

The Tsai-Wu Failure Criterion

Steve Tsai

Ed Wu (1939-2009)

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Tsai, S. W. and Wu, E. M. (1971). *A general theory of strength for anisotropic materials*. Journal of Composite Materials. vol. 5, pp. 58-80.

The Tsai-Wu Failure Criterion

Tsai-Wu Theory

Tsai-Wu theory is a simplification of Gol'denblat and Kapnov's generalized failure theory for anisotropic materials. It is stated as

$$f_i \sigma_i + f_{ij} \sigma_i \sigma_j = 1 \quad i, j = 1, 2, 3, 4, 5, 6$$

For plane-stress condition:

$$f_1 \sigma_1 + f_2 \sigma_2 + f_6 \tau_6 + f_{11} \sigma_1^2 + f_{22} \sigma_2^2 + f_{66} \tau_6^2 + 2f_{12} \sigma_1 \sigma_2 + 2f_{16} \sigma_1 \tau_6 + 2f_{26} \sigma_2 \tau_6 = 1$$

Shear strength is independent of sign of the shear stress, therefore all linear shear stress terms must vanish. Therefore we get

$$f_1 \sigma_1 + f_2 \sigma_2 + f_{11} \sigma_1^2 + f_{22} \sigma_2^2 + f_{66} \tau_6^2 + 2f_{12} \sigma_1 \sigma_2 = 1$$

The Tsai-Wu Failure Criterion (cont.)

Evaluation of Constants

(a) Longitudinal tension & compression tests:

$$f_1 = \frac{1}{F_{1t}} - \frac{1}{F_{1c}} \quad \text{and} \quad f_{11} = \frac{1}{F_{1t}F_{1c}}$$

(b) Transverse tension & compression tests:

$$f_2 = \frac{1}{F_{2t}} - \frac{1}{F_{2c}} \quad \text{and} \quad f_{22} = \frac{1}{F_{2t}F_{2c}}$$

(c) Shear tests:

$$f_{66} = \frac{1}{F_6^2}$$

(d) Interaction coefficient f_{12} is assumed as

$$f_{12} \cong -\frac{1}{2}\sqrt{f_{11}f_{22}} \quad \text{or} \quad f_{12} = -\frac{1}{2}\sqrt{\frac{1}{F_{1t}F_{1c}F_{2t}F_{2c}}}$$

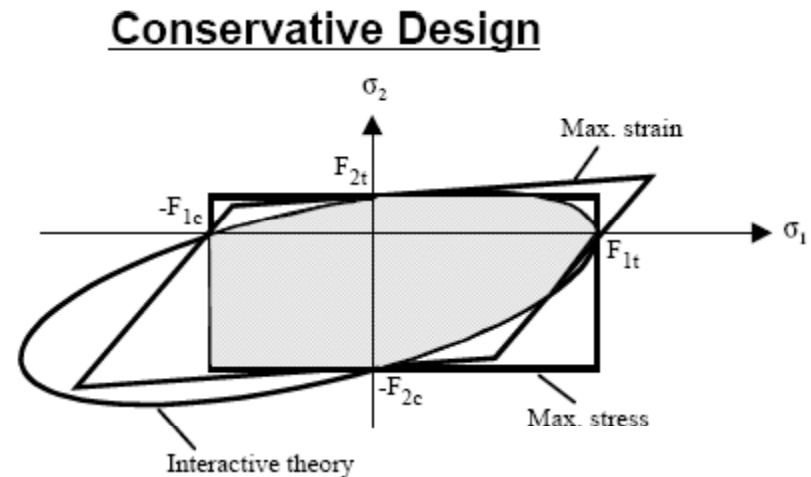
Cairns' Comments

Tsai and Wu developed their tensor polynomial failure criterion independent of Gol'denblat and Kapnov. In any event, Gol'denblat and Kapnov stated their criterion in the form of an infinite series, which is of little practical use. Tsai and Wu developed the present form, including interaction parameters to a useful level for composites. The only region which it does not work very well is for fiber compression failure, which is actually a geometrically driven failure, and not a true material property. For that type of failure, one of the "micro-buckling" types of criteria seem to be more accurate.

Use in Design

3.4 Comparison of Failure Theories

Theory	Physical basis	Operational convenience	Required operational convenience
Maximum stress	Tensile behaviour of brittle material	Inconvenient	Few parameters by simple testing
Maximum strain	Tensile behaviour of brittle material Some stress interaction	Inconvenient	Few parameters by simple testing
Deviatoric strain energy (Tsai-Hill)	Ductile behavior of anisotropic materials "Curve fitting" for heterogeneous brittle composites	Can be programmed Different functions required for tensile and compressive strengths	Biaxial testing is needed in addition to uniaxial testing
Interactive tensor polynomial	Mathematically consistent Reliable "curve fitting"	General and comprehensive; operationally simple	Numerous parameters Comprehensive experimental program needed



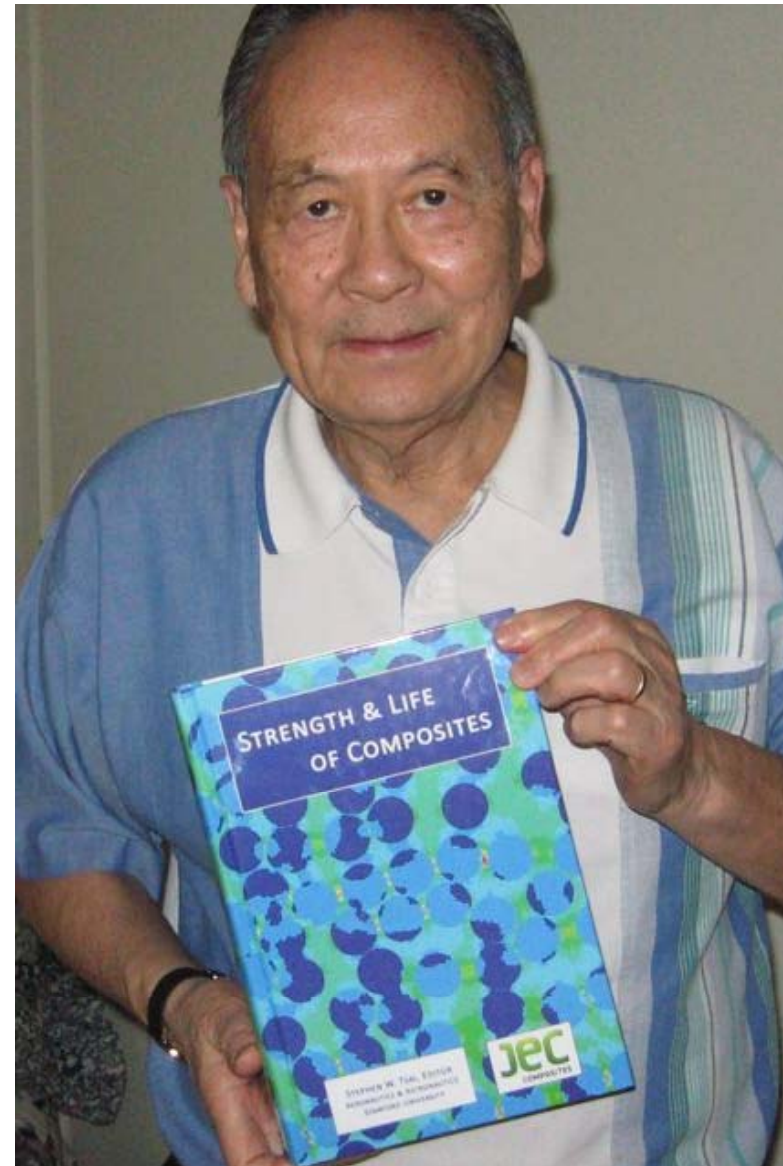
Steve Tsai

Doug,
We put in the mail a
copy of our latest
book based on
micromechanics
and viscoelastic
matrix for life
prediction.

Hope you find it useful
in your research
and teaching.

Steve

10/12/2008



Ed Wu and Steve Tsai (Late 2008)



Dear All,

Ed Wu passed away this evening at 8 PM. I spent the day in the hospital with Becky, his wife, and daughter Susan.

After his liver transplant in January, he had a good month. They chemical imbalance and toxicity of various medicines took their tolls. He was going down hill. Yesterday, he was told that there was nothing that doctors could do for him. His kidney stopped functioning. Other organs were also breaking down.

When I saw him today, he was struggling with breathing. The end was near.

He was a good man. Very meticulous experimentalist. Knew Lekhnitskii, and fracture mechanics. We will all miss him.

He left this world with many friends and colleagues. He enjoyed many sports like bicycle, motor cycle, flying, photography, and eating. The picture below was taken at China Stix last October when he was full of good spirits and appetite. During our January 2009 workshop at Stanford he stopped by to say hello.

Steve

Stephen W. Tsai
steve.tsai@mac.com

June 9, 2009

Ed Wu's Obituary

to be published in technical journals

Edward M. Wu, Professor Emeritus of Aeronautics at the Naval Postgraduate School in Monterey, CA, died on June 3 of complications following a liver transplant. He leaves his dear wife of 47 years, Rebekah, who was his partner, in both family and professional life. His most prized technical publication was co-authored with his wife and he often referred to the technical team of Wu and Wu. He was very proud of his daughters, Susanne (Robert) Gong-Guy and Stephanie (Kenneth II) Geremia, who went on to professional careers in dentistry and accounting, respectively. He leaves six grandchildren; Mark, Kathryn, Jonathan, and Stephen Gong-Guy of San Jose, CA, and Kenneth III and Kaitlyn Geremia of Jupiter, FL. He is survived by two brothers, Robert Wu of Woodstock, NY and Michael Wu of Berkeley, CA and two sisters, Marlene Yang of New Port Beach, CA and Hannah Wu Lam of Seattle, WA. He was preceded in death by his daughter, Stephanie.

Ed Wu's Obituary

to be published in technical journals

Professor Wu was born and raised in Hong Kong. He attended the University of Illinois at Urbana/Champaign where he earned a BS in Mechanical Engineering in 1960 and a MS (1963) and PhD (1965) in Theoretical and Applied Mechanics. After a short stay at the University of Illinois as Assistant Professor of Theoretical and Applied Mechanics, Prof. Wu moved to Washington University in St. Louis in 1967 where he and Professor Steve Tsai developed and experimentally verified the Tsai-Wu Failure Criterion for anisotropic materials, which is now presented in composite materials textbooks and remains widely adopted in advanced composite design in handbooks such as the FAA Composite Materials Handbook and the Air Force/NASA Composite Design Guide. His work on understanding fracture mechanics in composite materials has also been incorporated into textbooks on composite materials. During this time he also pioneered the computer-controlled, multi-axial testing of anisotropic composite materials. In 1976, Professor Wu moved to the Lawrence Livermore National Laboratory where he set up state-of-the-art laboratories for static and life testing of composite materials in order to support his research on statistical modeling and the quantification of composite reliability.

Ed Wu's Obituary

to be published in technical journals

In 1984, he moved to the Naval Postgraduate School (NPS) in Monterey, CA to teach and do research on computer-aided certification of large critical composite structures ranging from aerospace parts to civilian infrastructure components. His research and publications at the NPS focused on the statistical nature of failure in composite materials and structures. He developed the concept of statistically-defensible reliability for composite and aerospace structures. He retired from NPS with Emeritus rank in 2001 and continued his consulting work and original research for both industry and Government until his death.

Professor Wu served on the Editorial Board of the Journal of Composite Materials and was a member of the American Institute of Aeronautics and Astronautics, the American Society of Mechanical Engineers, the American Society for Testing and Materials, Pi Tau Sigma, Sigma Tau and Tau Beta Pi.

Ed Wu's Obituary

to be published in technical journals

In the academic portion of his career, he earned the respect and admiration of hundreds of undergraduate and graduate students. At the Naval Postgraduate School, Professor Wu mentored a generation of Naval aviators, whose impressions of him ranged from “a spirited pioneer who challenged us in the classroom” to “he was the father I wish I could have had”. An avid sportsman and pilot, he loved flying and eventually arranged a supersonic flight with one of his naval aviator students.

Ed was also a very competitive criterium and road bicycle racer. Among his friends and colleagues, he is remembered by all for his “sharp wit and his ability to use humor to make a point”.

Throughout his professional life, Professor Wu's combination of humor, fairness, sensitivity, loyalty, intellect, and honesty endeared him to his family, his students, and his colleagues alike.

Ed Wu and MSU

- Ed visited MSU many times since Dr. Cairns came here in 1995.
- He gave special seminars and guest lectures in ME463 Composites, ME464 Mechanical Behavior of Materials, ME 550 Failure of Materials, and ME 551 Advanced Composites
- We were making plans for Ed to be here Fall 2009 to deliver lectures in Reliability, and to work with undergraduate students for failure of composite materials and structures

One of Ed's Great Loves

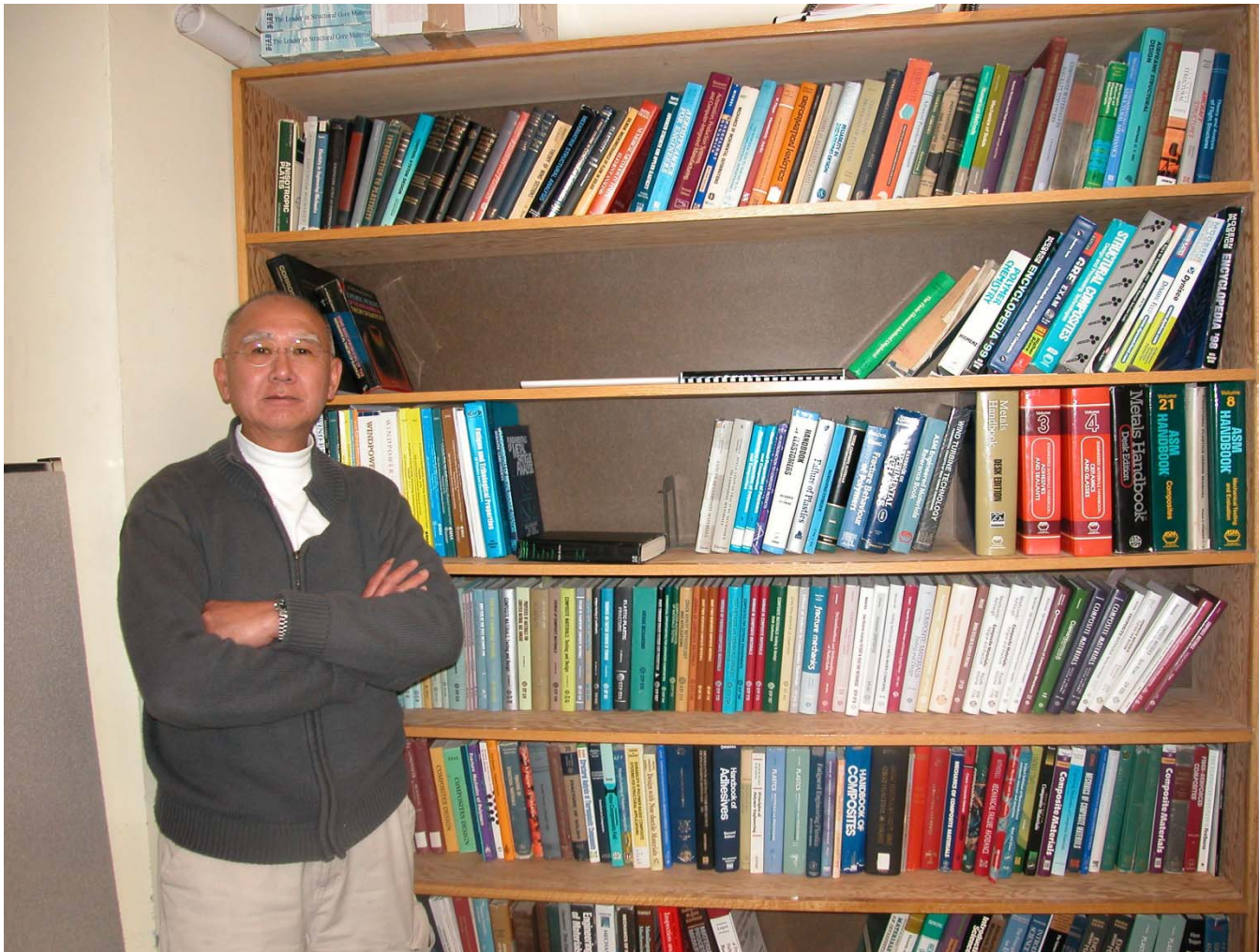
Ed and his
Ducati Paso



Ed, Doug, and Ed



Ed Donating Books for Our Composites Library (many more to come from his estate)



Ed at Our Cabin (Circa 2003)

