Case 3 Elliptical Polarization:
this is the most general case.

$$\bar{u} = a_x \hat{x} + a_y \hat{y}$$

$$= a_L \hat{L} + a_r \hat{R}$$

(this is where the sign error was!)

from case 2; $\hat{L} = \dfrac{\hat{x} + e^{j\pi/2}\hat{y}}{\sqrt{2}} = \dfrac{\hat{x} + j\hat{y}}{\sqrt{2}}$

$$\hat{R} = \frac{\hat{x} + e^{-j\pi/2}\hat{y}}{2} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}}$$

so $\vec{u} = a_L \dfrac{(\hat{x} + j\hat{y})}{\sqrt{2}} + \dfrac{a_r(\hat{x} - j\hat{y})}{\sqrt{2}}$

$$= \frac{(a_L + a_r)\hat{x}}{\sqrt{2}} + \frac{j(a_L - a_r)\hat{y}}{\sqrt{2}}$$

{note: $|a_L| > |a_r| \Rightarrow$ LHEP, left hand elliptical polarization.
and $|a_L| < |a_r| \Rightarrow$ RHEP }

also $\bar{u} = a_x \hat{x} + a_y \hat{y}$

so $a_x = \dfrac{a_L + a_r}{\sqrt{2}}$, $a_y = \dfrac{j(a_L - a_r)}{\sqrt{2}}$

$a_L = \frac{1}{\sqrt{2}}(a_x - j a_y)$ $\quad a_r = \frac{1}{\sqrt{2}}(a_x + j a_y)$

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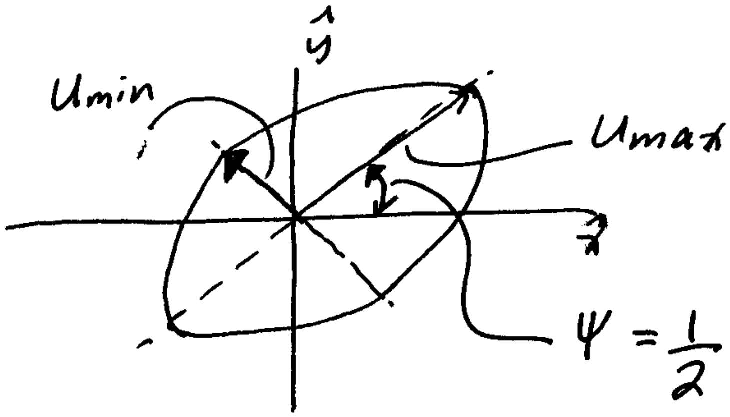
<u>Axial</u> <u>Ratio</u> $\triangleq AR = \left| \dfrac{|a_L| + |a_r|}{|a_L| - |a_r|} \right|$

usually given in $AR_{dB} = 20\log(AR)$

note: $a_L = a_r \Rightarrow AR = \infty \Rightarrow$ Linear Polarization



$AR = \dfrac{U_{max}}{U_{min}}$

$\psi = \frac{1}{2}(\delta_R - \delta_L) + m\pi$

$m = 0, \pm 1, \pm 2 \ldots$

Sense of Polarization.

$|a_r| > |a_L|$ RHEP

$|a_L| > |a_R|$ LHEP

$|a_L| = |a_r|$ LP $\rightarrow$ Linear pol. - Less expensive
used for short range.
example $\rightarrow$ dipole.

$|a_r| = 0$ $|a_L| \neq 0$ LHCP

$|a_L| = 0$ $|a_r| \neq 0$ RHCP

CP more expensive - used
for long distances
example - satellites

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example of CP

$$\vec{u} = \hat{x}\cdot\frac{1}{\sqrt{3}} + \frac{(1+j)\hat{y}}{\sqrt{3}} = a_x\hat{x} + a_y\hat{y}$$

$$a_x = \frac{1}{\sqrt{3}} \qquad a_y = \frac{1+j}{\sqrt{3}}$$

$$a_L = \frac{1}{\sqrt{2}}\left(a_x - j a_y\right) = \frac{2-j}{\sqrt{6}} \qquad |a_L| = \sqrt{5/6}$$

$$a_r = \frac{1}{\sqrt{2}}\left(a_x + j a_y\right) = \frac{j}{\sqrt{6}} \qquad |a_r| = \frac{1}{\sqrt{6}}$$

so $|a_L| > |a_r| \Rightarrow$ LHEP

$$AR = \left|\frac{|a_L| + |a_r|}{|a_L| - |a_r|}\right| = \left|\frac{1+\sqrt{5}}{1-\sqrt{5}}\right| = 2.62 = 8.36\,dB$$

(not very good)

find ψ:

$$\delta_L = \tan^{-1}\left(\frac{-1}{2}\right) = -26.6^\circ \qquad \delta_R = \tan^{-1}\left(\frac{1}{0}\right) = 90^\circ$$

$$\psi = \frac{1}{2}\left(\delta_R - \delta_L\right) + m\pi \rightarrow m=0 \rightarrow \psi = 58.3^\circ$$

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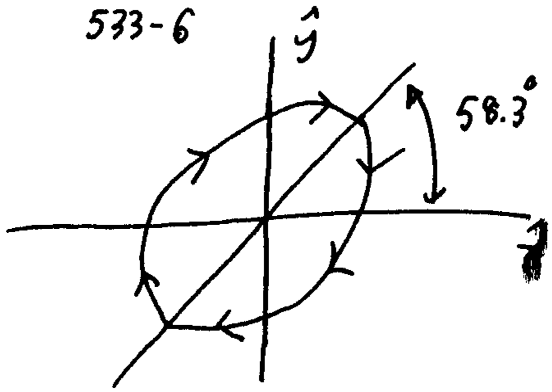

$$\frac{u_{max}}{u_{min}} = 2.62.$$

Cross-polarization level (example cont'd)

relates to power missmatch

terms:

co-pol : intended polarization.

X-pol : unintended polarization.

$$\text{Cross-pol} \triangleq \left| \frac{a_{x\text{-pol}}}{a_{co\text{-pol}}} \right|^2$$

for example: Let LHCP be co-pol
RHCP be x-pol

$$\left| \frac{1/\sqrt{6}}{\sqrt{\frac{5}{6}}} \right|^2 = 1/5 = -6.99\,dB$$

note: 20 is a good number

(example: SatComm satellites → even transponders & odd transponders.)

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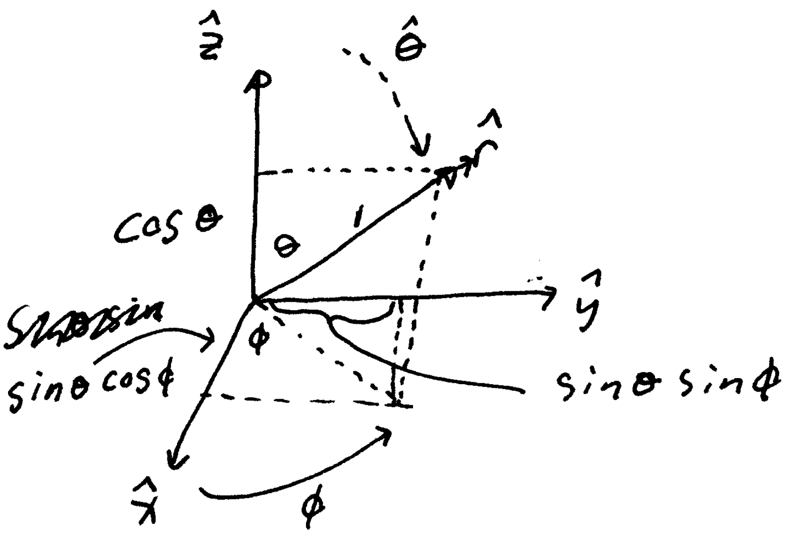
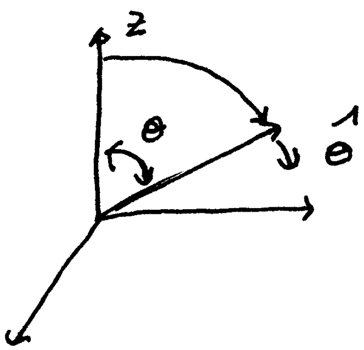
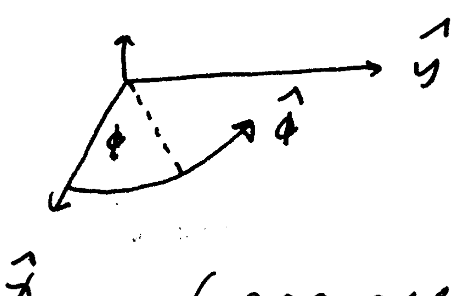
Spherical Coordinates



$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$



$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$



$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

(see appendiz VII in the text for a more complete Review)

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$$\begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix}$$

Far field Approximation (Fraunhofer zone)

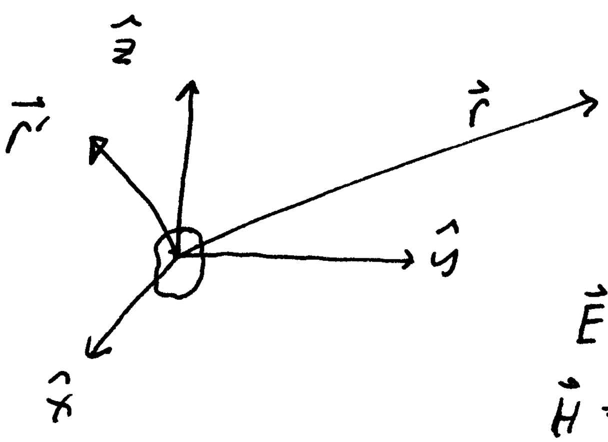
(see fig 2.4 in text)

$\Updownarrow$

$\left(R > \dfrac{2D^2}{\lambda} \right.$ p35 text.$\left.\right)$

Let $\vec{r}$: observation point

$\vec{r}'$: source point



Far field - spherical wave

$$\vec{E} = E_r\hat{r} + E_\theta\hat{\theta} + E_\phi\hat{\phi}$$
$$\vec{H} = H_r\hat{r} + H_\theta\hat{\theta} + H_\phi\hat{\phi}$$

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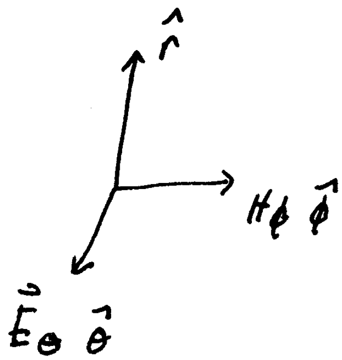
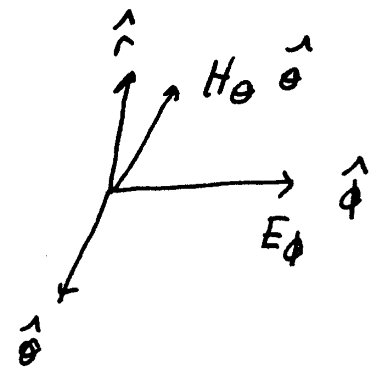
in far field $\dot{E}_r, H_r \cong 0$ so plane wave approximation holds.

$$E_\theta = \eta H_\phi$$





$$E_\theta \, (= \eta H_\phi) = -jK\eta A_\theta - jKF_\phi$$

$$E_\phi \, (= -\eta H_\theta) = -jK\eta A_\phi + jKF_\theta$$

$$\eta = 120\pi = \sqrt{\frac{\mu}{\varepsilon}} = 377\Omega$$ free space.

$$K = \frac{2\pi}{\lambda}$$

magnetic vector potential

$$A(\bar{r}) \cong \underbrace{\frac{e^{-jKr}}{4\pi r}}_{\text{observaition distance. } r} \underbrace{\iiint_{\text{volume}} \vec{J}(\bar{r}') e^{j\vec{K} \cdot \vec{r}'} \, dr'}_{\text{source distribution at } r'}$$

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electric vector potential

$$\vec{F}(\vec{r}) \tilde{=} \frac{e^{-jkr}}{4\pi r} \iiint_V \vec{m}(\vec{r}\,') \, e^{j\vec{k}\cdot\vec{r}\,'} \, d\vec{r}\,'$$

$\vec{k} = |\vec{k}|\hat{k}$ $\quad |\vec{k}| = k$ $\quad \hat{k} = \hat{r}$ in the direction of propogation.

$\vec{J}(\vec{r}\,')$ electric current dist at source.

$\vec{m}(\vec{r}\,')$ magnetic current dist at source.

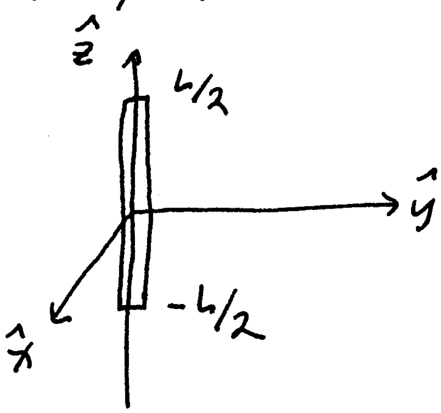
$\vec{r}\,'$ Location of the source point

$\vec{k} = k\hat{r} = k(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

$\hat{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$

$$\vec{k}\cdot\vec{r}\,' = k\left[x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta\right]$$

Example: ideal dipole Antenna in <u>Far Field</u>.



$$J(r') = I_o\,\delta(x')\,\delta(y')\,h(z')\,\hat{z}$$

$$h(z') = \begin{cases} 1 & -\frac{L}{2} \le z' \le \frac{L}{2} \end{cases}$$

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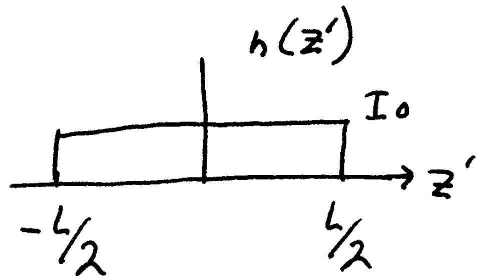

note:
$\vec{m} = 0, \ \vec{F} = 0$

$$\vec{A}(\vec{r}) = \frac{e^{-jkr}}{4\pi r} \underbrace{\int dx' \int dy' \int dz'}_{dv' = dx'dy'dz'} \vec{J}(\vec{r}\,') \, e^{j\vec{k}\cdot\vec{r}\,'}$$

$$e^{j\vec{k}\cdot\vec{r}\,'} = \exp\left[jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta)\right]$$

$$A(\vec{r}) = \frac{I_0 \, e^{-jkr}}{4\pi r} \hat{z} \left[\int_{-\infty}^{\infty} \delta(x') dx' \, e^{jkx'\sin\theta\cos\phi}\right] \Rightarrow [\,] = 1$$

$$\cdot \left[\int_{-\infty}^{\infty} dy' \, \delta(y') \, e^{jky'\sin\theta\sin\phi}\right] \Rightarrow [\,] = 1$$

$$\cdot \left[\int_{-\infty}^{\infty} dz' \, h(z') \, e^{jkz'\cos\theta}\right]$$

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$$\vec{A}(\bar{r}) = \frac{e^{-jkr}}{4\pi r}\,\hat{z}\,I_0 \underbrace{\int_{-L/2}^{L/2} e^{jkz'\cos\theta}\,dz'}$$

$$= \frac{1}{jk\cos\theta}\, e^{jkz'\cos\theta}\Big|_{-L/2}^{L/2}$$

$$\vec{A}(\vec{r}) = \hat{z}\frac{e^{-jkr}}{4\pi r} I_0 \frac{1}{jk\cos\theta}\underbrace{\left[e^{\frac{jkL\cos\theta}{2}} - e^{-jk\frac{L}{2}\cos\theta}\right]}_{2j\sin\left(\frac{kL}{2}\cos\theta\right)}$$

$$\vec{A}(\vec{r}) = \hat{z}\, I_0 L \frac{e^{-jkr}}{4\pi r}\,\frac{\sin\Psi(\theta)}{\Psi(\theta)}$$

where $\Psi(\theta) \triangleq \frac{kL\cos\theta}{2}$

now find <u>far</u> field E_θ

$\Rightarrow \vec{F} = 0$

$E_r,\ H_r = 0$

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$$E_\theta = -jk\eta A_\theta$$

$$E_\phi = -jk\eta A_\phi = -j\eta H_\theta$$

$$\vec{A} = A_z \hat{z} = \frac{e^{-jkr}}{4\pi r} I_0 l \frac{\sin\Psi(\theta)}{\Psi(\theta)} \hat{z}$$

$$\vec{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_z \hat{\phi}$$

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$$\hat{z} = \cos\theta \hat{r} - \sin\theta \hat{\theta}$$

$$A_\phi = 0 \qquad A_r = \frac{e^{-jkr}}{4\pi r} I_0 l \cos\theta \frac{\sin\Psi(\theta)}{\Psi(\theta)} \hat{r}$$

$$A_\theta = \frac{+e^{-jkr}}{4\pi r} I_0 l(-\sin\theta) \frac{\sin\Psi(\theta)}{\Psi(\theta)} \hat{\theta}$$

note: vector potential is used for convience....