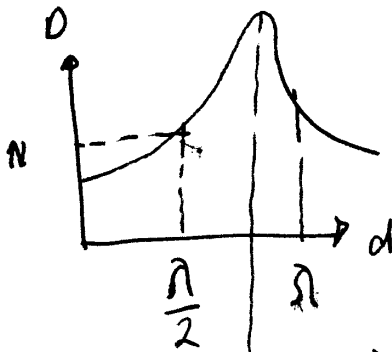
$$k = \frac{2\pi}{\lambda} \Rightarrow \theta_{max} = \cos^{-1} \left[\frac{-\alpha}{2\pi d/\lambda} \right]$$

Summary

- ① when N increases, directivity goes up.
- ② " " " , side lobe level goes down
($\propto \frac{1}{N}$)
- ③ as d increases, the directivity increases up to a certain point

$\alpha = 0$ is a "broadside pattern" $\Rightarrow$ 

(see section 6.31)



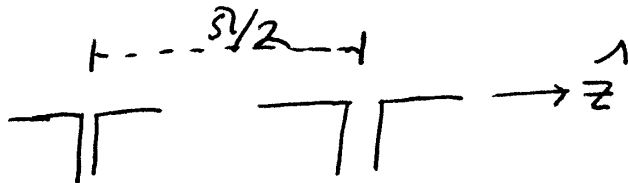
(see fig 6.6, 6.7)

additional side lobes appear.

EE 533-17. 10/10/08

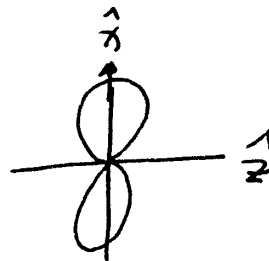
Orientation of the array elements has to be considered.

Example: Consider 2-element array:



for the above:

Element factor

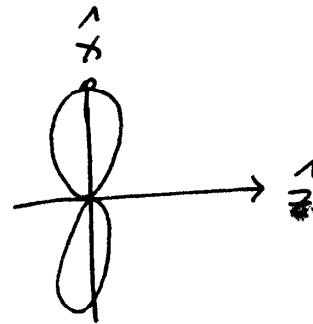


Array factor

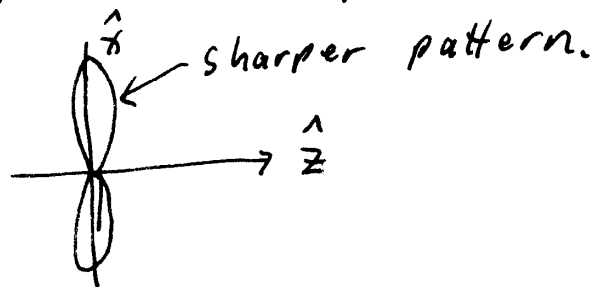
$$N=2 \quad d=\frac{\Omega}{2}$$

$$kd = \frac{2\pi}{\Omega} d = \pi$$

$$f(\theta) = \frac{\sin(\pi \cos \theta)}{\sin\left(\frac{\pi}{2} \cos \theta\right)}$$

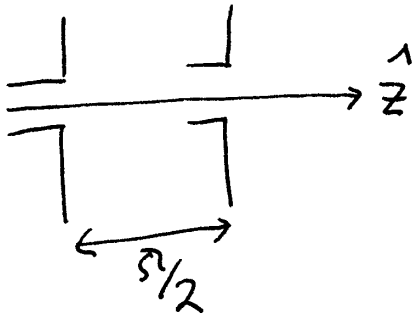


multiply for total pattern.

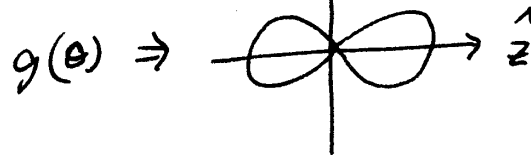


EE533-17

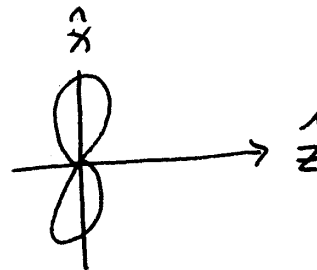
now turn the 1 elements



the element

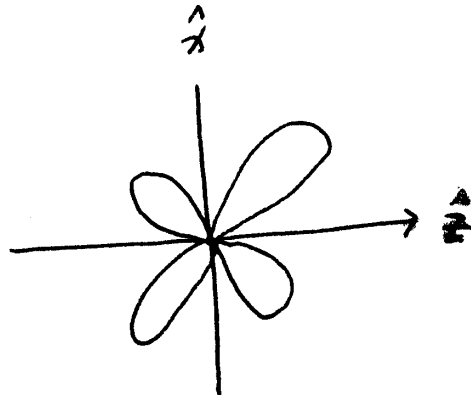


the array factor
is the same.



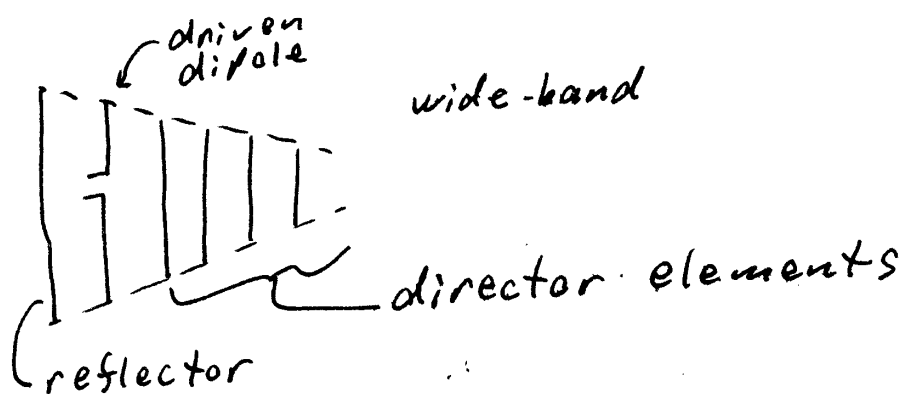
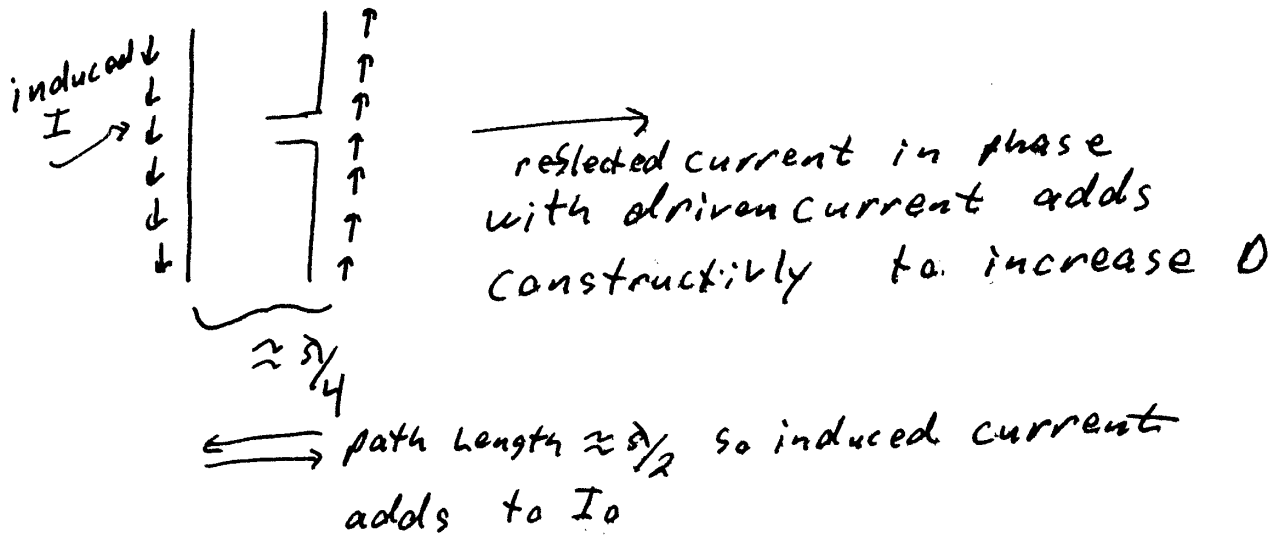
So the total pattern

is now $F(\theta) = g(\theta)f(\theta)$



EE533-17

variations on the dipole (original work done in the 1920's!)



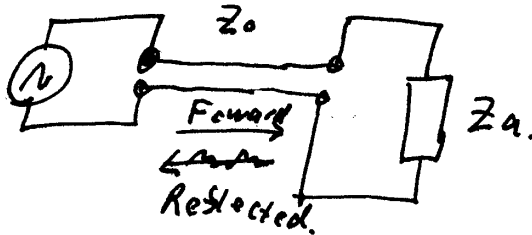
called Yagi-Uda Antenna. (see text)

Directivity $> 10\text{dB}$

Uda did the work - student, but Yagi was his teacher $\rightarrow$ Japan custom to credit the teacher!

EE533-18.

Antenna matching and Feed Lines.



To connect a transmitter or Receiver to the antenna we use a transmission Line. Small Antennas may use microstrip Line on the pcb $\rightarrow$ but we still have a transmission Line.

The Reflection coef. is.

$$\Gamma = \frac{Z_a - Z_0}{Z_a + Z_0}$$

$0 \leq |\Gamma| \leq 1 \leftarrow$ all power reflected $\rightarrow$ non radiated.
 $\uparrow$ ideal - no reflection loss

~~Antennas are specified:~~

Antenna match is usually specified by Return loss: $RL = -20 \log |\Gamma|$ or VSWR:

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

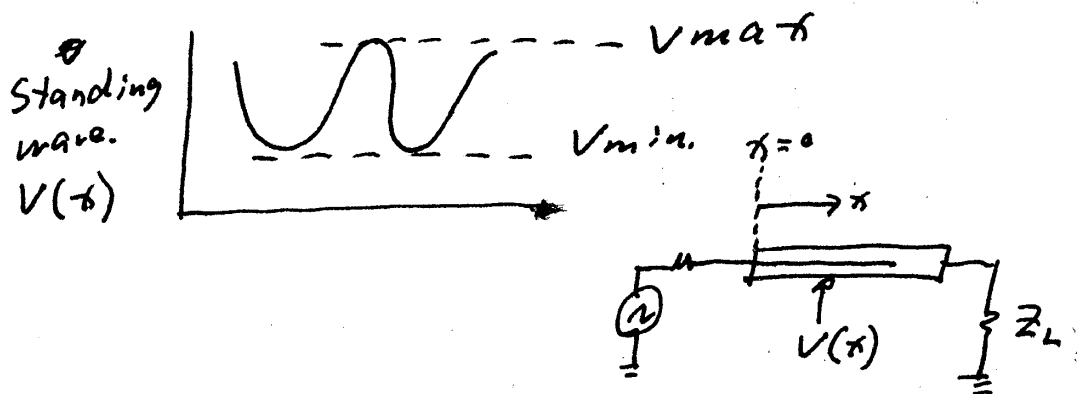
$$1 \leq VSWR \leq \infty$$

↳ total reflection.

↳ ideal

Transmitter antennas, $VSWR < 1.5$
 Receive antennas, $VSWR < 2.0$.

$VSWR \Rightarrow$ "Voltage standing wave Ratio" on the line.



$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{V_{max}}{V_{min}}$$

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

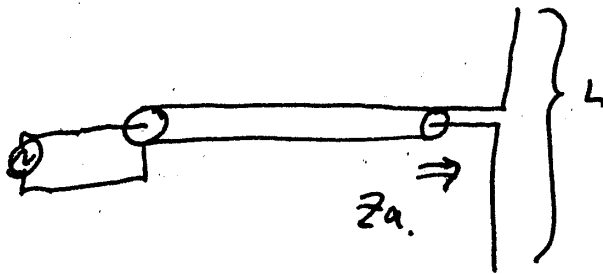
$|\Gamma|^2$ is the fraction of incident power reflected from the input of the Antenna $\Rightarrow$ wasted power!

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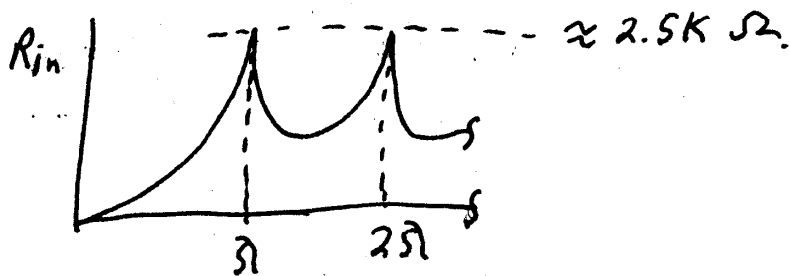
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$V_{SWR} = 2 \Rightarrow \Gamma = \frac{1}{3} \Rightarrow 11\% \text{ of power}$
 is not radiated $\rightarrow$
 it is reflected back
 down the transmission
 line.

example - real dipole. - see plots in text.



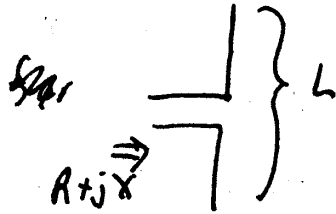
$$Z_a = R + jX.$$



Similar plot for X

EES33-18.

for dipoles: (see 8.62 in text)

Empirical Expression for R .for
 $0 \leq L < \frac{\lambda}{4}$

$$R = 20\pi^2 \left(\frac{L}{\lambda}\right)^2$$

uniform
current
infinitesimal
dipole.

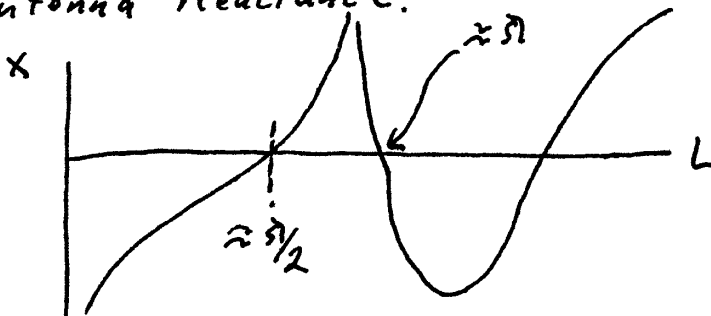
 $\frac{\lambda}{4} \leq L < \frac{\lambda}{2}$

$$R = 24.7 \left(\frac{\pi L}{\lambda}\right)^{2.4}$$

short
dipole.
 $\rightarrow I$

$$\frac{\lambda}{2} \leq L \leq 0.637\lambda \quad R = 11.14 \left(\frac{\pi L}{\lambda}\right)^{4.17}$$

antenna Reactance.

Element diameter effects the
length where $X=0$

533-18

 $\longleftrightarrow 2a$ Lengths for $j\chi = 0$:

$$\text{very thin} \Rightarrow \frac{L}{2a} = 5000 \Rightarrow L = 0.49 \lambda$$

$$\text{Thick: } \frac{L}{2a} = 10 \Rightarrow L = 0.455 \lambda$$

$$\text{Thin: } \frac{L}{2a} = 50 \Rightarrow L = 0.475 \lambda$$

see the text for the formal treatment
of input Z ----

"Wire" antennas are often made from

tubing $\rightarrow$ (check out a TV Antenna)

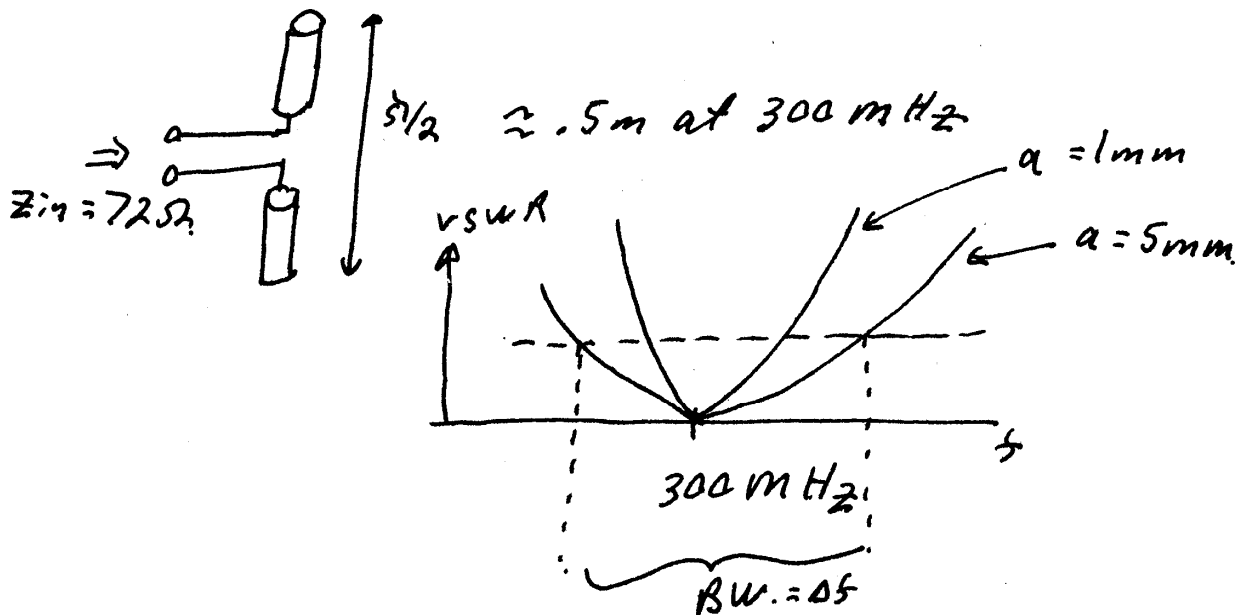
$\rightarrow$ light weight + larger "a" $\Rightarrow$ wider BW

EE533-19.

Bandwidth $\Rightarrow$ thick rods give wider BW.

BW $\triangleq$ frequency range over such that $V_{SWR} < 2:1$, i.e. $|r|^2 < 11\%$ power loss.

example $\rightarrow$ dipole.



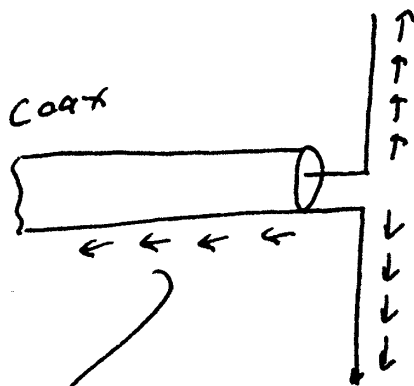
$$a = 0.005 \rightarrow \frac{L}{2a} = 50 \rightarrow \Delta f = 48\text{ MHz}.$$

$$\text{Bandwidth} = \frac{\Delta f}{f_0} = \frac{48}{300} = 16\%$$

EE533-19

matching Balun.

Problem: coax feed to a balanced ant.



induced currents on shield of coax
results in the currents being unbalanced.

this is corrected using a "Balun"
"Balanced to unbalanced"

