Introduction to Geometric Modeling

Learning Objectives:

By the end of the lecture the student should be able to:

- Outline and explain seven main issues with validation of geometric model representation
- Outline and explain four basic types of geometric modeling
- Perform calculation on Bezier Curve
- Outline and explain the differences between different types of surface models
- Perform solids construction using C-Rep Technique
- Perform solids construction using B-Rep technique
- Ensure/calculate the “enclosure” of solid objects using Euler’s Formula
- Highlight the differences between C-Rep and B-Rep

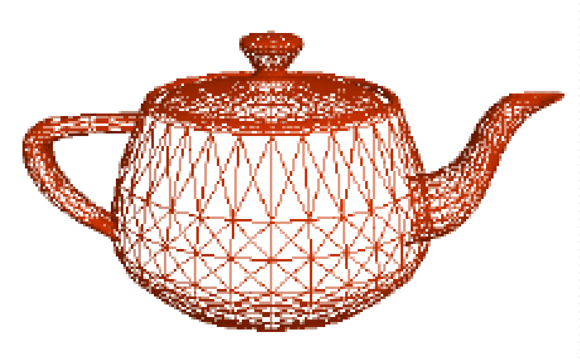
NOTE: Materials used to create this presentation were supplied from:

Lecture notes designed by Dr. Gary Chen, Northern Illinois University

Lecture notes designed by Professor Darek Ceglarek, University of Wisconsin – Madison.

Geometric Modeling: The Foundation of CAD

- A Geometric Modeling System can be characterized as an application component responsible for creating, inspecting, analyzing and distributing geometric models.
- CAD Geometric Modeling
  - Computer-based representation of geometry and related information needed for supporting various computer-based applications in engineering design, analysis, manufacturing and related areas.
  - Approaches to geometric modeling vary in content and capability of supporting the full range of geometric computations.



Geometric Modeling: Validation

A good geometric modeling representation should address the following seven issues:

- ***Domain***: While no representation can describe all possible solids, a representation should be able to represent a useful set of geometric objects.
- ***Unambiguity*****:** When you see a representation of a solid, you will know what is being represented without any doubt. An unambiguous representation is usually referred to as a ***complete* one.**
- ***Uniqueness*****:** That is, there is only one way to represent a particular solid. If a representation is unique, then it is easy to determine if two solids are identical since one can just compare their representations.

Geometric Modeling: Validation

A good geometric modeling representation should address the following seven issues:

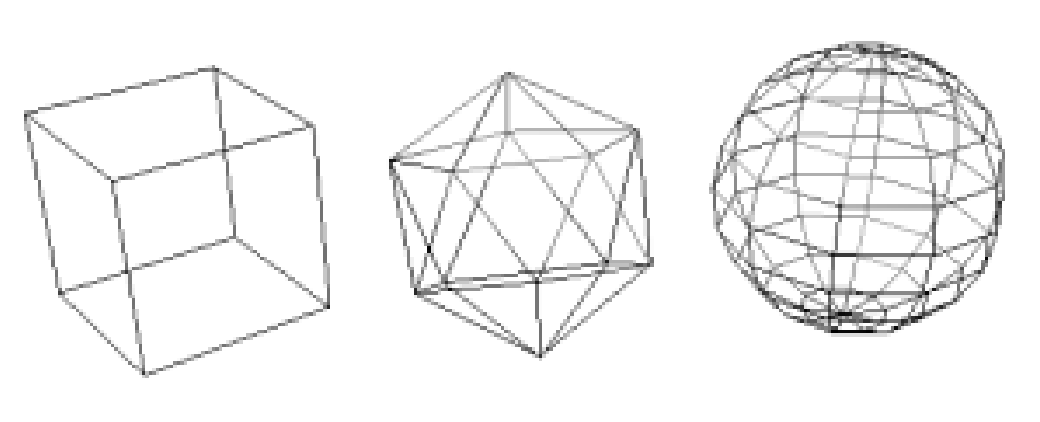
- ***Accuracy*:** A representation is said *accurate* if no approximation is required.
- ***Validness*:** This means a representation should not create any invalid or impossible solids. More precisely, a representation will not represent an object that does not correspond to a solid.
- ***Closure*:** Solids will be transformed and used with other operations such as union and intersection. "Closure" means that transforming valid solids always yields valid solids.
- ***Compactness and Efficiency*:** A good representation should be compact enough for saving space and allow for efficient algorithms to determine desired physical characteristics.

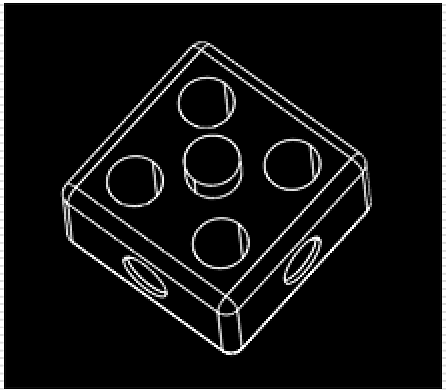
Geometric Modeling

- Involves study of data structures, algorithms and formats for creating, representing, communicating and manipulating geometric information of parts and processes.
- There are 4 major types of geometric models:
  - Graphical models → Wireframe Model
  - Curve models → Hermite Cubic Splines; Bezier Curves; B-Splines, NURBS
  - Surface models → Non-parametric; Parametric Surfaces (Bicubic, Bezier, ..., Ruled Surfaces)
  - Solid models

→ Quadtrees/ Octrees (2D/3D, orthogonal partitioning);
→ Binary Space Partitioning (BSP; Non-orthogona lines/surfaces);
→ Constructive Solid Geometry (CSG: Primitives/Boolean operations)
→ Boundary Representation (B-Rep: Primitives/Euler Operations)

Wireframe Models





Picture source: http://en.wikipedia.org/wiki/Wire_frame_model

Curve Models: Boundary Conditions

- Start & end points of the curve
- The tangent vectors at the start & end points of the curve

Curve Models - Bezier Curves

- Developed by P.Bezier for use in the PolySurf CAD system for Renault Automobile Co. in France (1962)
- Example on board

(4 control points)

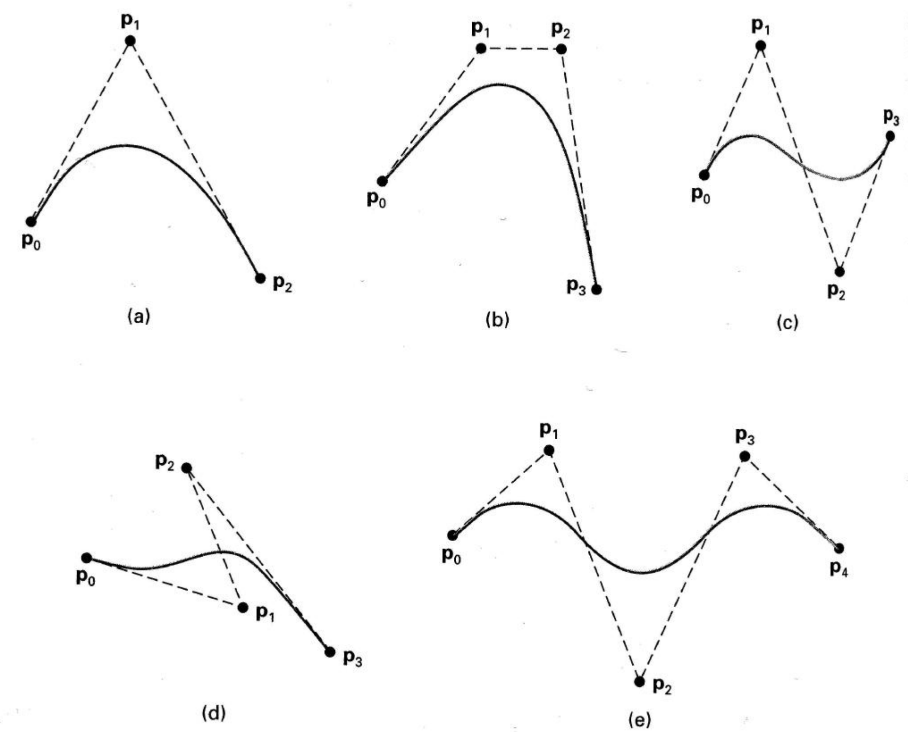
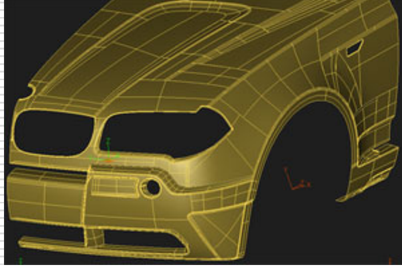


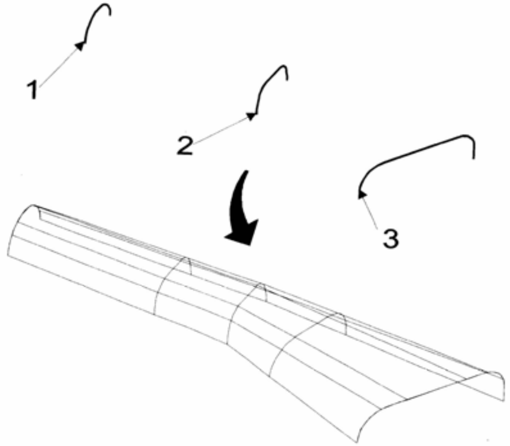
Figure 10-34
Examples of two-dimensional Bézier curves generated from three, four, and five control points. Dashed lines connect the control-point positions.



Surface Modeling in MasterCam

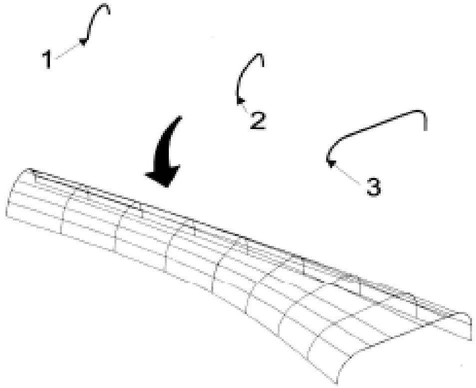
Ruled

The Ruled surface function creates a surface by transitioning between two or more chains of curves in the order that you select them and by using linear blending between each section of the surface. It is important to select each chain of curves at the same relative position to each other. The following picture shows the surface created when you select at positions 1, 2, and 3. You can create this surface type by choosing **Main Menu, Create, Surface, Ruled**.



Loft

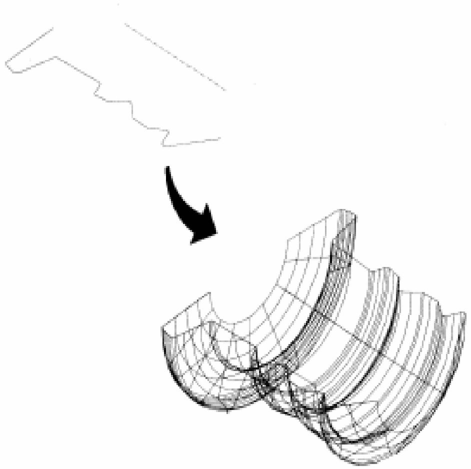
The Loft surface function creates a surface by transitioning between two or more chains of curves in the order that you select them and calculating a smooth blend by considering all the section chains at once. It is important to select each chain of curves at the same relative position to each other. The following picture shows the surface that is created when you select at positions 1, 2, and 3. Notice the difference between the Loft surface and the Ruled surface on the previous page using the same wireframe geometry. You can create this surface type by choosing **Main Menu, Create, Surface, Loft**.



Surface Modeling in MasterCam

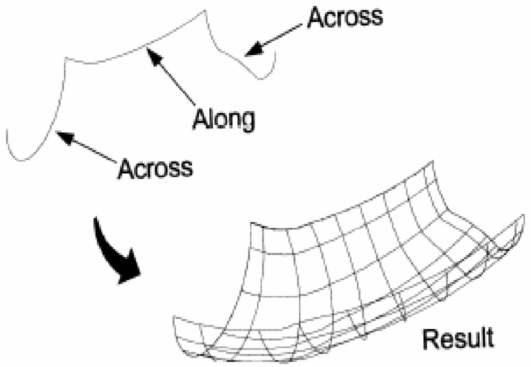
Revolved

The Revolved surface function creates a circular surface by driving the shape of a selected chain of curves about an axis using given start and end angles. Use Revolved when a cross-section and an axis can describe a surface, as shown in the following example. You can create this surface type by choosing **Main Menu, Create, Surface, Revolve**.

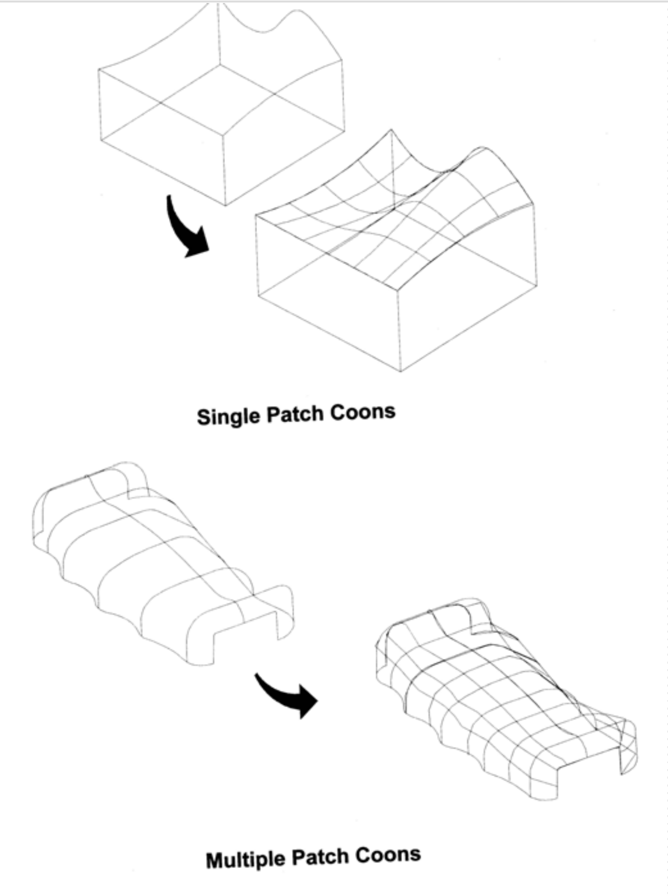


Swept

The Swept surface function creates many different surface configurations depending on the curves that you select. The system sweeps chains of curves called "across contours" over other chains of curves called "along contours." You can select any number of across curves if you are using one along curve. This surface type is shown in the following three pictures. You can create this surface type by choosing **Main Menu, Create, Surface, Sweep.**

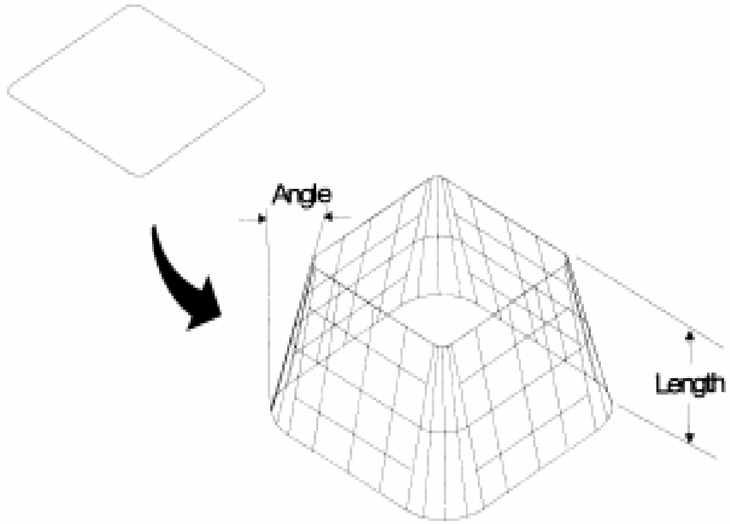


Surface Modeling in MasterCam



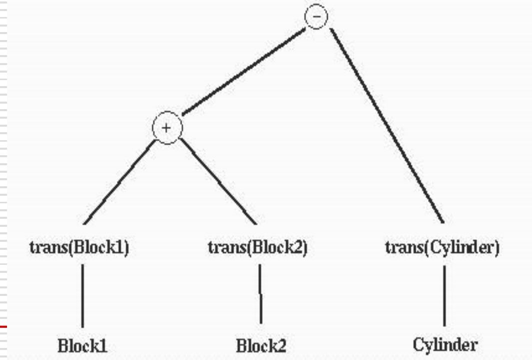
Draft

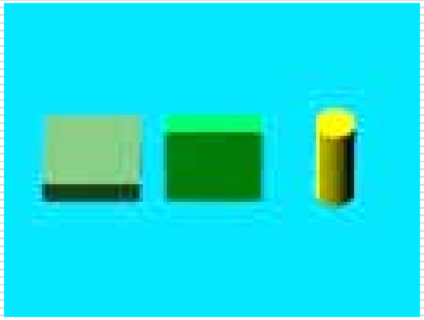
The Draft surface function creates a surface that has angled (or tapered) walls defined by a given length and angle. The following picture shows the draft surface that is created from a single chain of curves. You can create this surface type by choosing **Main Menu, Create, Surface, Draft**.

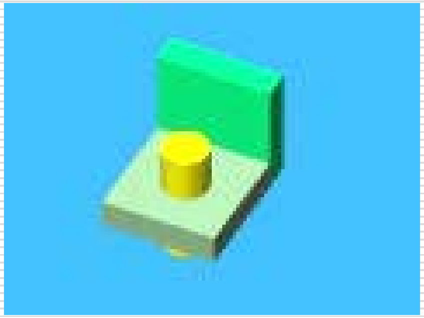


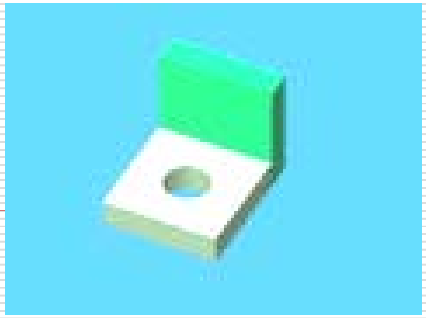
Constructive Solid Geometry (CSG or C-Rep)

- Simple Primitives are joined by means of Boolean operations using Binary Tree
- Boolean operations
  - UNION, INTERSECTION, DIFFERENCE
- Data Structure based on the Binary Tree
  - Primitives Definition
    - A = CUBOID (L_A, W_A, H_A) AT (0, 0, 0)
    - B = CUBOID (L_B, W_B, H_B) AT (0, 0, 0)
    - H = CYLINDER (D_H, H_H) AT (0, 0, 0)
  - Object Construction Procedures
    - $B_1 = B \times$ TRANS (0, 0, Z_1)
    - $H_1 = H \times$ ROT (Y, 90^0)
    - $H_2 = H_1 \times$ TRANS (0, Y_2, Z_2)
    - O_1 = A U B_1
    - O_2 = O_1 DIFF H_2









Solids Modeling
Boundary Representation (B-Rep)

- Boundary surfaces are combined to form a solid model "Enclosure"
- CAD system example - IDEAS
- To ensure the "Enclosure" of the solid model, Euler's formula has to be satisfied
- Euler's formula
  - Relationship among number of vertices, edges, and faces of a simple polyhedron
  - Necessary but not sufficient condition

Euler's Formula

- Euler's formula
  - $V - E + F = 2$

    where V = the number of vertices

    E = the number of edges

    F = the number of faces
- Generalized Euler's formula for objects with holes
  - $V - E + F = 2 + R - 2H$

    where V, E, F are as before

    R = the number of disconnected interior rings

    H = the number of through holes

Comparisons of CSG & B-Rep

- In Modeling
  - CSG advantages
    - Significant procedural advantage in initial modeling
    - Easy to construct a precise solid model out of regular solid primitives
  - B-Rep advantages
    - Construct unusual shape with complex contour surfaces, i.e., aircraft body, wings, auto body, etc.