Lab #3

Diffraction

Contents:

1. Pre-Lab Activity

Write the equation for the irradiance (intensity) distribution in a diffraction pattern from a uniformly illuminated grating of period Λ. Calculate the angle expected for the 1st-order spot of HeNe laser light (λ = 633 nm) diffracted by a grating with 40 lines/mm.

2. Introduction

Diffraction effects play an important role in virtually every aspect of wave propagation, and any electro-optical device or system must be designed to accommodate this fundamental behavior of light. In this lab we will observe and measure the diffraction pattern that arises from a periodic grating and from a circular aperture. We will restrict our attention to the important Fraunhofer region, which is the far-field diffraction pattern. The reason is not merely for convenience of analysis (since the Fraunhofer diffraction region is the simplest to express mathematically), but also because it is the Fraunhofer diffraction pattern that appears in the focal plane of a positive lens. It is precisely this diffraction pattern that limits the resolution of a "perfect" imaging instrument – the so-called diffraction limit.

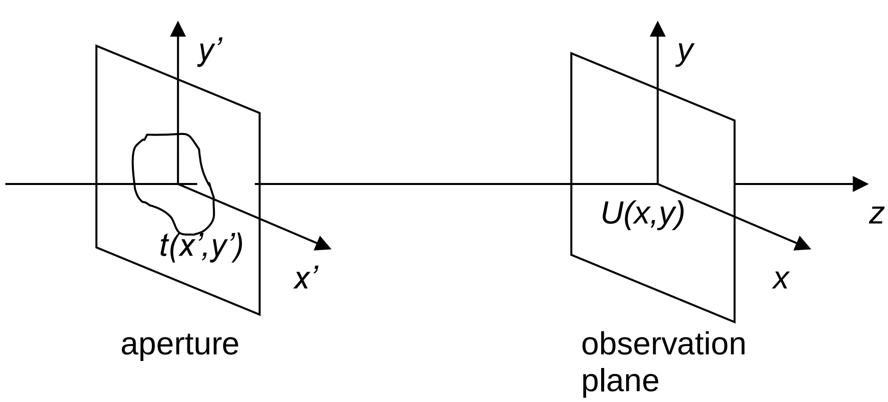
Far field limit

We saw in class that when the observation plane is sufficiently far from the aperture plane, the diffraction pattern takes on the simple form of a Fourier transform relationship to the aperture function. The condition for this was seen to be $z >> \pi D^2/\lambda$ where *D* is the dimension of the aperture. Under these conditions the field pattern *U(x)* may be represented by

$$U(x,y) = C \int_{\text{aperture}} t(x',y')\,e^{-j2\pi(x'f_x + y'f_y)}\,dx'dy' \,,$$

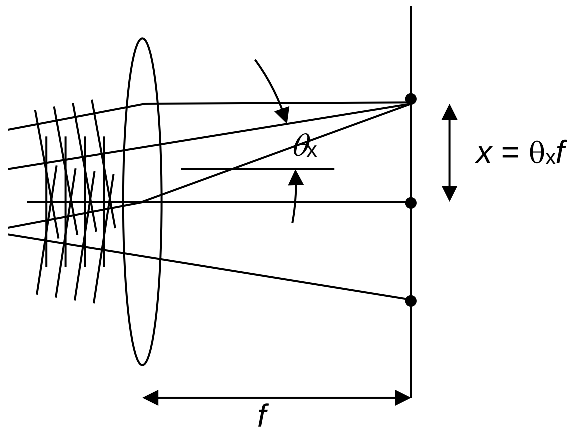
where $f_x = x/\lambda z = \theta_x/\lambda$ and $f_y = y/\lambda z = \theta_y/\lambda$

which is seen to be the two-dimensional Fourier transform of the aperture field transmission function *t(x',y')*. For a simple hole in a screen, *t(x,y)* = 1 inside the hole and *t(x,y)* = 0 everywhere else. More complicated transmission functions can include variable magnitude and phase terms across the aperture.



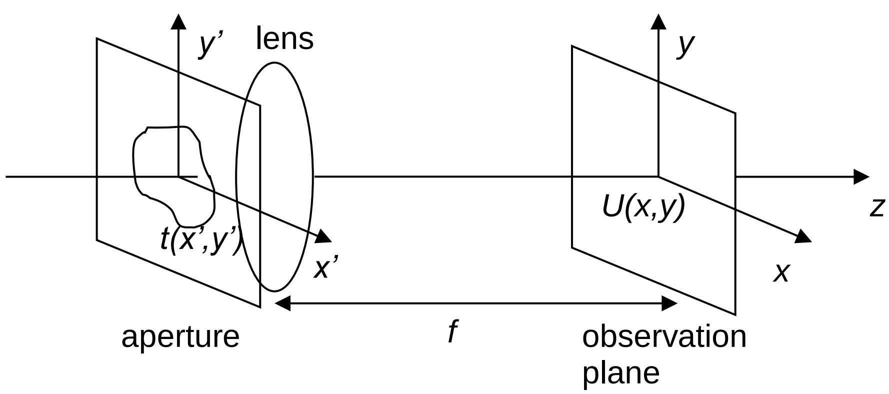
Effect of a lens

The effect of a lens is to take a plane wave (a wave focused at infinity) and bring it to a focus in the lens back focal plane. If we take the view of the above far-field diffraction pattern as a collection of plane waves making angles θ_x and θ_y with respect to the optic axis, then we may expect that the far-field pattern will be replicated in the focal plane of the lens. The transverse position of the features of the diffraction pattern are related to the diffraction angles according to $x = \theta_x f$ and $y = \theta_y f$.



If the lens is placed behind the aperture in the previous arrangement we have the following relationship, after substituting for $\theta_x = x/f$ and $\theta_y = y/f$:

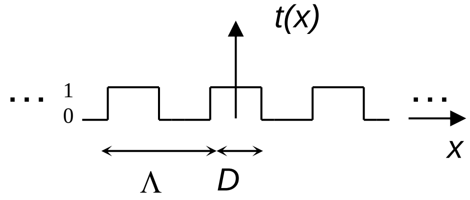
$$U(x,y) = C \int_{\text{aperture}} t(x',y')\, e^{-j2\pi(x'f_x + y'f_y)} dx'dy' \text{, where } f_x = x/\lambda f \text{ and } f_y = y/\lambda f \text{ .}$$



We will use these basic results to interpret the diffraction patterns produced in the lab.

3. Diffraction Grating

In this section you will measure the diffraction pattern from a periodic transmission grating. The grating is a transmission function varying in only one dimension, so only a one dimensional Fourier transform is required for its analysis. Consider the transmission function *t(x)* specified below:

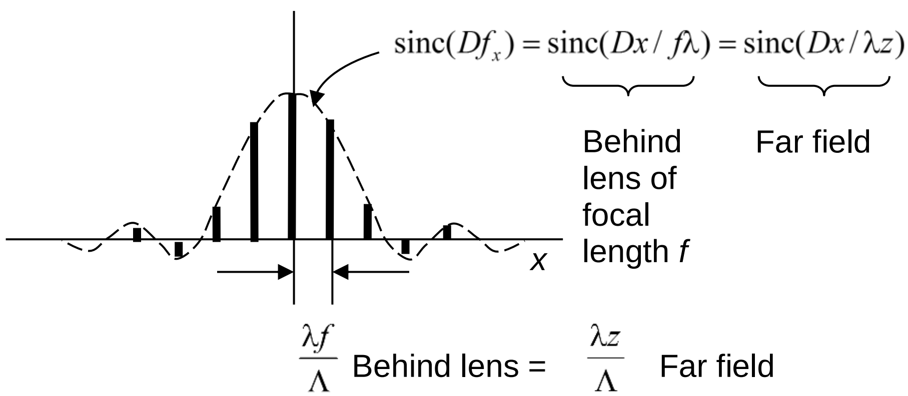


$$t(x) = \mathrm{rect}(x/D) * \mathrm{comb}(x/\Lambda),$$

$$\mathrm{rect}(x/D) = \begin{Bmatrix} 1, |x| \le D/2 \\ 0, |x| > D/2 \end{Bmatrix},$$

$$\mathrm{comb}(x/\Lambda) = \sum_n \delta(x - n\Lambda)$$

We know from Fourier theory that if $A \overset{F}{\Leftrightarrow} a$ and $B \overset{F}{\Leftrightarrow} b$ then $AB \overset{F}{\Leftrightarrow} (a * b)$, where the double arrow indicates a Fourier transform relationship. Furthermore, we know that $\mathrm{comb}(x/\Lambda) \overset{F}{\Leftrightarrow} |\Lambda| \mathrm{comb}(f_x \Lambda)$ and $\mathrm{rect}(x/D) \overset{F}{\Leftrightarrow} |D| \mathrm{sinc}(Df_x)$ where we define $\mathrm{sinc}(x) \equiv \dfrac{\sin(\pi x)}{\pi x}$. The above pattern for *t(x)* can be represented as a a rect function convolved with a comb function, so the Fourier transform of the pattern must look like a sinc function multiplied by another comb function. The result is illustrated below.



a) Use a Ronchi ruling as a diffraction grating for this experiment. Use a rotation stage, and mount the grating perpendicular to the laser beam with vertically oriented grooves. The rotation stage axis should be vertical, parallel to the grooves. <u>Observe the diffraction pattern in the far field. Describe the diffraction pattern.</u>
b) <u>Measure the angle of a high order diffracted spot, and calculate the grating pitch (in lines/mm) and the grating period.</u>
c) Rotate the grating and observe the change in the diffraction pattern. <u>What rotation angle corresponds to a spread of the diffraction pattern by a factor of two?</u> What is

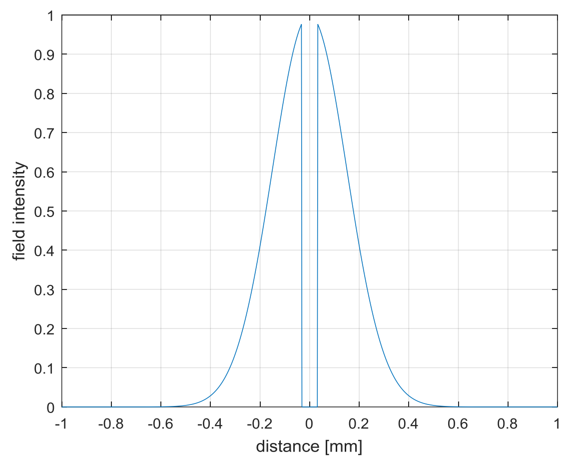
the corresponding grating period, measured normal to the beam propagation direction?

d) Return the grating to beam normal. Place a long focal length lens (200 – 250 mm focal length) behind the grating, and image the diffraction pattern onto a CCD camera. Describe what you see. Compare the focused pattern to the far field pattern.

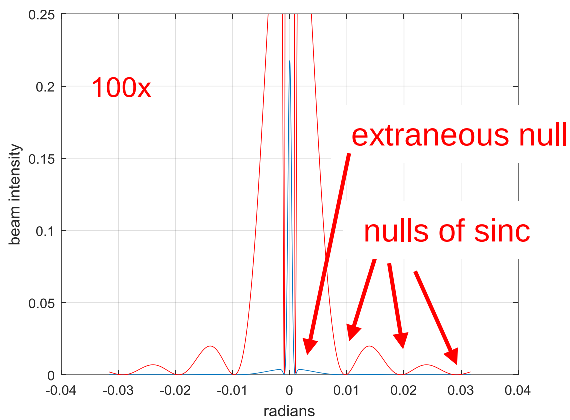
e) Measure the optical power of the first few diffracted orders (out to order 5). You may make this measurement using the camera image, or by measuring each diffracted order directly using the power meter. Comment on how well (or poorly) this agrees with the predicted intensity. Base your prediction on the Fourier relationship between the diffraction pattern and the grating, with the assumption of an intensity grating with equal dimension lines and spaces. Remember that the above relationships refer to the optical field, and that you are measuring irradiance (intensity).

4. Measure the Width of Your Hair

In this section you will measure the diffraction pattern from a strand of hair. Like the grating, a long hair has a transmission function varying in only one dimension, so a one dimensional Fourier transform is all that is required for its analysis. A hair (the darker the better) may be modeled as an obstruction of width *D*, with $t(x) = 1 - \mathrm{rect}(\frac{x}{D})$. The field behind the hair may be written $U(x) = e^{-\frac{x^2}{w^2}}(1 - \mathrm{rect}\left(\frac{x}{D}\right))$, which is plotted below.



The Fourier transform of this field produces a far-field amplitude $U(\theta) = w\sqrt{\pi}e^{-\left(\frac{\pi w\theta}{\lambda}\right)^2} - D\,\mathrm{sinc}(\frac{D\theta}{\lambda})$. Below is a plot of the far-field intensity $I(\theta) = |U(\theta)|^2$. The pattern is characterized by a strong undiffracted beam, with a superimposed sinc pattern due to the hair. The nulls of the sinc function occur when $\frac{D\theta}{\lambda} = 1,2,3$ ....

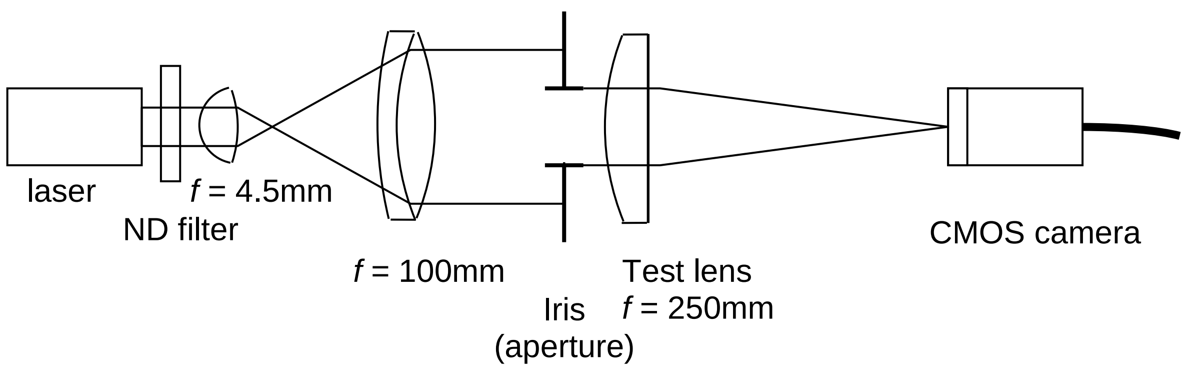


f) Tape a hair from your head to a card or a glass slide. Illuminate with the laser beam, and observe the diffraction pattern in the far field on a card or the wall (or you may use a CMOS camera for this). Describe the diffraction pattern.

g) Measure the angle of the first few nulls of the diffraction pattern, and then compute the width of your hair.

5. Focusing with a lens

a) You will use a CCD camera to measure the shape and width of a focused laser beam in this section. The manual for the camera we are using is linked on the course webpage. You will find there that the pixel pitch for our cameras is 5.3 um.



Diffraction limited spot size

b) You will need to expand the laser beam for these measurements. Use a short focal length lens (such as Thor Labs C230TM-B asphere (f=4.5mm)) as the primary lens and a 100mm focal length achromatic doublet lens as the secondary. If these lenses are not available, build a beam expander with the lenses at hand, but try for at least 10x expansion. Pay attention to lens orientation for minimum aberration (highly curved face away from the beam focus). Use an adjustable iris as the aperture stop for your measurements. Place a long focal length lens (250 mm) directly behind the iris as the test lens for your measurement. With small apertures, the lens will be "diffraction limited", meaning that aberrations introduced by the lens are small enough that the spreading of the spot will be dominated by diffraction effects. Place the camera in the focal plane of the lens. You will need to reduce the intensity of the laser until the CMOS array is in its linear range of operation (not saturated) by inserting ND filters or crossed polarizers in front of the laser.

c) Make measurements of the diffracted spot size for aperture (iris) diameters in the range of 1-3 mm. Begin with the largest aperture, and adjust the beam intensity until the center of the diffracted spot is in the mid-range in the camera response. Make a note of the ND filters used for this and the subsequent measurements. How does the diffracted spot size change as a function of the aperture diameter? Do your measurements agree with theory? Why are we using such a long focal length lens? (How many pixels are in the focused spot?) How does the intensity at the center of the focused spot depend on the aperture diameter? Can you explain this with a simple geometrical argument?

d) If time permits, you may experiment with non-circular apertures or objects, and observe the diffraction pattern. For the best Fourier imaging relationship, place the objects one focal length in front of the lens, and observe the diffraction pattern one focal length behind the lens.

6. Fresnel Lens (optional, with time and if we can find the Fresnel lens!)

The Fresnel lens operates on the combined effects of refraction and diffraction. It consists of refracting segments separated by step discontinuities in the refracting surface, with the OPL change at the discontinuities being an integer number of wavelengths so that each segment, or “zone” adds constructively with the next. The classic Fresnel lens has step heights of a single wavelength, and therefore the zone widths decrease for zones further from the center of the lens, where the slope of the refractive surface is steeper. The Fresnel lens in the lab is constructed with steps that are several wavelengths high, and with zone widths that are nearly uniform. In this case, the height of the discontinuities is increasing for zones that are further from the center of the lens.

A well designed Fresnel lens can produce a diffraction limited spot when the beam is centered on the lens and the lens tilt is minimal, and when used at the design wavelength. The imaging performance decreases rapidly when the illumination wavelength differs from the design wavelength, and for images made away from the optic axis, or with increased amounts of tilt. The plastic lenses in the lab are not probably diffraction limited even under the best conditions.

a) Using the expanded beam from the HeNe, measure the approximate focal length of the Fresnel lens. With the camera, measure the shape and size of the spot in the focal plane, and compare to a “diffraction limited” spot. Comment on appropriate uses for a Fresnel lens, and its advantages over conventional lenses.