EE 533 Lec 3 9/8/08

Maxwell diff forms.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ → Faraday's Law

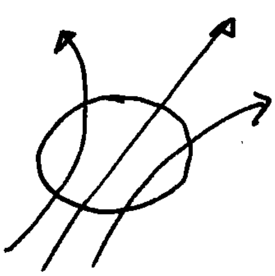
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$ – Amperes Law

$$\varepsilon_0 \vec{E} = \vec{D}$$

$\nabla \cdot \vec{B} = 0$ no magnetic Monopole

$\nabla \cdot \vec{D} = \rho$ Gauss' Law

Integral Forms.



$$\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi}{dt} \qquad \Phi = \int \vec{B} \cdot d\vec{s}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed} + \underbrace{\frac{d}{dt}\int \vec{D} \cdot d\vec{s}}_{\text{displacement current}}$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{D} \cdot d\vec{s} = Q_{enclosed}$$

usually interested in AC i.e. Harmonic excitation

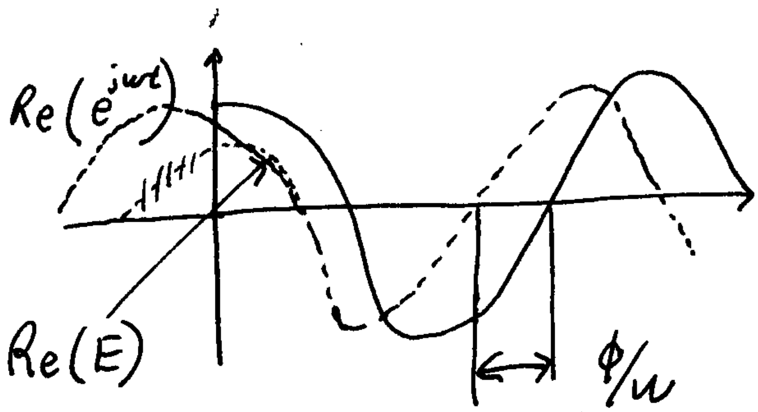
$\vec{E}, \vec{H} \propto e^{j\omega t}$ from Euler's Identity

$$e^{jx} = \cos x + j\sin x$$

$$E = E_0 \, e^{j\omega t}$$

$E_0 \rightarrow$ complex $E = |E_0|\, e^{j\phi}$

$$E = |E_0|\, e^{j\phi} e^{j\omega t} = |E_0|\, e^{j\omega(t+\phi/\omega)}$$

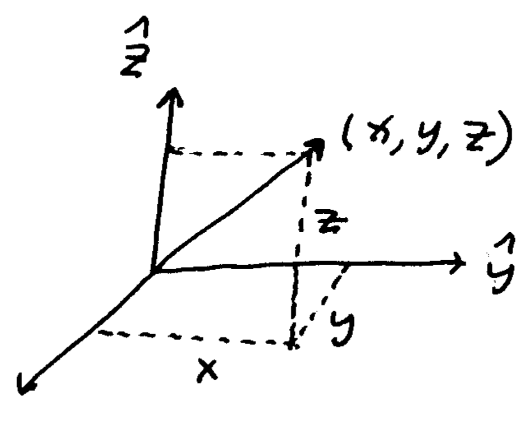


using Harmonic Excitation:

$$\vec{E}(\bar{r},t) = \vec{E}(\bar{r})\, e^{j\omega t}$$

$$H(\bar{r},t) = \vec{H}(\bar{r})\, e^{j\omega t}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$



$$\frac{\partial \vec{E}}{\partial t} = j\omega\vec{E}$$

$$\frac{\partial \vec{H}}{\partial t} = j\omega\vec{H}$$

then

$$\nabla \times \vec{E} = -j\omega H \qquad \nabla \times \vec{H} = j\omega\varepsilon \vec{E} + \vec{J}$$

$$\vec{B} = \mu H \qquad \vec{D} = \varepsilon \vec{E}$$

μ_i = permeability

ε_i = permitivity

Let $\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$

divergance $\nabla \cdot \vec{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$ scaler

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

Current within a Conductor:

Current density $\vec{J} = \sigma \vec{E}$

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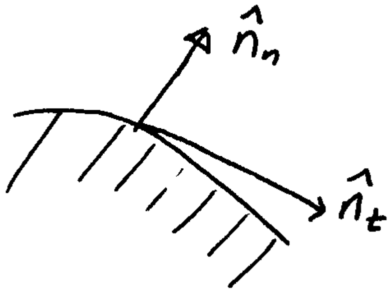
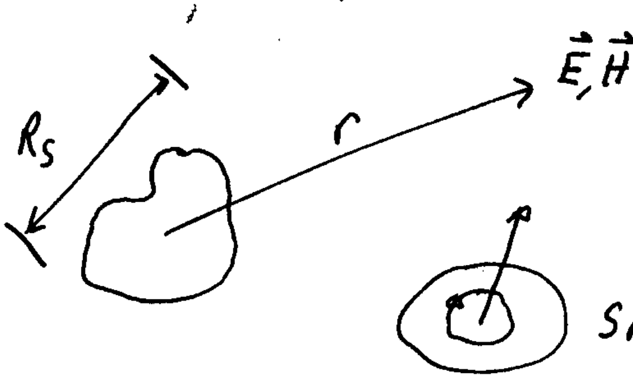
Boundry Conditions of Conducting Surfaces.



$\vec{E}_t = 0$ tangential component of E vanishes.

$\vec{H}_t = \vec{J}_s$ surface charge density

Radiation.



For $R_S << r$

$$E, H \propto \frac{1}{r} e^{-jKr}$$

spherical wave.

Poynting Theorem

$\vec{P} = \vec{E} \times \vec{H}$ $\qquad \hat{P} = \frac{\vec{P}}{|\vec{P}|}$ direction of Energy flow

$|\vec{P}|$ = power density → our Book uses $\vec{W} \Leftrightarrow \vec{P}$

$$W_{av} = \frac{1}{2} Re [\vec{E} \times \vec{H}^*) \quad w/m^2$$

in far field $W_{av} \approx \frac{1}{2} | \vec{E} \times \vec{H} |$

reactive part is ≈ 0

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The $\frac{1}{2}$ is from $\vec{E} = E_0 \hat{x}\, e^{j\omega t} e^{-k_x x}$

average. ~~rms~~ of a sine wave. – ie remember rms?

Lecture 4 Maxwell in free space.

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \qquad \vec{E}, \vec{H} \propto e^{j\omega t}$$

$$\vec{B} = \mu \vec{H}$$

$$= -j\omega\mu_0 \vec{H}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = j\omega\varepsilon_0 \vec{E}$$

$$\nabla \cdot \vec{D} = 0, \quad \nabla \cdot \vec{E} = 0, \quad \nabla \cdot \vec{B} = 0, \quad \nabla \cdot \vec{H} = 0$$

Curl of Both sides:

$$\nabla \times \nabla \times \vec{E} = -j\omega\mu_0 \nabla \times \vec{H} \quad \text{use}$$

$$\nabla(\cancelto{0}{\nabla \cdot \vec{E}}) - \nabla^2 \vec{E} = \omega^2 \mu_0 \varepsilon_0 \vec{E} \qquad c \triangleq \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\nabla^2 \vec{E} + \omega^2 \mu_0 \varepsilon_0 \vec{E} = 0 \qquad \nabla^2 E + \frac{\omega^2}{c^2} \vec{E} = 0$$

Let $\vec{E} = E_x \hat{x}$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + 0 + 0 = 0 = \frac{\partial E_x}{\partial x}$$

$$\nabla^2 E_x + \frac{\omega^2}{c^2} E_x = 0 \qquad k \triangleq \frac{\omega}{c}$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial E_x}{\partial y^2} + \frac{\partial E_x}{\partial z^2} + k^2 E_x = 0 \qquad \left\{ \begin{array}{l} \text{assume } \dfrac{\partial E_x^2}{\partial y^2} = 0 \\ \text{to} \\ \text{simplify} \end{array} \right\}$$

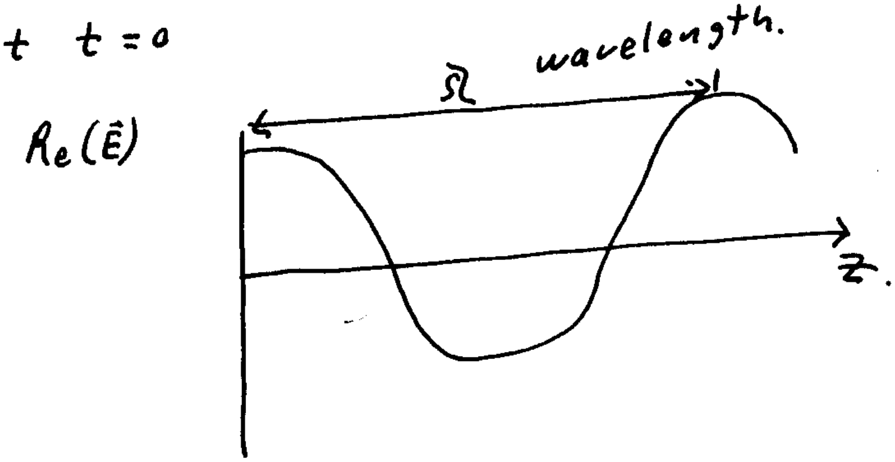
$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \Rightarrow \vec{E} = e^{j\omega t} E_x(z)\,\hat{x}$$

so $E_x = \cos kz,\ \sin kz \Rightarrow e^{jkz},\ e^{-jkz}$

$\vec{E} = E_0\, e^{j\omega t}\, e^{+/-jkz}\,\hat{x} \Rightarrow \mathrm{Re}(\vec{E})$ is measured in the Lab.

$$\mathrm{Re}(\vec{E}) = E_0\,\hat{x}\,\mathrm{Re}\left[e^{j(\omega t - kz)}\right] = E_0\,\hat{x}\cos(\omega t - kz)$$

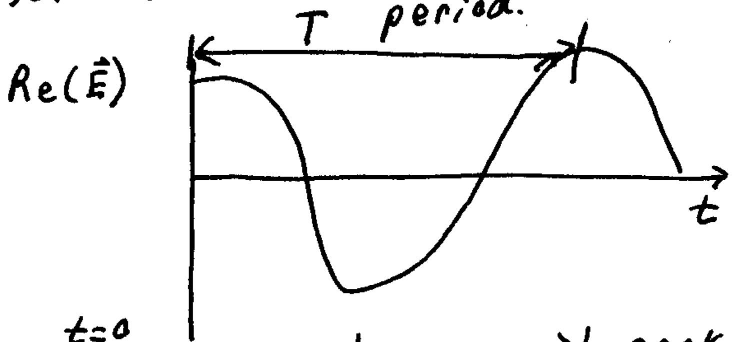
at $t = 0$



$\lambda = \dfrac{2\pi}{k} \qquad k = \dfrac{2\pi}{\lambda} \Rightarrow$ wave number.

units m^{-1}

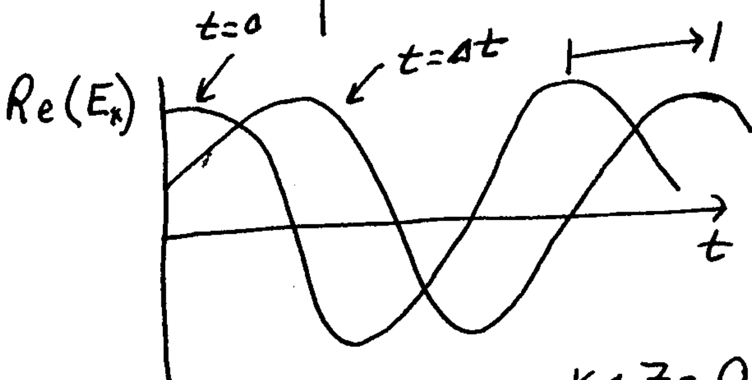
for $z = 0$



$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$ Angular frequency

$\omega = 2\pi f \quad f = \frac{1}{T}$



peak moves to right Δz

set $\omega \Delta t - K \Delta z = 0 \rightarrow$ constant phase point

$$\frac{\Delta z}{\Delta t} = \frac{\omega}{K} \triangleq \text{phase velocity}$$

$$\frac{\omega}{c} = K \Rightarrow \frac{\Delta z}{\Delta t} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec}$$

in free space.

$\vec{B} = \mu_0 \vec{H} \quad \vec{D} = \epsilon_0 \vec{E}$

e^{-jKz} propogates in $+z$ direction.

533-4 (plane wave contd)

$$E = E_0 e^{-jkz}\hat{x} \qquad \nabla\times\vec{E} = -j\omega\mu_0\vec{H}$$

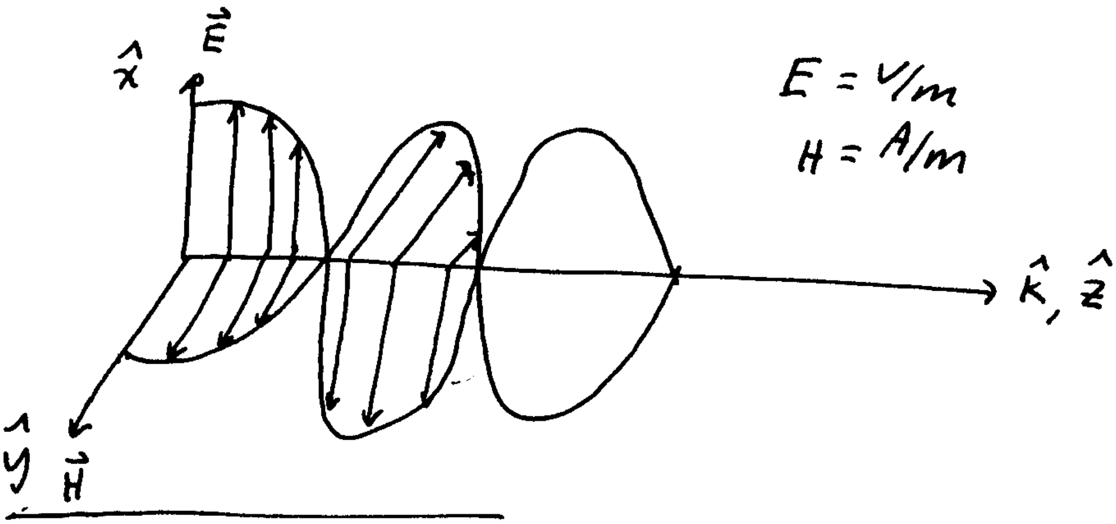
$$\nabla\times\vec{E} = \frac{\partial}{\partial z}E_0 e^{-jkz}\hat{y} = -\hat{y}\,jkE_0 e^{-jkz}$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0 e^{-jkz} & 0 & 0 \end{vmatrix} \Rightarrow \vec{H} = \frac{k}{\omega\mu_0}E_0 e^{-jkz}\hat{y} \quad \begin{cases} \vec{E}\perp\vec{H} \\ |\vec{E}| = \frac{1}{\eta_0}|\vec{H}| \end{cases}$$

$$\eta_0 \triangleq \frac{\omega\mu_0}{k} \qquad k = \frac{\omega}{c}$$

$$\eta_0 = \mu_0 c \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \boxed{\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega}$$

$$|\vec{E}| = \eta_0|\vec{H}|$$



$E = V/m$
$H = A/m$

Summary:

$\vec{E}\perp\vec{H}$, $\vec{E}\perp\hat{k}$, $\vec{H}\perp\hat{k}$ $\qquad \hat{k}$ direction of propagation

$\vec{E}\times\vec{H} \parallel \hat{k} \qquad \frac{|E|}{|H|} = \eta_0 = 120\pi = 377\Omega.$

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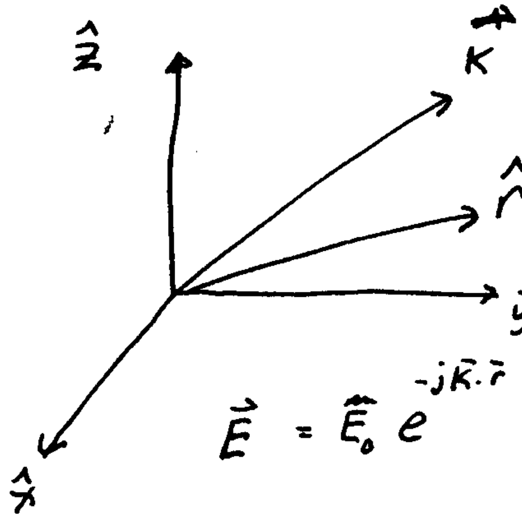
$\vec{E} \times \vec{H} = \vec{P}$ (or $\vec{w}$ in text) Poynting Vector.

$\vec{E} = \eta_o \vec{H} \times \hat{k}$

E & H in "peak" values.

$\frac{1}{2}|P| \Rightarrow$ time averaged Power density, i.e. Intensity

in general:



$\vec{K} = K_x \hat{x} + K_y \hat{y} + K_z \hat{z}$

$\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$

$\vec{E} = \vec{E}_o e^{-j\vec{K}\cdot\vec{r}}$ $\quad \vec{H} = \vec{H}_o e^{-j\vec{K}\cdot\vec{r}}$

$\bar{K}\cdot\bar{r} = K_x x + K_y y + K_z z$ $\quad \vec{E}_o = \eta_o \vec{H}_o \times \vec{K}$

$|\bar{E}_o|^2 = |E_x|^2 + |E_y|^2 + |E_z|^2$

Lecture 5, 9/12/08

Polarization

$\vec{E} = E_x \hat{x}$ or $E_x \hat{x} + E_y \hat{y}$ when $\hat{k} = \hat{z}$

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$$\vec{E} = E_o \vec{u}\, e^{-jkz} \qquad H = \frac{E_o}{\eta} \hat{z} \times \vec{u}\, e^{-jkz}$$

$\vec{u}$ is the <u>polarization</u> vector - complex

$$\vec{u} \cdot \vec{u}^* = 1$$

$$\vec{u} = a_x \hat{x} + a_y \hat{y} \qquad a_x = |a_x| e^{+j\delta_x}, \quad a_y = |a_y| e^{j\delta_y}$$

Example

$$a_x = \frac{1+j}{\sqrt{2}}, \; a_y = 0 \Rightarrow a_x = 1\, e^{j45^\circ}$$

$$|a_x| = 1 \quad \delta_x = 45$$

$$|a_y| = 0$$

add time dependance

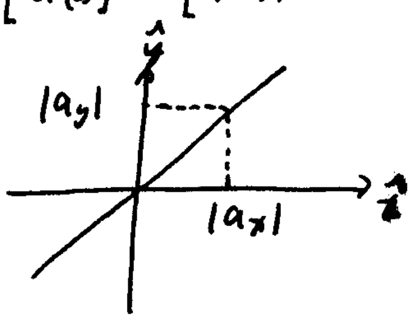
$$\vec{u}(t) = \vec{u}\, e^{j\omega t} = \hat{x} |a_x| e^{j\delta_x} e^{j\omega t} + \hat{y} |a_y| e^{j\delta_y} e^{j\omega t}$$

Physical quantity measured is $Re(\vec{u}(t))$

$$Re(\vec{u}(t)) = |a_x| \cos(\omega t + \delta_x) \hat{x} + |a_y| \cos(\omega t + \delta_y) \hat{y}$$

<u>Case I</u> $\delta_x = \delta_y = \delta_o$

$$Re[\vec{u}(t)] = [\,|a_x|\hat{x} + |a_y|\hat{y}] \cos(\omega t + \delta_o)$$



Linear Polarization.

δ_o = starting phase.

<u>Case 2</u> $\delta_y - \delta_x = \pm\frac{\pi}{2}$ (used in satellites)

$\bar{u}\cdot\bar{u}^* = 1$, $|a_x| = |a_y| = \frac{1}{\sqrt{2}}$, $|a_x|^2 + |a_y|^2 = 1$

Let $\delta_0 = 0 \Rightarrow \delta_y = \pm\frac{\pi}{2}$

$$\bar{u} = \left[\frac{\hat{x} + e^{j\pi/2}\hat{y}}{\sqrt{2}}\right] e^{j\delta_x}$$

$\hookrightarrow$ starting phase (not important)

or

$$\bar{u} = \frac{\hat{x} + e^{-j\pi/2}\hat{y}}{\sqrt{2}}$$

$$\bar{u}(t) = \frac{\hat{x} + e^{j\pi/2}\hat{y}}{\sqrt{2}} e^{j\omega t}$$

$$\mathrm{Re}[\vec{u}(t)] = \frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{x} + \cos(\omega t + \pi/2)\,\hat{y}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{x} - \sin\omega t\,\hat{y}\right)$$

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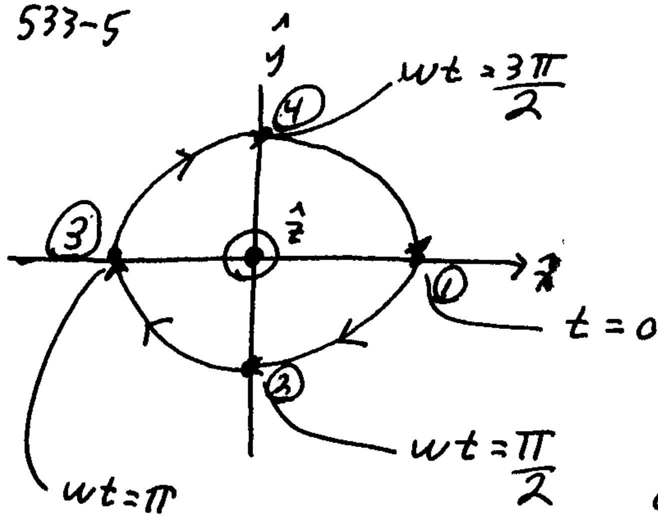


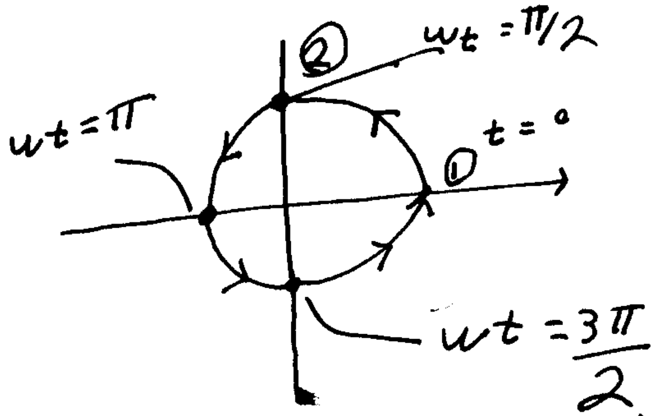
$\hat{z}$ axis out of paper toward you.

Convention:
This is Left-handed circular polarization (LHCP) for $+\hat{z}$ direction of propagation.

direction would reverse to RHCP if propagation is $-\hat{z}$

$$\delta_y - \delta_x = -\frac{\pi}{2} \quad \text{then} \quad \mathrm{Re}(\bar{u}(t)) = \frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}\right)$$

this gives RHCP for $+\hat{z}$ propagation.



Addendum: Per our book on page 75, Reverse the sense of Polarization so that what the notes define as LHCP will be RHCP. Balanis defines the sense of polarization as viewed traveling <u>AWAY FROM THE OBSERVER.</u> This will make the notes consistent with the text. avo 22/08

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Case 3 Elliptical Polarization:
this is the most general case.

$\bar{u} = a_x \hat{x} + a_y \hat{y}$

$= a_L \hat{L} + a_r \hat{R}$

(this is where the sign error was!)

from case 2; $\hat{L} = \frac{\hat{x} + e^{j\pi/2} \hat{y}}{\sqrt{2}} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}}$

$\hat{R} = \frac{\hat{x} + e^{-j\pi/2} \hat{y}}{2} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}}$

so $\vec{u} = a_L \frac{(\hat{x} + j\hat{y})}{\sqrt{2}} + \frac{a_r(\hat{x} - j\hat{y})}{\sqrt{2}}$

$= \frac{(a_L + a_r)\hat{x}}{\sqrt{2}} + \frac{j(a_L - a_r)\hat{y}}{\sqrt{2}}$

{note: $|a_L| > |a_r| \Rightarrow$ LHEP, left hand elliptical polarization.
and $|a_L| < |a_r| \Rightarrow$ RHEP }

also $\bar{u} = a_x \hat{x} + a_y \hat{y}$

so $a_x = \frac{a_L + a_r}{\sqrt{2}}$, $a_y = \frac{j(a_L - a_r)}{\sqrt{2}}$

$a_L = \frac{1}{\sqrt{2}}(a_x - j a_y)$ $\quad a_r = \frac{1}{\sqrt{2}}(a_x + j a_y)$