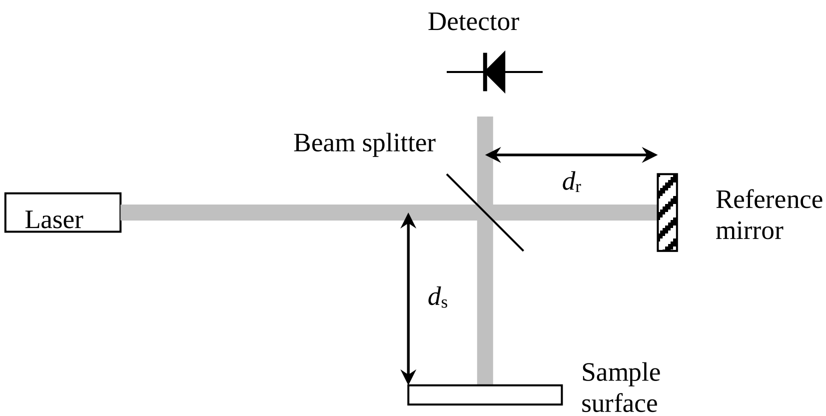
Interference



1) You wish to build a laser profilometer (an instrument to measure the height profile of a surface) using a Michelson interferometer. You have available a 1 mW single frequency Helium Neon laser, with a center frequency of 633 nm. After you construct your interferometer, you place your sample a distance d_s from the beamsplitter. Then you play around with the position d_r of the reference mirror.

a) Sketch the output of the photodetector, in relative units, as d_r is scanned over a distance of a few wavelengths. You may want to plot the output in terms of the difference length $d = d_s - d_r$.

b) Indicate on your drawing the position of the reference mirror for maximum sensitivity to small changes in the position of the surface, d_s.

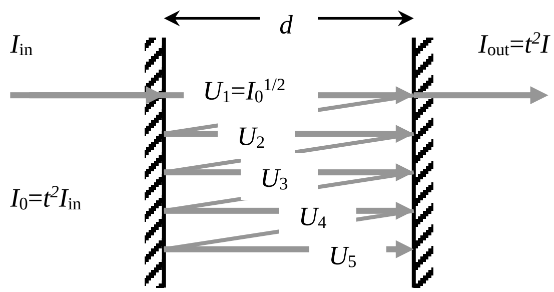
c) Leave your interferometer adjusted for maximum sensitivity. If the intensity noise of your laser/detector combination is 1 μW rms, what is the minimum height change in the surface that your system can measure? Assume the minimum height gives you a signal-to-noise ratio of 1.

2) You've grown tired of measuring surface variations with nanometer precision, and you decide to use your interferometer to monitor traffic on your street. You point the sample arm of your HeNe interferometer out the window and find to your delight that you get a measureable backreflection from a vehicle that is coming your way. (Please don't try this – your HeNe is **not** eyesafe!)

a) At the detector, you measure a doppler frequency of 45 MHz. If the speed limit is 25 MPH (11.3 m/s), was the vehicle speeding? Speed is dd_s/dt, in the above interferometer.

b) Calculate what doppler frequency you would measure if you had used an invisible (and also eye safe) beam with wavelength 1.3 um instead. What beat frequency would the vehicle have generated, and what frequency corresponds to your neighborhood speed limit of 25 MPH?

3) A Fabry-Perot interferometer may be constructed with two highly reflective mirrors facing one another. Usually one or both mirrors are curved to provide a stable cavity (this is discussed in the text in chapter 1) but we will neglect the specifics of the mirror curvatures here. Each mirror has a field reflectivity r, and the phase delay for one round trip in the interferometer is $\varphi=2kd$, so the round trip field amplitude complex "gain" is $r^2e^{j\varphi}$. The field in the cavity is the superposition of an infinite number of waves, corresponding to multiple passes through the cavity, so that each wave has a constant phase difference from the previous one, and an amplitude reduced by r^2.



$$U = U_1+U_2+U_3 \ldots$$
$$= I_0^{1/2} + I_0^{1/2}r^2e^{j\varphi} + I_0^{1/2}(r^2e^{j\varphi})^2 + I_0^{1/2}(r^2e^{j\varphi})^3 + \ldots$$

$$I = |U|^2$$

The intensity I inside the cavity is found by evaluating the geometric series for the field in the resonator. Equation 2.5-18 in the text gives the expression for the intensity within the resonator. (note that $|h|=r^2$ in this case)

$$I = \frac{I_{\max}}{1+(2\mathcal{F}/\pi)^2\sin^2(\varphi/2)}$$

$$I_{\max} = \frac{I_0}{(1-r^2)^2} \quad , \quad \mathcal{F} = \frac{\pi\, r}{1-r^2}$$

The parameter $\mathcal{F}$ is called the finesse of the resonator. The input and output intensities are related to the intensity inside the resonator according to $I_0=t^2I_{in}$, and $I_{out}=t^2I$, where t is the mirror field amplitude transmission coefficient, such that $|r|^2+|t|^2=1$.

We wish to use this Fabry-Perot cavity as a spectrum analyzer.

a) Using the relationships above, write an expression for I_{out} in terms of I_{in}, and in terms of the frequency ν of the optical wave.

b) Sketch the transfer function $H = I_{out}/ I_{in}$ as a function of frequency ν, showing a few cycles of the periodic function. Use $\mathcal{F}=10$.

c) What is the maximum intensity transmission coefficient of the resonator, in terms of $R=|r|^2$ and $T=|t|^2$? What is the minimum? Is the maximum transmission of the resonator higher or lower than T, the intensity transmission coefficient of one of the mirrors?

d) If d = 10 cm, what is the "free spectral range" of the resonator, meaning the frequency difference between transmission maxima.

e) What finesse is required if the spectrum analyzer is to have a resolution $\Delta\nu$ of 100 kHz? (You may use the "large $\mathcal{F}$" approximation.) What value of R, the mirror intensity reflectivity, is required for this frequency selectivity?