Chapter 16 notes: Simultaneity bias/reverse causation

16.1 The Nature of Simultaneous Equation Models

Basic model: $y_i = \beta_0 + \beta_1 x_i + u_i$

Example: Police and Crime

Estimate of β_1 will be biased if

(1) there is some omitted var V that affects both y and x

(previous example: ability causes both education and wages) or

(2) reverse causation x→y and y→x (e.g., police→crime crime→police)

Police force and crime are **determined simultaneously**
2 actors making choices at the same time
→ Observe a set of (crime, police) points
Are these criminals' actions or city's? Both.

- potential criminals choosing crime based on how # of police
- city officials choosing police based on how high crime is.

→ model the decisions of these actors together to understand the actions of one

Example: D& S. Suppose we are interested in measuring labor supply. Observe a set of points: hours and wage. How are these determined?

- hours supplied = β0 + β1wage + β2nonlabor income + ui
- hours demanded = = γ0 + γ1wage + γ2price of capital + ei

Again, if observe set of points, these are net results of actions of 2 actors.

NOTE: simultaneity does NOT mean one person whose choice of one action affects their choice of another. For example,

- hours supplied = β0 + β1wage + β2nonlabor income + β3leisure + ui
- leisure hours = γ0 + γ1wage + γ2nonlabor income + γhours suplied + ei

This doesn't make sense. Can't think about how hours supplied changes when wage changes, holding leisure constant.

→Each equation should have a behavioral, ceteris paribus interpretation on its own

16.2 Simultaneity bias in OLS

We have shown how <u>omitted vars</u> lead to biased coefficients.
Now let's look at how <u>simultaneity</u> leads to a bias. Should make sense intuitively.

Write out the STRUCTURAL MODELS
(equation derived from economic theory--model in terms of causal effects):

(1) $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$ zs are exogenous variables

(2) $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$

Know that will not estimate α_1 correctly if y_2 is corr with u_1. Is it?

$y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$

$y_2 = \alpha_2(\alpha_1 y_2 + \beta_1 z_1 + u_1) + \beta_2 z_2 + u_2$ [obvious y2 corr with u1]

$(1-\alpha_2\alpha_1)y_2 = \alpha_2\beta_1 z_1 + \beta_2 z_2 + u_1 + u_2$

Assume that $1-\alpha_2\alpha_1 \neq 0$
Divide through by $1-\alpha_2\alpha_1$
→ (3) $y_2 = \pi_{21} z_1 + \pi_{22} z_2 + v_2$

This is the REDUCED FORM—the reduced form parameters are nonlinear functions of the structural parameters

Several important points:

- v_2 contains both u_1 and u_2 → y_2 is correlated with u_1
  BUT—zs are not correlated with these STRUCTURAL errors

- The interpretation of the parameters in the STRUCTURAL model is different from the REDUCED form. What is held constant in (3) is not same as what is held constant in (1). Will not get same coeff for β_2 as for π_{22} (though they are related). Why not?

- Under certain special assumptions, OLS estimates of (1) may be fine. What are these assumptions? IF $\alpha_2 = 0$, u1 and u2 are uncorrelated, then y2 is not corr. with u1.

- Sign of the bias

16.3 Identifying and Estimating a Structural Equation

Just as in case of endogenous variables in Ch 15 (perhaps due to omitted variable), OLS estimates are biased/inconsistent—estimates are not measures of causal effect

Use Two Stage Least Squares Estimators—Like before

Sometimes may not be able to identify entire system, but only one equation.

Demand and supply example best illustration of this:

Supply: $q=\alpha_1 p + \beta_1 z_1 + u_1$
Demand: $q=\alpha_2 p + u_2$
q=per capita milk consumption, p=price milk, z=price of cattle feed

Which of these equations can we identify? Demand—can use z_1 as an instrument for p in demand equation
→ changes in Z1 will shift supply→ changing price.

Show this graphically.

What would enable us to identify supply? Demand shifter.

General 2-equation model:

$y_1 = \beta_{10} + \alpha_1 y_2 + z_1\boldsymbol{\beta_1} + u_1$ U's are structural errors
$y_2 = \beta_{20} + \alpha_2 y_1 + z_2\boldsymbol{\beta_2} + u_2$

y's are the endogenous variables.
Zs are matrices (sets) of exogenous variables. betas are vectors.

$z_1\boldsymbol{\beta_1}= \beta_{11}z_{11}+ \beta_{12}z_{12}+. . . + \beta_{1k1}z_{1k1}$
$z_2\boldsymbol{\beta_2}= \beta_{21}z_{21}+ \beta_{22}z_{22}+. . . + \beta_{2k2}z_{2k2}$

The z's might partially overlap—might be some of same z's in both equations (example, income in city might determine both criminal activity and number of police)

- When can we solve for reduced form? (y1,y2 as linear functions of all exogenous vars + structural errors?) $\alpha_1\alpha_2 \neq 1$ as before

- When can we identify the equations? Need EXCLUSION RESTRICTIONS: some exogenous variables need to be different in each equation for identification

  Recall conditions from before for valid instrument:

  (1) Exogenous—not correlated with error. Means not affected by ys, not correlated with omitted vars
  (2) Relevant—has to be included in other equation

  Add a third here to make more precise:

  (3) Validly excluded—only effect on y1 is through effect on y2

<u>Order condition</u>: (necessary) 1st equation has at least one variable that is excluded--total number of exogenous vars must be at least as great as total number of explanatory vars

<u>Rank condition</u>: (necessary and sufficient) 1st eqn in a 2 eqn model is identified IFF 2nd equation contains at least 1 exogenous variable not in 1st equation (validly excluded) AND the coeff on that variable is non-zero in the 2nd equation (relevant)

Proceed by 2SLS—the excluded variable is the instrument
Police and crime example—Levitt instrument is point in election cycle