ECNS 502 Macroeconomic Theory
Montana State University – Depart. of Ag. Econ. & Econ.
Course Number 30020
Spring 2017 Course Packet
Dr. Gilpin

Course Schedule

Week	Date		Topic	Term Paper Due Dates	Homework Due Dates
1	Jan	12	Syllabus & Lecture 1		
2		17	Lecture 2		
2		19	Lecture 3 & Lit. review guidelines		
3		24	Lecture 3 con.	Topic	
3		26	SAS Module 1: Importing/exporting data		
4		31	Lecture 3 con.		
4	Feb	2	Lecture 3 con.		
5		7	Lecture 4	Step 2 (references)	
5		9	SAS Module 2: Cleaning data		
6		14	Lecture 5		
6		16	Lecture 5 con.		
7		21	Lecture 5 con.		
7		23	Lecture 5 con.		
8		28	SAS Module 3: Merging data		
8	Mar	2	**Exam I: Lectures 1- 5 (LINH 306 – during class time)**		
9		7	Lecture 6	Step 3 (summaries)	
9		9	Lecture 6 con.		
10		14	*Spring Break (no class)*		
10		16	*Spring Break (no class)*		
11		21	Lecture 7	Step 4 (paper outline)	
11		23	Lecture 7 con.		
12		28	Lecture 8		
12		30	Lecture 8 con.		
13	Apr	4	Lecture 8 con.		
13		6	Lecture 8 con.	1st draft of research paper	
14		11	Lecture 9		
14		13	Lecture 9 con.		
15		18	Lecture 9 con.		
15		20	Student Presentations	Present research	
16		25	Student Presentations	Present research	
16		27	Review for exam		
17	May	1	**Final Exam: Lectures 6 – 9 (LINH 406 2-3:50 pm)**	Final draft	

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ECNS 502: Macroeconomic Theory
Dr. Gilpin

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Montana State University – Depart. of Ag. Econ. & Econ.
ECNS 502 Macroeconomic Theory (3 credits)

Instructor: Dr. Gilpin	**Office:** 313B Linfield Hall
Office Hours: M - R: 9 – 10 am & by appt.	**Phone:** 994-5628
Class Website: dropbox.com (will receive invitation)	**Email:** gregory.gilpin@montana.edu

Course Description

This course provides the bridge from undergraduate to introductory Ph.D.-level macroeconomics. The foundation of modern macroeconomics and dynamic economic analysis are rigorously presented. The macroeconomic workhorses: the Solow growth model, the Neoclassical growth model, the overlapping generations model, and models of technological change and technology adoption are surveyed along with many extensions to those models. Additional topics include time inconsistency, dynamic efficiency, and the Lucas critique. The official prerequisite for this course is a passing grade in ECNS 302. However, basic calculus skills, such as the ability to use derivatives and integrals, and the knowledge of optimization are required.

Student Learning Outcomes

Upon completion of this course, students should be able to:

- Assess various models that explain how countries grow, both in the short and long run.
- Apply mathematical tools necessary to analyze modern macroeconomic models.
- Explain the effects of fiscal and monetary policies on the macroeconomy.

Course Material - There are no required texts for this course.

Grades

The course grades will be determined by one midterm exam (25%), homework assignments (20%), a project (20%), and a final exam (35%). The project is based on a presentation (4%) while the paper is (16%) of your final grade. Please inform me ahead of time if the exam date has an unavoidable conflict. There is no extra credit work assigned or accepted during the semester.

Grading Scale

A (93-100), A- (90-92), B+ (87-89), B (83-86), B- (80-82), C+ (77-79), C (70-76), D (69-60), F(<60).

Homework

There are homework assignments throughout the semester. Homework assignments are workout problems similar to those discussed in-class. These will prepare you for the midterm and final exams.

Expectations

- Don't be late to class.
- Review the previous class's notes and do any assignments.
    - Try to work as independently as possible on the homework.
- Participate in class discussions and problem solving. Ask all of the questions you have.
    - Don't eat, text, or otherwise engage in activities that draw your attention away from the lecture.
    - Multitasking learning with trivial matters is stupid.
- Final grades reflect outputs, not inputs.

Additional Information

1. If you desire classroom/testing accommodations for a disability, please contact me outside of class to present the written supporting memorandum of accommodation from the office of *Disability Student Services*. Requests for accommodations for disability must be received and authorized by me in written form no less than two weeks in advance of need in order to allow adequate time to review and make appropriate arrangements. No accommodation should be assumed until authorized.

2. I follow the policies in the *MUS Policy and Procedure Manual* with regard to cheating, plagiarism, and academic dishonesty. A grade of zero will be given for any assignment or examination on which the student is in violation of a policy, and the incident will be reported to the Office of the Dean of Students. I reserve the right to refuse to sign a drop form if cheating has been committed on an examination.

3. I expect you to follow the *MSU Student Conduct Code*. No form of harassment is tolerated.

4. I adhere to the policies in the *MSU Catalog* on assigning grades of incomplete. I only award a grade of incomplete upon the student proving there was such a hardship that would render it unjust to hold the student to the time limits previously fixed for the completion of his/her work.

5. Any student desiring to observe a religious holiday must contact me before the absence. Regarding assignments due on a religious holiday, it is preferable that they be submitted before observance; however, the due date will be extended to the next school day following the holiday if the student chooses.

6. I adhere to the policies in the *MSU Catalog* on change of grades. Final grades will not be changed except in the case of clerical error on my part or if it was fraudulently obtained. A change of final grade does not mean allowing additional time to complete the work of a course or allowing the student to submit work or to take or to retake examinations after the conclusion of the semester.

7. The information in this syllabus, including the schedule and grading information is not concrete. Any changes to the syllabus will be discussed in class.

8. You are required to attend the writing center to obtain feedback on all written assignments.

Lecture 1: Introduction to Macroeconomics

1. Modern macroeconomics

Macroeconomics is the study of the performance of the economy as a whole. There are five pillars of modern macroeconomics:

1. Output: the production of goods and services.
2. Prices: the cost of purchasing inputs and outputs over time.
3. Employment: the utilization of economies' key resource.
4. Finance: how credit markets create economic efficiency and improve wellbeing.
5. Government policy: how government policies effect the economy.

These five pillars are typically analyzed over three different time periods:

1. Long run: decade to decade changes in the economy.
2. Medium run: Referring to the business cycle (recessions and expansions).
3. Short run: year to year changes in the economy.

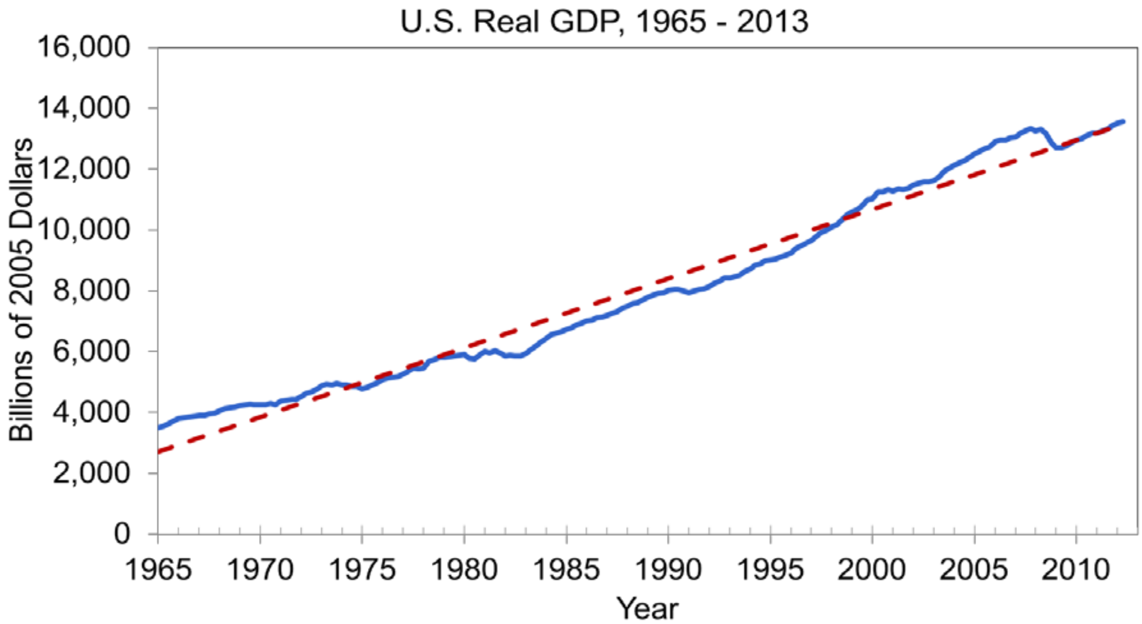
Due to inflation, macroeconomic variables are categorized by whether they net out price changes.
1. Real: Variables that net out changes in price (inflation).
2. Nominal: Variables that are not adjusted for changes in price.

1.1 Aggregate output

Aggregate output is the total quantity of goods and services produced in an economy during a particular time period.

The main measures of aggregate output are Real Gross Domestic Product (RGDP or Real GDP) and Real Gross Domestic Product per Capita (RGDPP or Real GDPP).

Over the last 100 years:
1. U.S. RGDP has grown by 3.4% annually.
2. U.S. RGDPP has grown by 2.2% annually.



Source: U.S. Department of Commerce

1.1.1. Relevant questions on aggregate output

Some questions that arise when studying aggregate output are:

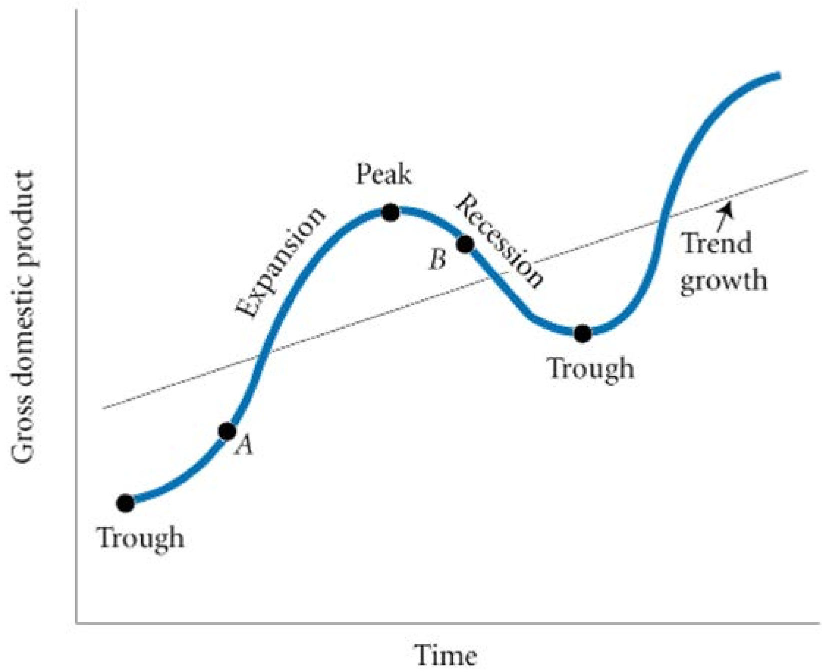
1. What factors allow sustained growth?
2. Can government actions assist in sparking or sustaining growth?
3. What causes business cycles?
4. Can government policies smooth the business cycle? Should they attempt to do so?

1.2. The business cycle

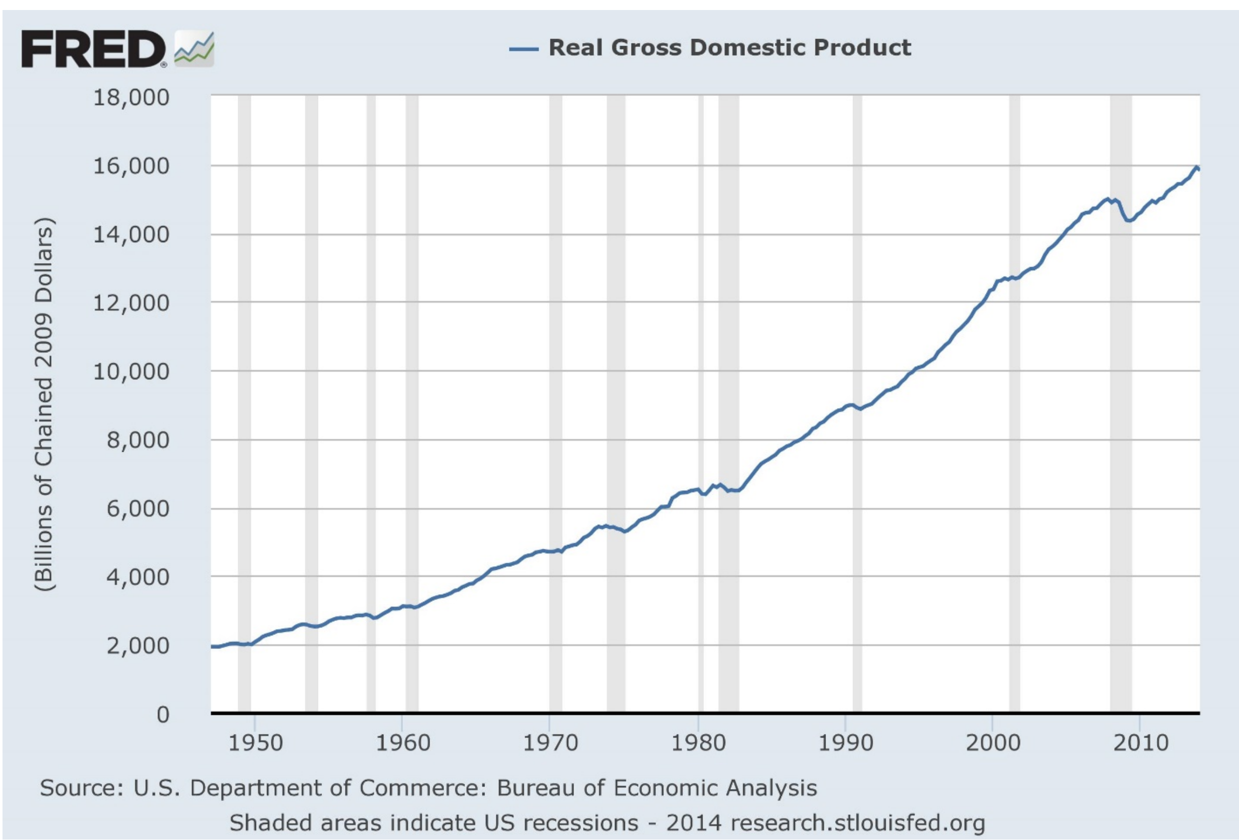
During the short-run, many of the indicators of the economy move together and form a pattern. That is, when the economy is doing well, economic growth occurs, unemployment is low, prices seem to be stable and not rising too fast, the financial system is sound, and government policies assist the economy. These co-movements in macroeconomic indicators are called the *business cycle*.

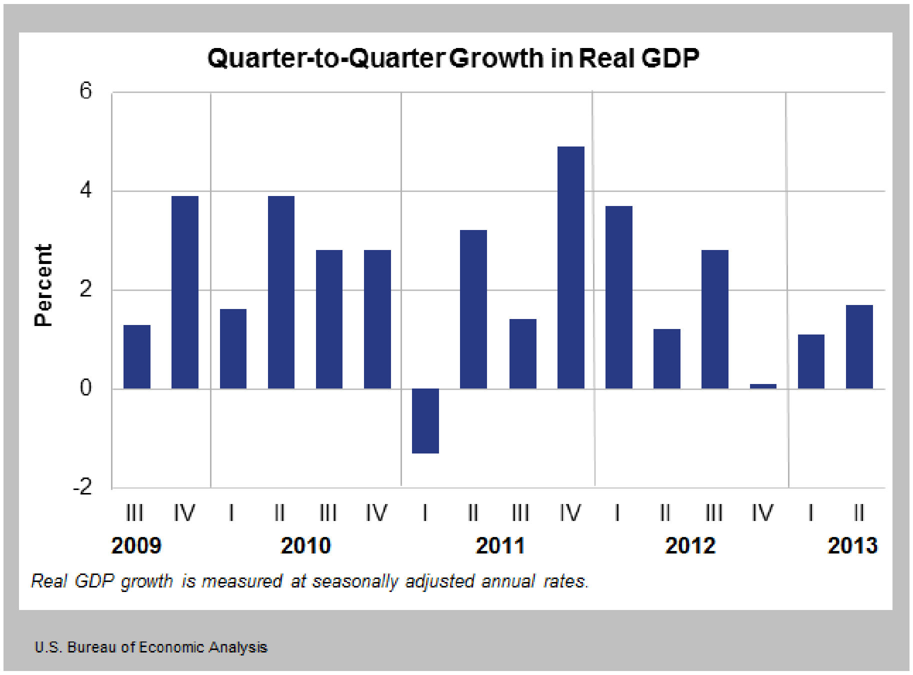
There are four distinct phases of the business cycle:

1. Expansion (or boom): Relatively rapid economic growth.
2. Peak: Period in which economic growth plateaus.
3. Recession (or contraction): Period of relative stagnation or decline in economic growth.
4. Trough: Lowest point in decline of economic growth.



- Fluctuations are often measured using the change in RGDP.
- Despite being labeled a cycle, most fluctuations in economic activity do not follow a mechanical or predictable frequency.



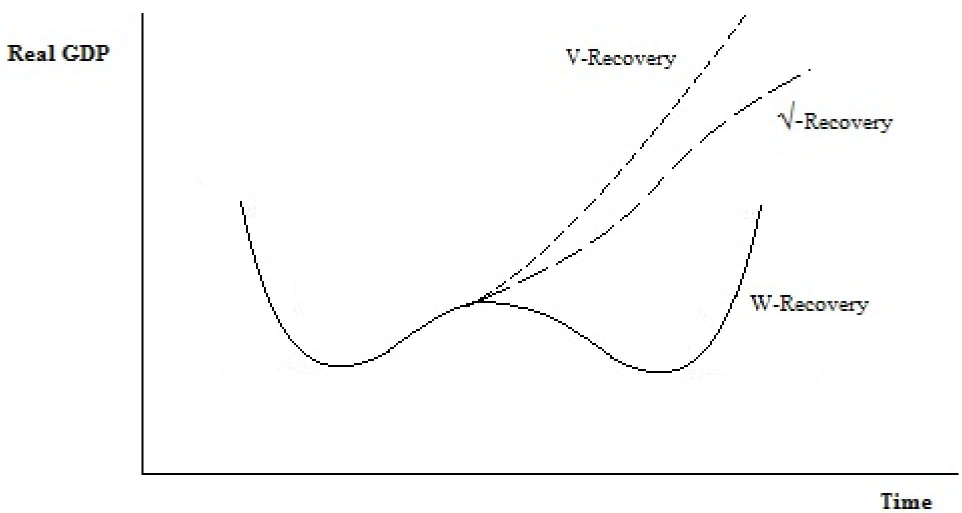


- Prior to the 2008 recession, recent U.S. recessions were not severe.
- Over the last 30 years, U.S. recessions have occurred less frequently.
- In a recession, most seek to know when the economy has entered the trough.

1.2.1. Recovering from Recessions

There are three ways economies recover from recessions:

1. <u>Rebound</u>: Time path of RGDP looks V-shaped.
2. <u>Stagnant</u>: Time path of RGDP after recession is flat.
3. <u>Double dip</u>: Time path of RGDP after recession is another downturn.



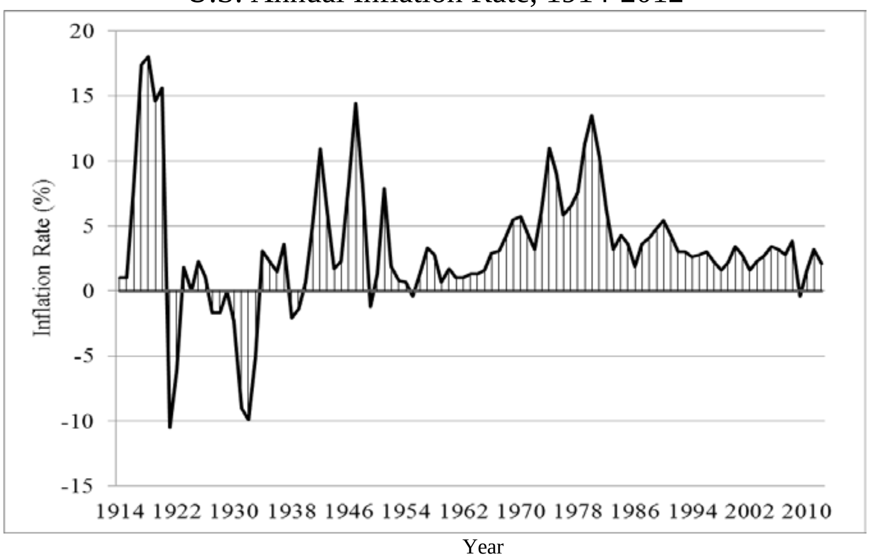
1.3. Inflation

Many consumers understand inflation as an overall increase in prices. Most economists view infrequent or one-time increases in prices as moot since economies adjust quite well to them. Most often economists discuss persistent inflation, which is continual increase in the price level.

Some definitions that aid in understanding inflation are:

1. Inflation: an increase in the overall price level (prices on all goods increase).
2. Price level: a measure of the average price to buy a basket of goods and services.
3. Inflation rate: the percentage increase in the price level from one year to the next.
4. Deflation rate: the percentage decrease in the price level from one year to the next.

U.S. Annual Inflation Rate, 1914-2012



Source: Bureau of Labor Statistics, CPI-U.

- Inflation is not inevitable, it is man-made.
- Fluctuations in inflation have decreased as a result of better monetary policy.

1.3.1. Relevant questions on inflation

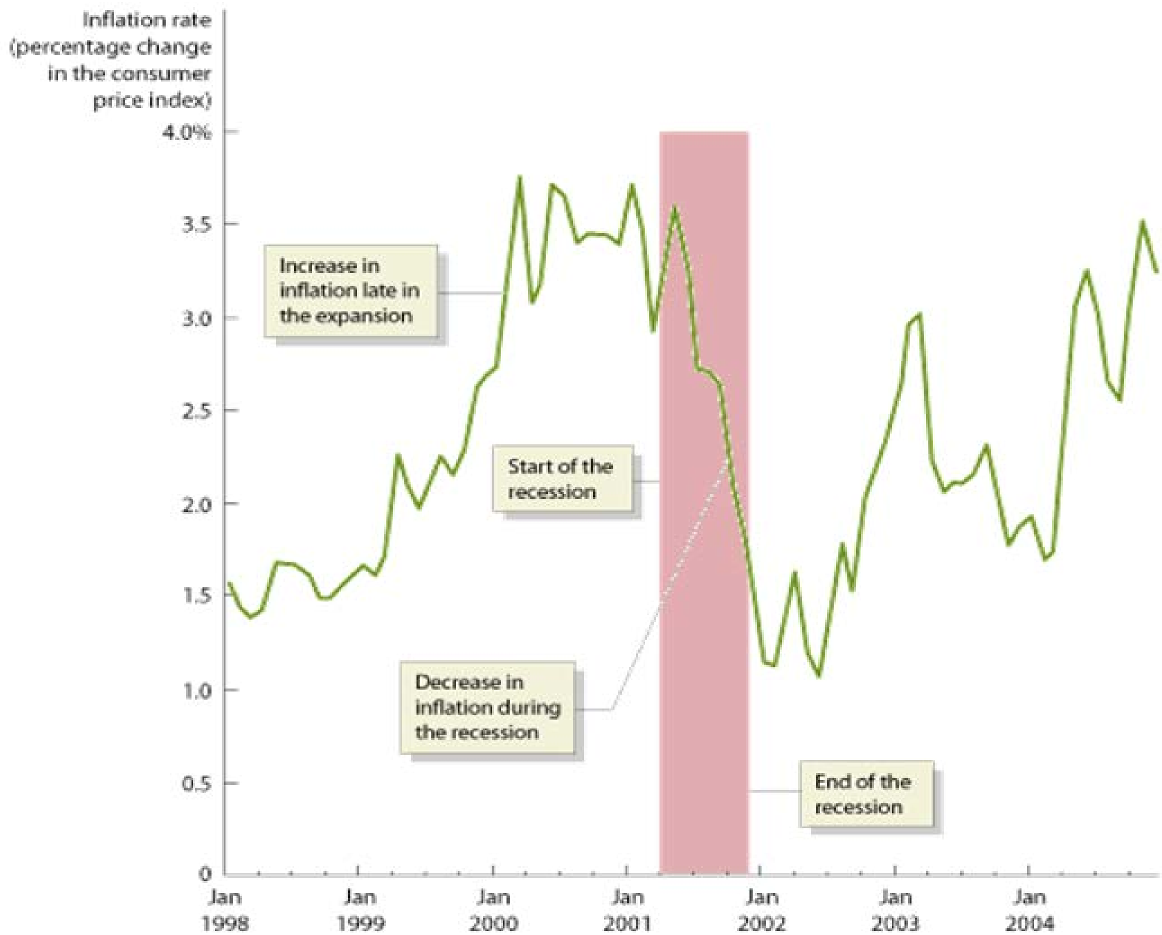
Some questions that arise when studying inflation are:

1. How does inflation affect the income distribution?
2. What problems are caused by anticipated inflation?
3. What problems are caused by unanticipated inflation?
4. How can governments control inflationary pressures?

1.3.2. Inflation and the business cycle

The data on inflation and aggregate output reveal that inflation and output tend to be procyclical – both increasing during expansions and decreasing during recessions. As an economy expands, prices tend to rise and cause inflation, and as it contracts, prices tend to decline leading to deflation.

U.S. Recession of 2001



Source: U.S. Department of Commerce

1.4. Unemployment

Unemployment occurs when a person is available and willing to work, but is currently without work. The prevalence of unemployment is usually measured using the unemployment rate, which is defined as the percent of those in the labor force who are unemployed.

Some definitions that aid in understanding unemployment are:

1. Labor force: the sum of employed and unemployed workers in the economy.
2. Unemployment rate: percentage of the labor force that is unemployed.
3. Discourage worker: individual available for work, but not currently looking.

U.S. Unemployment Rate, 1965-2014

1.4.1. The relationship between unemployment and output

Output, as measured by GDP, and the unemployment rate have an inverse relationship – when output is low, as measured by the deviation from the trend, unemployment is high, and when output is high, unemployment is low.

1.5. The financial system and government policies

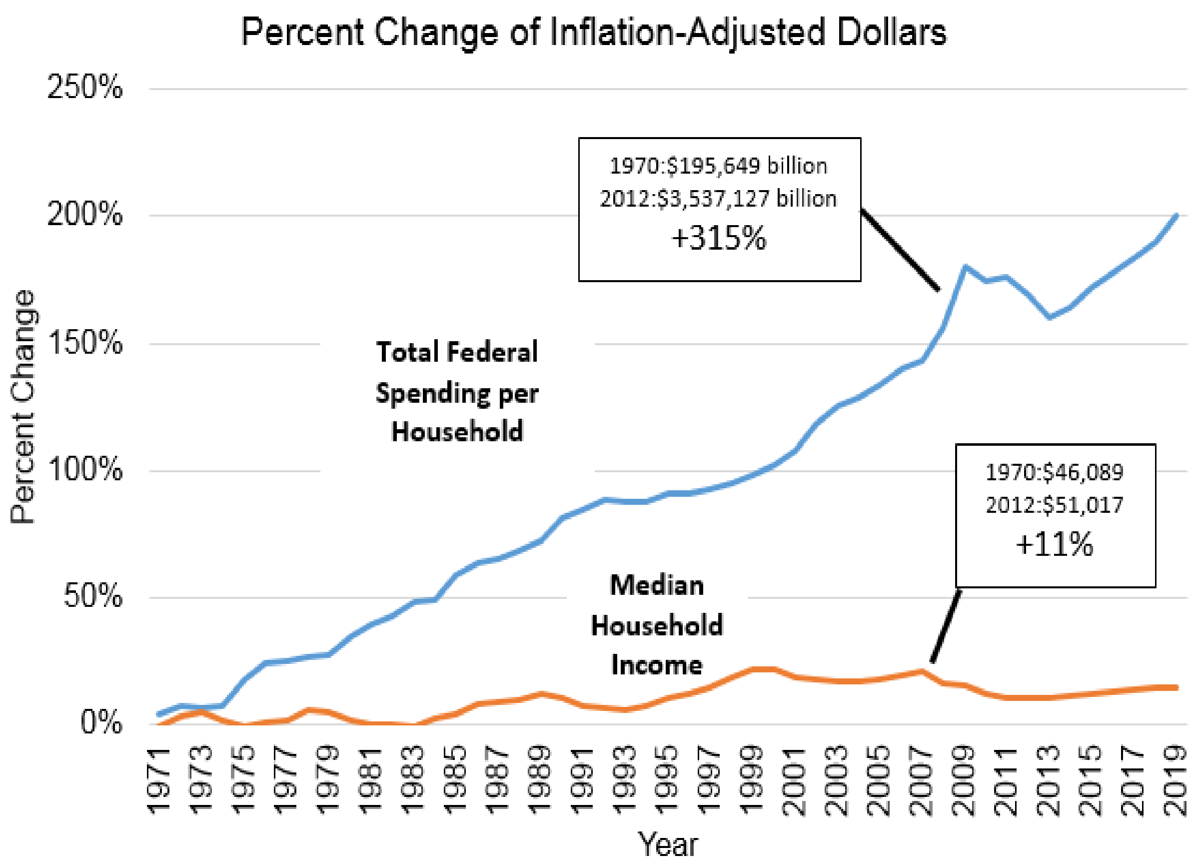
The financial system plays a vital role in the macroeconomy. In any given economy, the financial sector is four times the size of RGDP. We will explore this in-depth later on in the semester. In addition, government policies have profound effects on production capabilities, employment, and individuals' well-being.

2.0. The U.S. Federal budget: taxes and spending

U.S. Federal government spending comprises 20% of GDP even though just under 18% of GDP is raised in taxes. This implies that the government must borrow 2%, on average, every year. Fiscal policies are policymakers' decisions on how much and on whom to tax and on how much and on what to buy. While much of economics is positive, studying the way things are, much of fiscal policy is normative, being decided based on achieving some social goal.

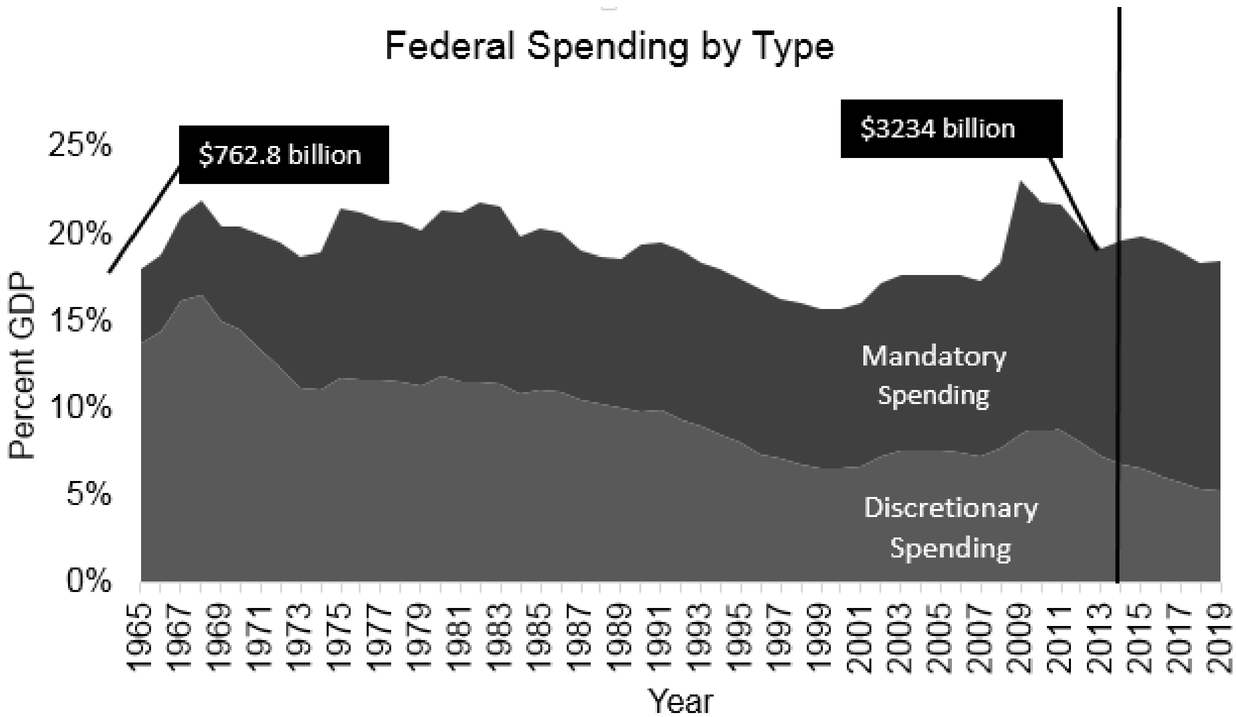
2.1. U.S. Federal spending

The U.S. Federal government spends approximately 3.5 trillion dollars a year. Recently, policymakers have suggested spending be cut by 2.4 trillion over 10 years. This amounts to a reduction in spending of about a quarter of a trillion dollars per year. Even with these cuts, federal spending has risen substantially more than household income over the last 40 years. The table below provides the percentage change in Federal spending per household and median household income. While median household income has only risen by 11%, total government spending has increased by 315%.



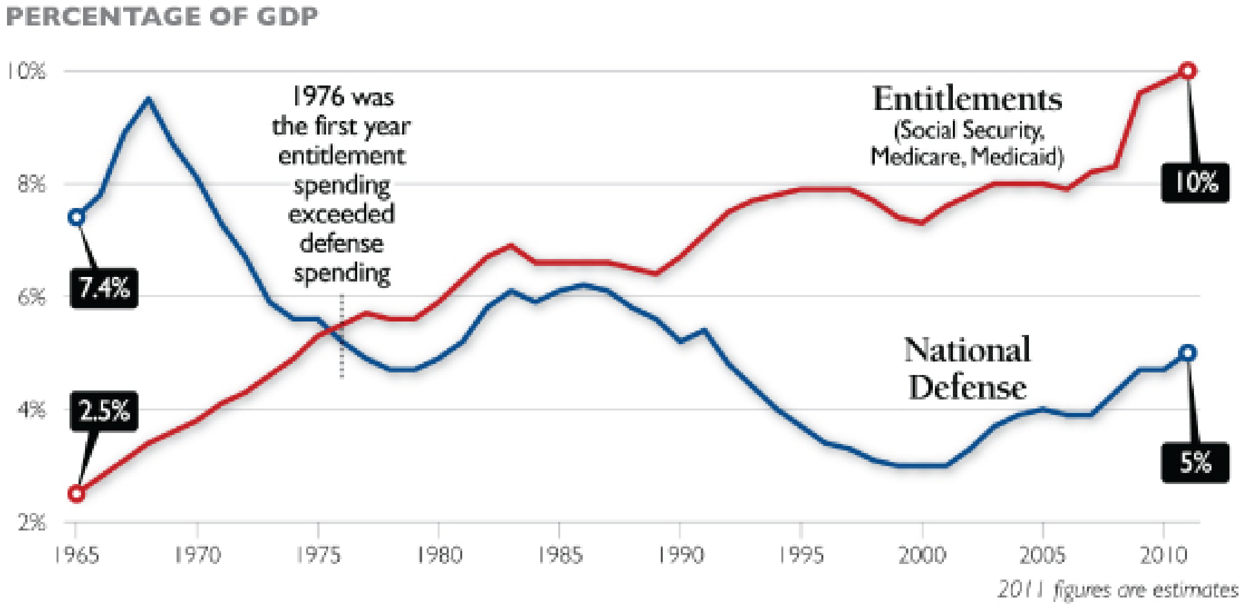
Source: U.S. Census Bureau, White House Office of Management and Budget, and Congressional Budget Office

One way to classify government spending is by whether the expenditure is 'discretionary' or 'mandatory'. Even though politicians choose mandatory spending items at some point, these are expenditures that run on autopilot and do not require yearly congressional approval once initiated. Discretionary spending is subject to annual budgets and accounts one-third of the federal budget. As the figure below demonstrates, mandatory spending has increased more than five times faster than discretionary spending.



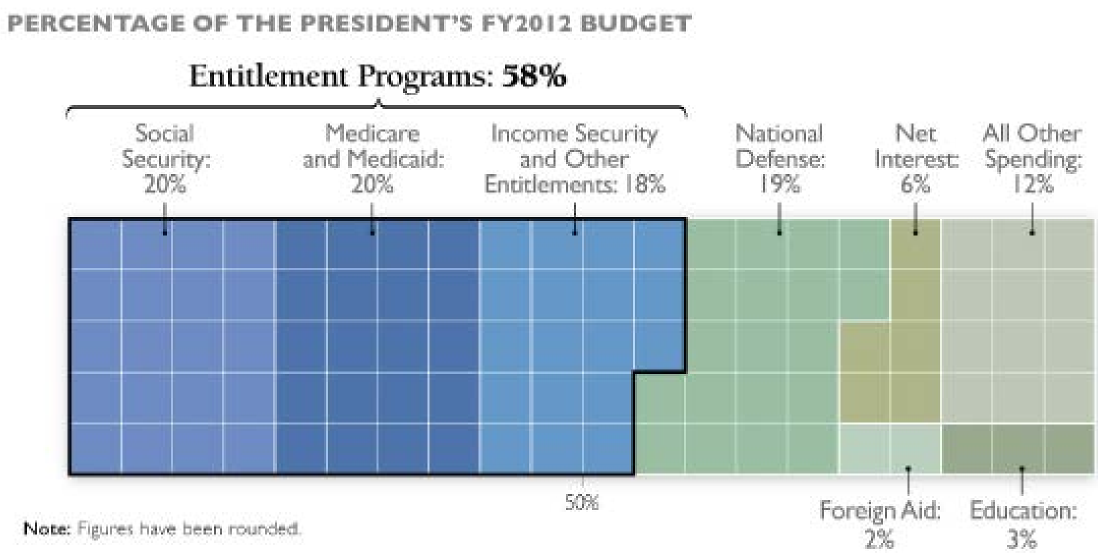
Source: White House Office of Management and Budget

National defense has always been an important line item of the federal budget even though it is not mandatory spending. Interestingly, defense spending has declined significantly over time, despite the recent wars in Iraq and Afghanistan. Spending on the three major mandatory spending items—Social Security, Medicare, and Medicaid—has more than tripled since 1965.



Source: White House Office of Management and Budget.

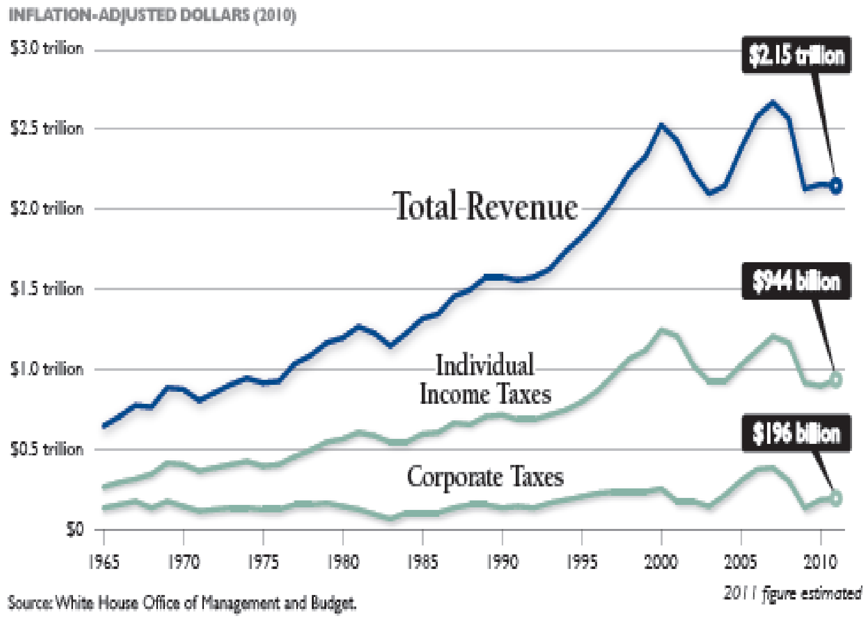
In combination with other entitlements, such as food stamps, unemployment insurance, and housing assistance, Medicare, Medicaid, and Social Security constituted 58% of the President's 2012 budget. In contrast, spending on foreign aid represents only 2%.



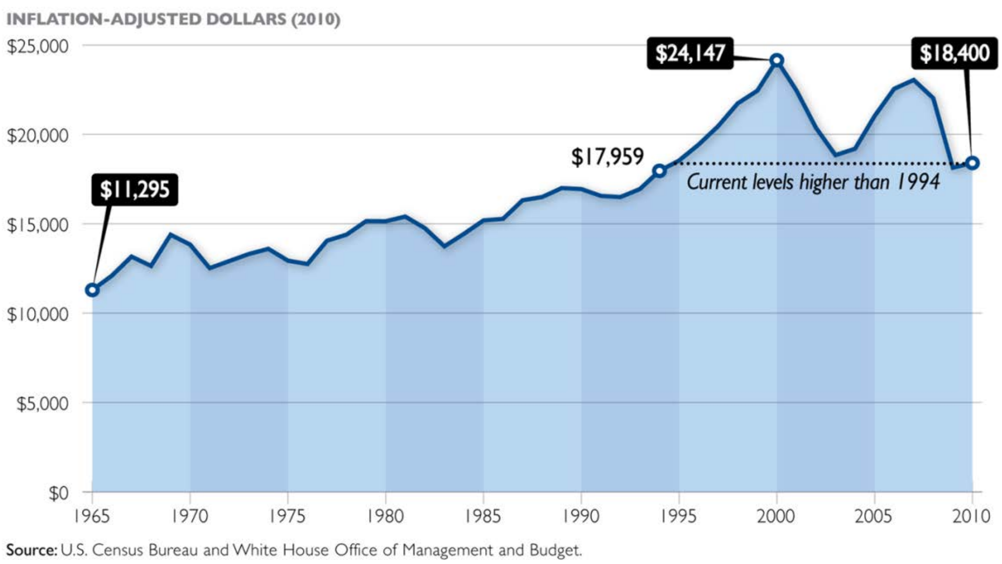
Source: White House Office of Management and Budget.

2.2. U.S. Federal revenue

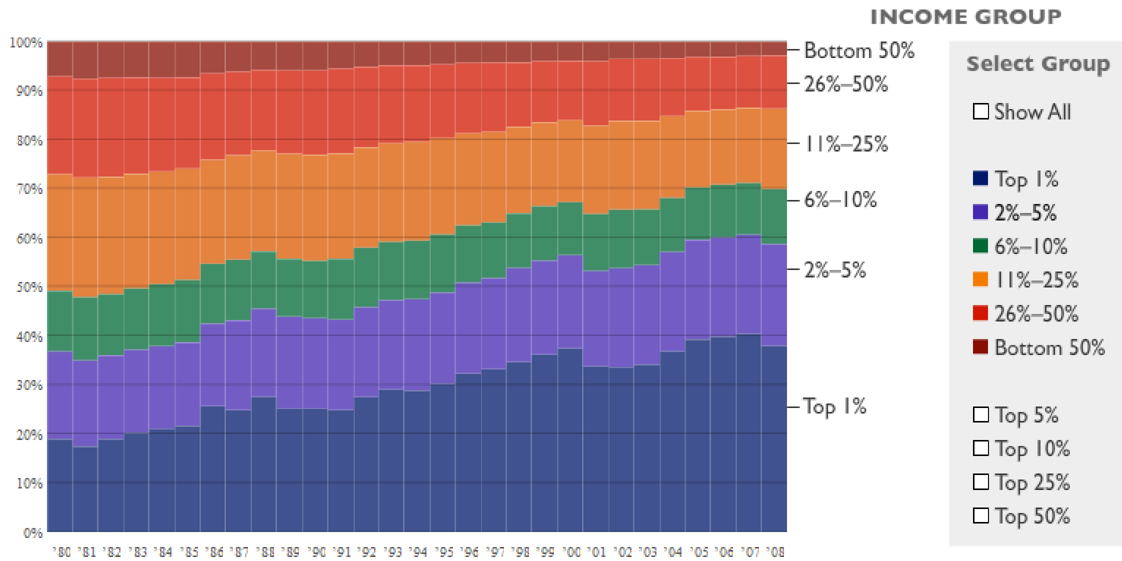
Total tax revenue has increase significantly over the last 45 years. The majority (82%) of tax revenue is from individual and payroll taxes. The Bush-era tax cuts dropped total revenue in the beginning of 2000.



Similar to total government expenditures, total tax revenue does not take into consideration population growth. On a per household basis, total revenue has increased, but not at the same rate as expenditures. In fact, there has only been a 100% increase in per household tax revenue. Currently, average tax revenue per household has remained the same as they were in 1994.



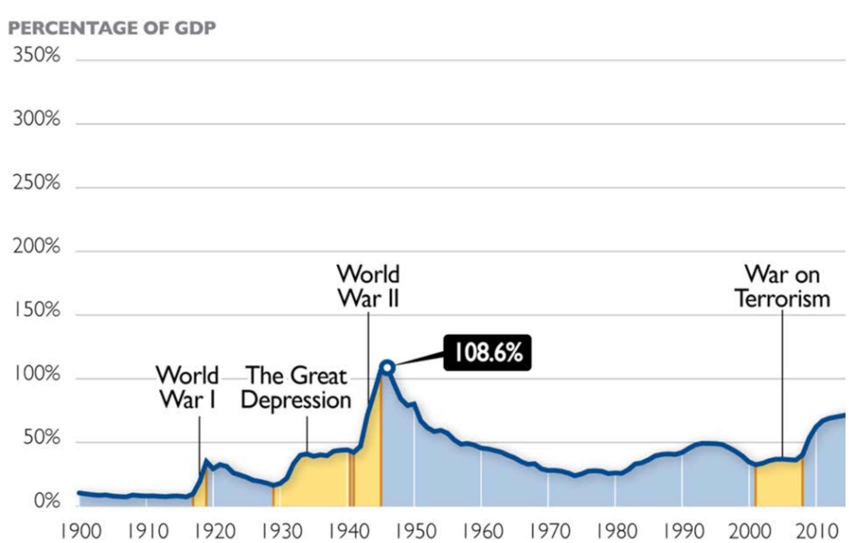
The tax burden is not shared equally across income brackets. In fact, the bottom 50% of U.S. taxpayers pay no taxes, while the top 1% pay 30% of all U.S. Federal income tax.



: Foundation and Internal Revenue Service.

2.3. Debt and deficits

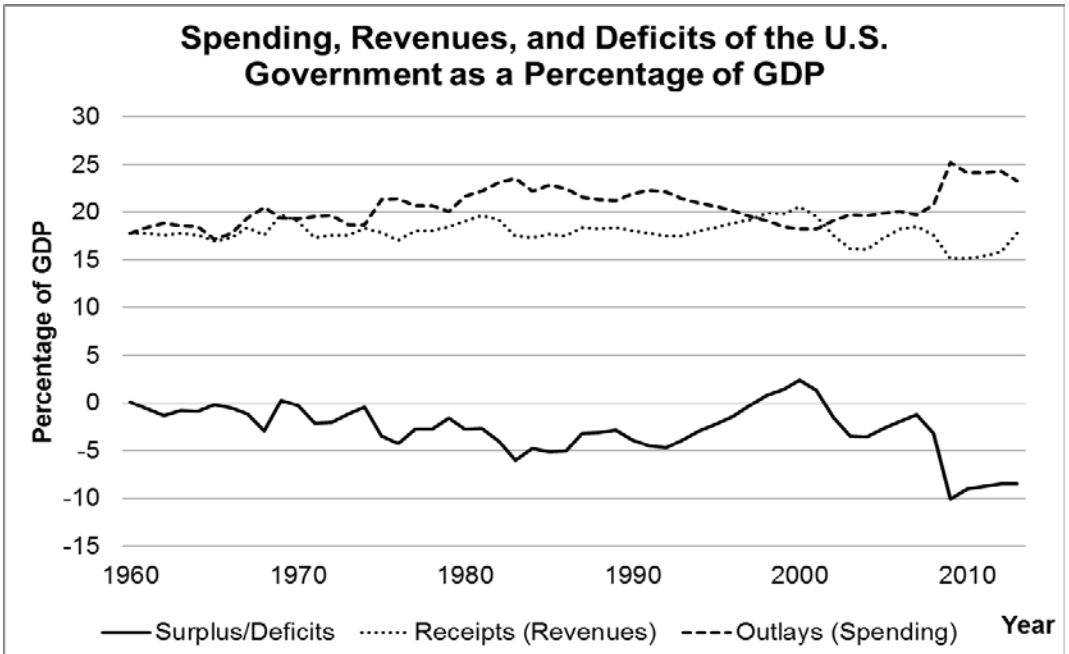
There are two ways to measure debt: overall debt and net debt. The overall debt measures how much a country owes its debtors regardless of how it is financed. Net debt subtracts off debt a country owes itself, and measures how much debt a country owes external debtors. Most economists are more interested in net debt metrics as countries can always pay themselves with another "IOU" or default. The figure below displays U.S. net debt as a percentage of GDP for the last 100 years. In the past, wars and recessions contributed to rapid but temporary increases in U.S. debt.



U.S. Department of the Treasury

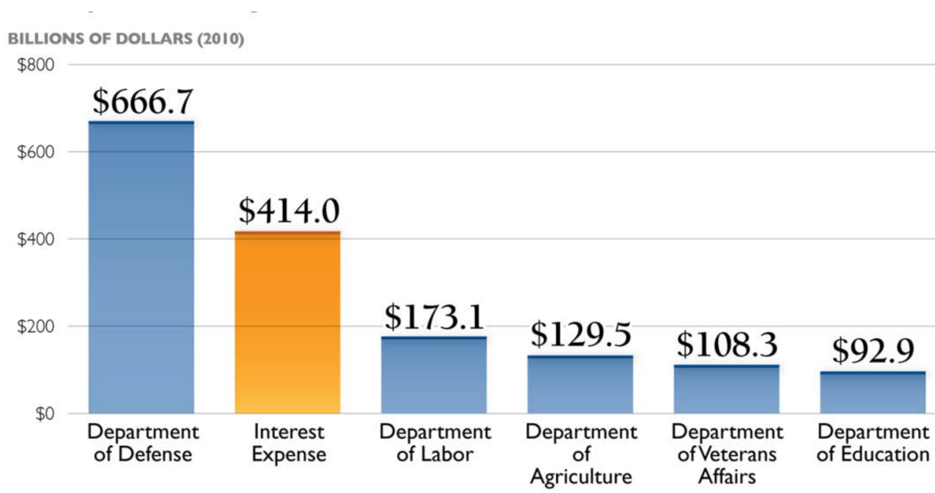
If debt levels remain the same, then, over time, the debt-to-GDP ratio shrinks if a country's economy is growing. For example, the large drop in the U.S. debt-to-GDP after World War II can be partly attributed to the launching of a stalled economy.

Debt accumulates from continual deficits. As the figure below demonstrates, revenue has been more constant than spending. Large gaps due to enormous increases in spending have had long-term consequences.



Source: Economic Report of the President, 2012

One consequence of mounting debt is the interest that must be paid on the debt. In 2010, the U.S. spent more on interest on the national debt than it spent on many Federal departments, including Education and Veterans Affairs.



Source: White House Office of Management and Budget.

When comparing U.S. net debt to other nations, the United States still has a very low net debt relative to the size of its GDP.

Country	Net Public Debt as a Percentage of GDP
Japan	200%
Euro Area	75%
Portugal	75%
Italy	115%
Ireland	66%
Greece	110%
Spain	52%
United Kingdom	66%
United States	66%

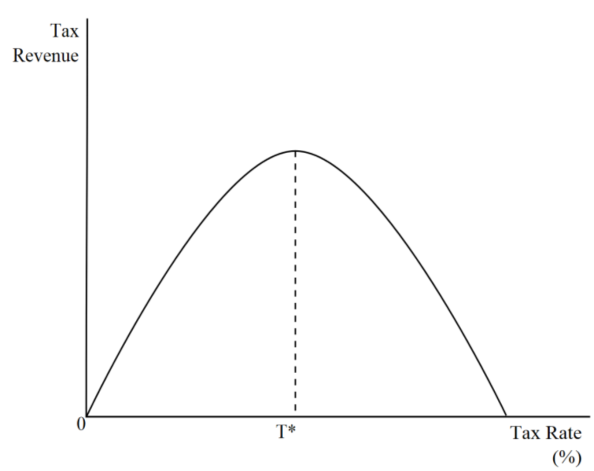
Sources: European Commission; U.S. Office of Management and Budget

Unlike businesses and households, the government does not have to *balance its budget* or be constrained by profit considerations. There are three main ways the government can finance its purchases:

1) Taxation
2) Print money
3) Borrow funds

For businesses and households, only (1) above is available when expenses exceed income. Since the government can always lower expenditures, raise taxes, and/or print money, defaulting on government debt is a choice, not a consequence.

There are some reasons why defaulting of debt seems to be the only option for some countries. First, a government may have maxed out its ability to increase tax revenue. This is illustrated by using the Laffer curve (see below). The tax rate (T*) provides the maximum amount of tax revenue. Beyond T*, all future increases in the tax rate lowers the total amount of tax revenue as individuals choose more leisure time and less time working to lower their tax bill.

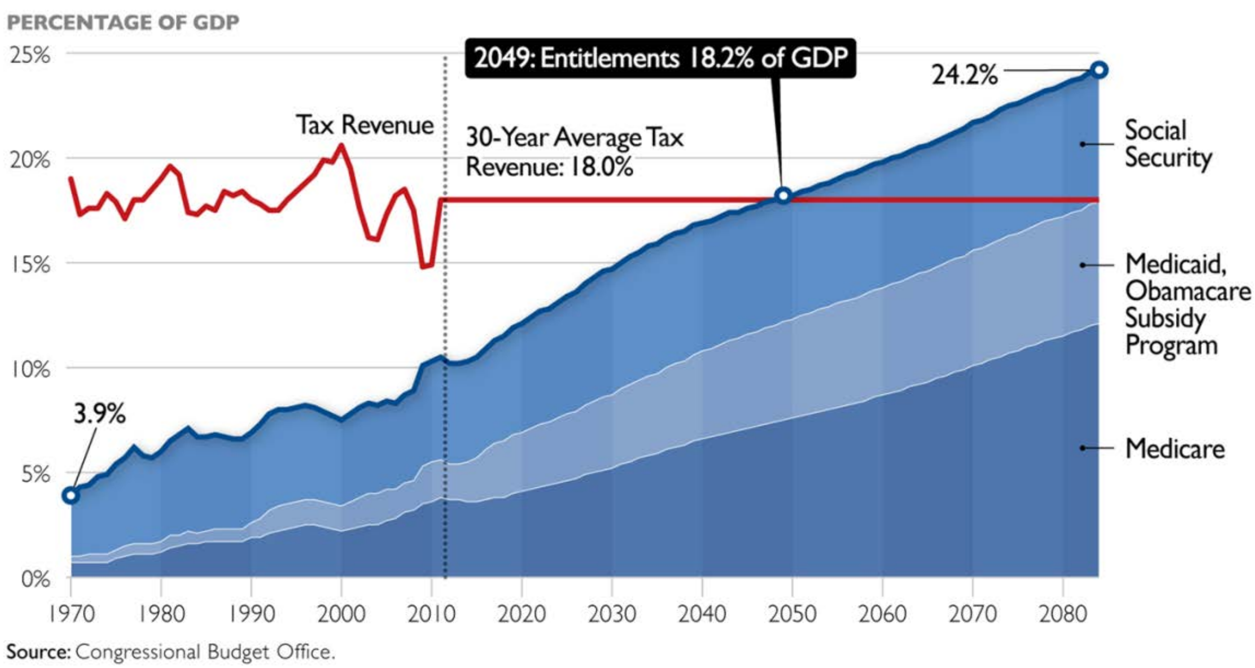


Second, cutting spending may cut into jobs or certain individuals' livelihoods that the government or society may desire to protect. Some countries' economies are comprised of up to 60% government spending. Economies with large government sectors have a hard time cutting spending as the government is doing the majority of the work in the economy. Furthermore, no one wants to see grandma suffer because we don't want to take care of her.

Third, a government may not be able to raise money through printing money. In high inflation economies, these countries produce little seigniorage off of printing money as inflation erodes the currency's value prior to it being spent. Other countries may not be able to print money if the central bank is independent of fiscal policymakers.

2.4. Entitlements

The majority of the U.S. Federal budget issues stem from future mandatory obligations promised to citizens. For example, Medicare and Medicaid are expanding rapidly. If the average historical level of tax revenue is extended into the future at the historical average of 18%, then spending on Medicare, Medicaid, the Obamacare subsidy program, and Social Security will consume all government revenues by 2049. Because entitlement spending is funded on autopilot, no revenue will be left to pay for other government spending.



3. A brief history of U.S. macroeconomics and the current field of macroeconomics

3.1. A brief history of U.S. macroeconomics

Macroeconomics is a relatively new field. Prior to 1934, no standard national accounts even existed

- Hard to know whether a country enters a recession without measuring aggregate output.

The Great Depression changed everything. Around the late 1929, a recession started. This recession deepened and at the height of the recession, the U.S. unemployment rate was 25%. Industrial output fell by 46%.

The biggest problem is that most economists believed the economy couldn't have gotten that bad considering Say's Law: production is the source of demand.

- According to Say's Law, when an individual produces a product or service, he or she gets paid for that work, and is then able to use that pay to demand other goods and services. Thus, if prices are sufficiently flexible, the market should correct itself over a certain period of time as individuals update their price expectations.

However, the economy didn't get better, it got much worse. The occurrence of the Great Depression sparked many new directions in macroeconomics. Examples include how to measure output, fiscal policy reform such as instituting social security in 1935 to aid those who lost everything during the Great Depression, and, later on, reforming monetary policy.

John Keynesian suggested that consumption is a function of income and that the government should attempt to boast incomes ($C = c(Y)$). He reasoned that when the government spends, individuals receive money for these goods and then they in turn spend. Thus, Keynes suggested the Great Depression was a substantial demand side shock (not supply). He also didn't believe that there was a stable money demand relationship ($M = f(P,Y)$) sufficient to conduct monetary policy. Given the responsiveness of the economy to fiscal stimulus, Keynesian economics was born and remains today.

During the 50s, 60s, and 70s, another group of economists lead by Milton Friedman became disgruntled that government intervention was required to make an economy get out of recessions and re-evaluated monetary policy during the Great Depression. Their analysis suggested that monetary and trade policies were to blame (and not some deep underlying problem with economies). The Monetarists (as they were labeled) provided economic theory that suggested that government involvement may exacerbate the business cycle and so the role of monetary policy should be put on autopilot with $M = f(\textit{permanent income, current income})$.

The Keynesians somewhat agreed that money is a function of income, and that if money was increased, then the economy could be stimulated. Thus, New Keynesian economics was born. To tune all of the policies (fiscal and monetary), large macroeconometric models were implemented that predicted the effect of policy changes using static rules. These models used in the 60s and 70s lead to high rates of inflation and unemployment.

The first attempt to explain coherently what was going wrong was Lucas (1977). This is now known as the Lucas Critique. The Lucas Critique suggests that because the parameters of macroeconometrics are not structural, i.e. not policy-invariant, they would necessarily change whenever policy changes. That is, individuals use rational expectations to forecast these policies and base decisions on them. Lucas summarized his critique:

> *Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.*

The Lucas Critique demonstrated that static rules based on historical data cannot be used to forecast the future because the rules were fixed in the historical setting when the outcomes were observed. Thus, there was no guarantee that the outcomes wouldn't change when the rules changed. For example, if a prison has never had a prisoner breakout in the past, this doesn't mean that the inputs to maintain security can be manipulated without consequence.

Unfortunately, the Lucas Critique doesn't say how to get things right. Out of the ashes of this critique was born three groups of macroeconomics (with an additional stepchild group – the Austrians) which now comprise Modern Macroeconomics.

3.2. New Classical Macroeconomics

New classical macroeconomics started under the direction of Ed Prescott and Chris Sims. These and other NCM economists are referred to as the freshwater economists (think Chicago and Minnesota).

Lucas suggested that micro-founded models were required and that the deep parameters be identified. This lead Prescott to develop the real business cycle (RBC) model based on the Ramsey model. The Ramsey model was an old work horse of the growth literature that used utility functions, profit functions, and general equilibrium.

- Much of Keynesian economics disappeared:
    - Nominal rigidities
    - Imperfect Info
    - Money
    - Philips curve (short-run relationship between unemployment and inflation relationship)
- Focused on stochastic properties such as technology shocks
- Goal: how far can we explain the business cycle without imperfections?
- <u>Problem</u>: Technology is typically diffused over time, not shocking the economy each quarter.
- <u>Answer</u>: Can't go very far in explaining business cycles with this class of models.

3.3. New Keynesian

New Keynesian macroeconomics combined existing RBC models and added 'realism' to the model. These economists are referred to as saltwater economists (think east and west coast). They reformed the existing Keynesian economic models.

- Nominal rigidities explain business cycles.
  - Credit market constraints
  - Efficiency wages
- They maintain three important equations:
  - Sticky price equation - monopolistically competitive markets so that firms can be price makers. Only a few firms can change prices at any one time.
  - An augmented Philips curve where inflation is a function of current and expected prices.
  - The Taylor rule
- Partial equilibrium

3.4. New Growth Theorists

The only thing new about the growth theorists is the complete abandonment of explaining the business cycle.

Focus solely on long-run growth.

- Public goods
- Nature of knowledge (public)
- Externalities in capital accumulation
- Fiscal policies

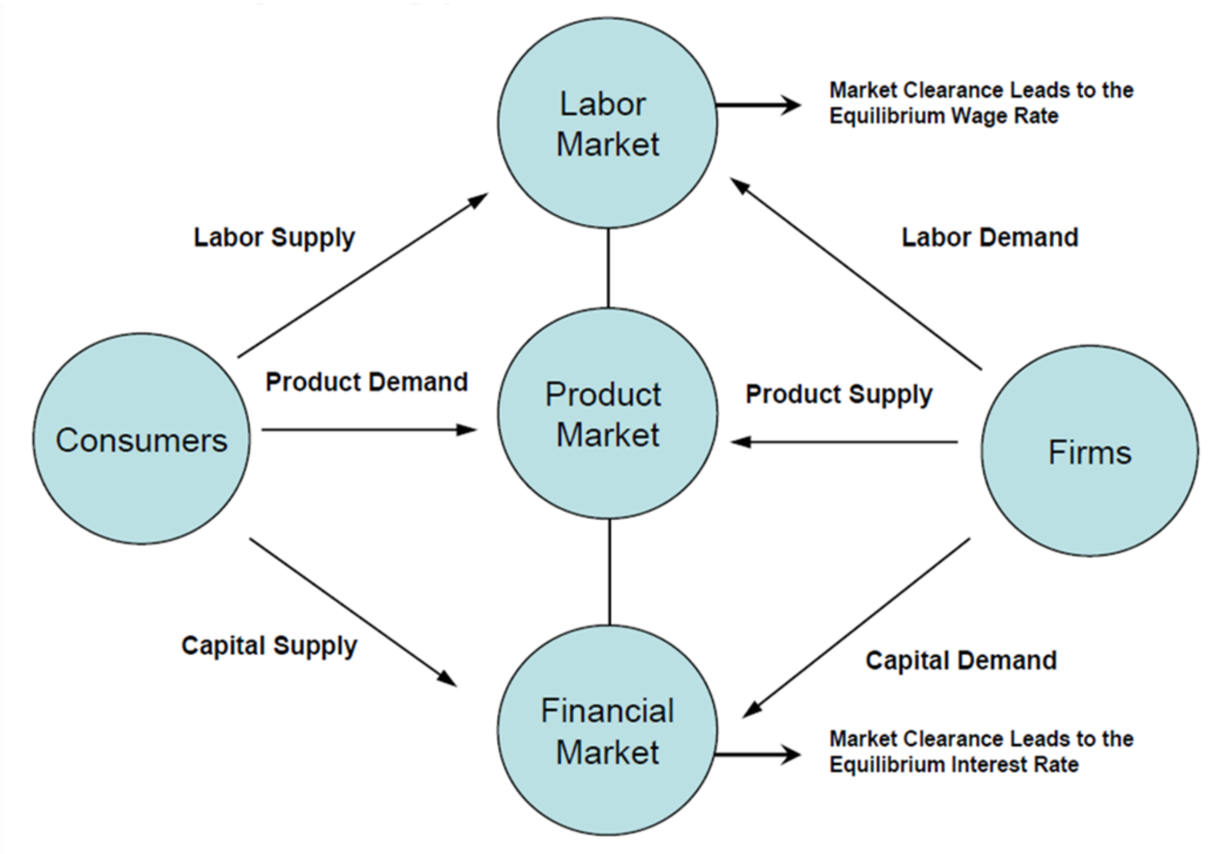
3.5. Austrian Economics

Continue to preach the identical argument that government intervention (mostly monetary policy) causes business cycles. Eliminate the government and the business cycle will go away.

3.6. The Current Issues of Macroeconomics

- How to incorporate financial markets
- Labor markets – how to introduce unemployment… maybe search models.
- Goods price mark-up and the business cycle
- Animal spirits [demand shocks]
- Anticipation (Peter Lynch and other that can somehow foresee markets)
- SR and LR effects of fiscal and monetary policy

3.6.1. A Topology of Macroeconomics



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Lecture 2: Macroeconomic Growth Empirical Facts

1.1. The Importance of Growth

There are very large differences in income per capita and output per worker across countries.

The following table provides the incomes per capita for a few countries.

Real GDP per capita[1]

United States	$33,330
Mexico	$8,000
China	$4,000
India	$2,500
Nigeria	$1,000

Even though these countries are some of the U.S.'s major trading partners, their incomes are quite low.

> **The dominant macroeconomic fact of developed economies for the last two centuries is that of output growth.**

The famous words of Nobel Prize Winner Robert Lucas sum up the biggest puzzle in macroeconomics:

> *Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the nature of India that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.*

What is interesting about this quote is that at the time, the Indian economy was growing faster than Indonesia's and Egypt's. It's just that the data did not reflect it yet.

[1] All figures are in 2000 U.S. dollars and are adjusted for purchasing power parity.

Interestingly, even small differences in growth rates over long periods of time can make a huge difference in final outcomes.

Exercise 1: What if the U.S. grew like other countries?

U.S. per-capita real GDP grew by a factor of 10 from 1870 to 2000. Average growth rate was approximately 1.8%.

Year	Real GDP per capita
1870	$3,340
2000	$33,330

In 2000 dollars.

- ❖ What if the U.S. had grown at .8% (like India, Pakistan or the Philippines)?

- ❖ What if U.S. had grown at 2.8% (like Japan or Taiwan)?

- ❖ At an annual rate of 1.8% GDP per capita growth, what year will the U.S. attain a per capita income of $121,000?

For many LDCs, it will require a high growth rate sustained for long periods of time to catch up with the U.S. and other G8 countries.

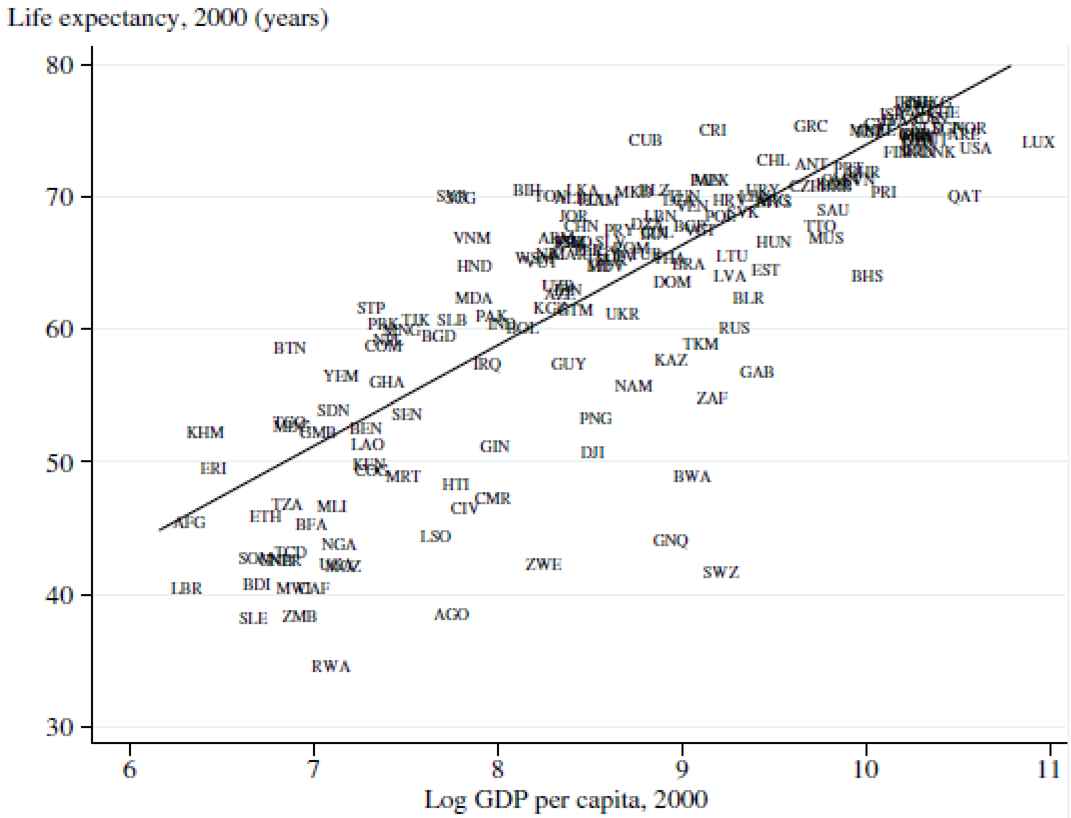
Exercise 2: How long would Tanzania take to catch up to the U.S. GDP per capita in the year 2000 if it currently has a GDP of $503 and grows at 1.8% per year?

- ❖ What about 2.8%?

- ❖ What about 6.4% (like Singapore or South Korea)?

❖ Do these differences translated into any meaningful difference in quality of life?

The figure below captures the difference in average life expectancy that occurs by per capita GDP.



- When comparing G8 countries to LDCs, there are striking differences in the quality of life, standards of living, and health.
    - Individuals in rich nations have life expectancies between 70 and 80 years while individuals in poor countries have expectancies between 40 and 50 years.

- When comparing G8 countries to themselves, there are subtle differences, but mostly clustered around the same area.
    - Increases in GDP per capita do not increase life expectancy.

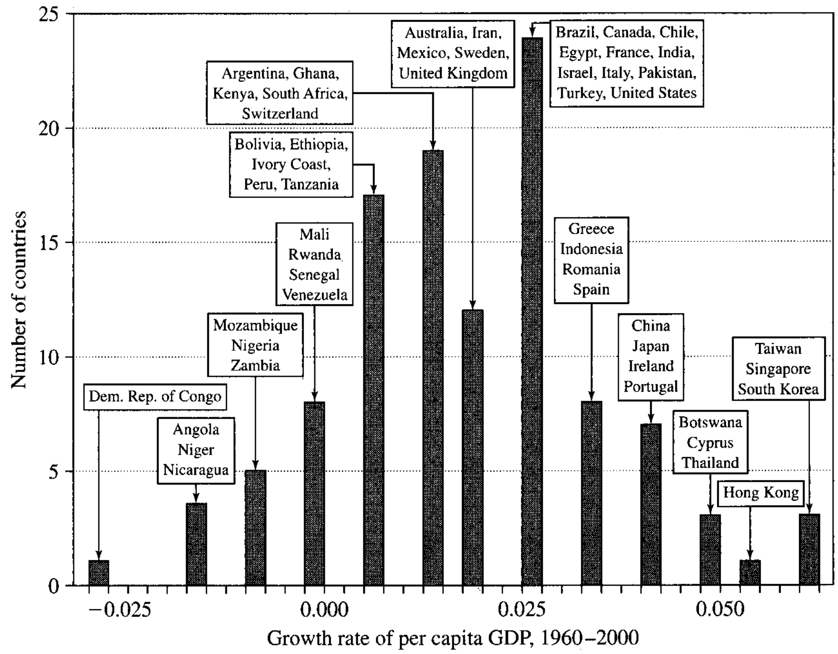
There is a health-income cycle that feedback onto itself (with diminishing returns).

Increase in GDP per capita => Increase in health care => healthier labor force => Increase in GDP per capita => ….

1.1. Cross-Country Income Differences

The previous section provides a sketch on growth rates and how growth *could* affect incomes over time. However, how growth *is* affecting incomes over time matters more.

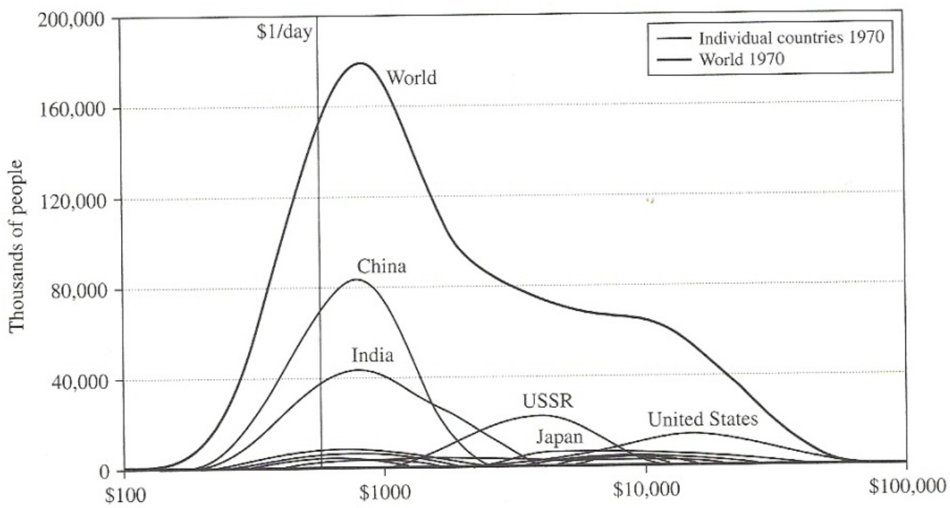
Many LDCs have high growth rates while the G8 countries have low growth rates. Below displays a histogram of average annual real per capita GDP growth rates between 1960 and 2000 for 112 countries.

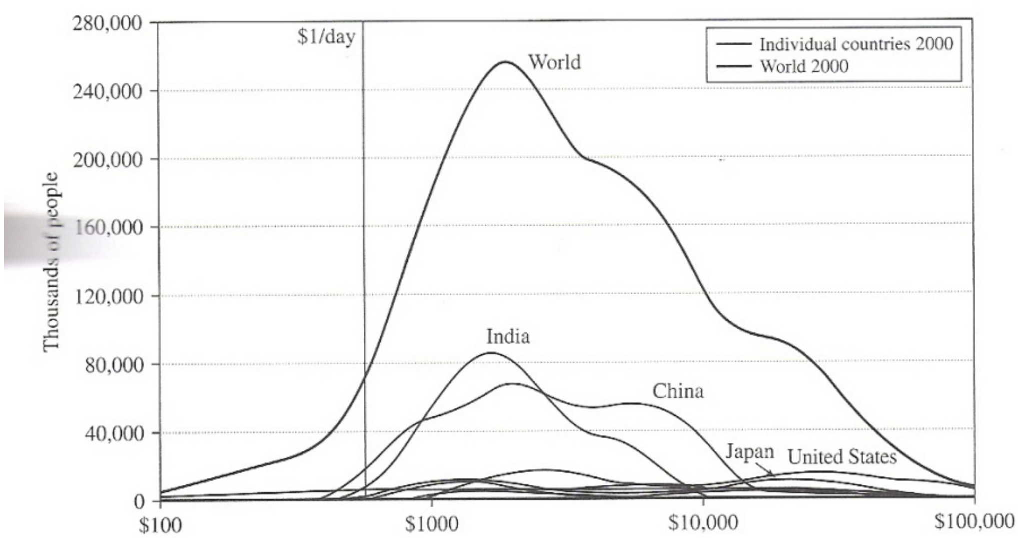


- The average growth rate is 1.8% per year and the range is -3.2% to 6.4%.
- Many countries have increased their levels of per capita GDP by a multiple of 7 in just 40 years.
- 12 countries have sustained negative economic growth; other than Nicaragua and Venezuela, all of these are comprised of sub-Saharan African countries.
- P.I.I.G.S. are especially troubling considering their recent downfall.

How has the world distribution of income been affected by having differential growth rates?

Below are displayed the distributions of world income in the year 1970 and 2000. The figures plot the number of people at each level of income (on a logarithmic scale) for 1960 and 2000, respectively.[2]





- ❖ Some important facts are revealed.

[2] Why logs? If a variable grows at a proportional rate, then the log of that variable grows at a linear rate. Thus, if two variables grow at the same proportional rate, then the difference of these variables in logs is constant.

- Are rising incomes bad if it creates larger inequality?

    - Economic growth is generally good for welfare but often creates winners and losers. Joseph Schumpeter's 'creative destruction' emphasizes this aspect of economic growth; productive relationships, firms, and sometimes individual livelihoods will be destroyed by the process of economic growth, because growth is brought about by the introduction of new technologies and creation of new firms replacing existing firms and technologies.

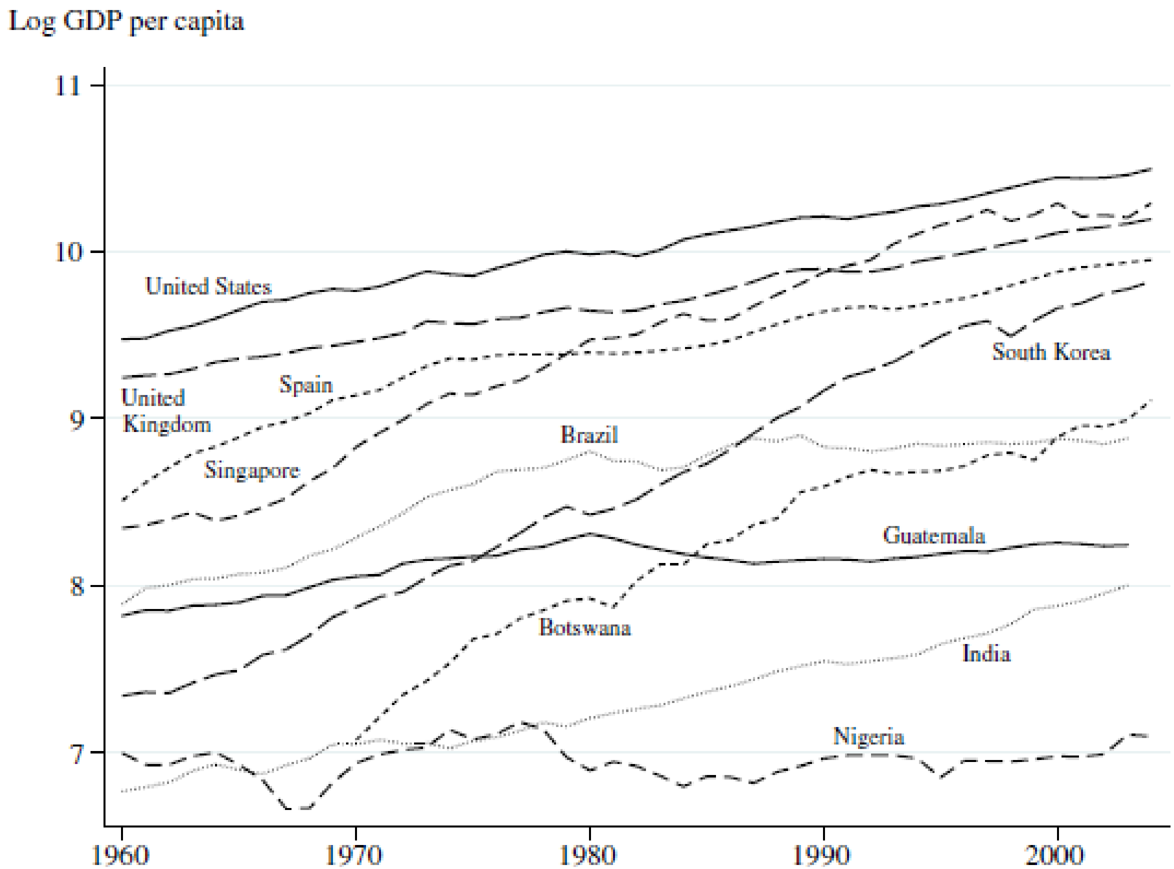
    - Most evidence on the industrial revolution suggests the living standard of the majority of workers may have fallen or at least remained stagnant. This may shed light on why certain segments of the population favor policies and institutions that do not encourage growth.

    - Rising income inequality does not imply the poor are getting poorer.

        - At a minimum, it means that some individuals' earns are rising faster than others.
        - Baumol's disease may actually be a benefit to society more than a cost.

The above graphs suggest that countries' incomes may be converging. Over time, have incomes converged, diverged, or remained the same?

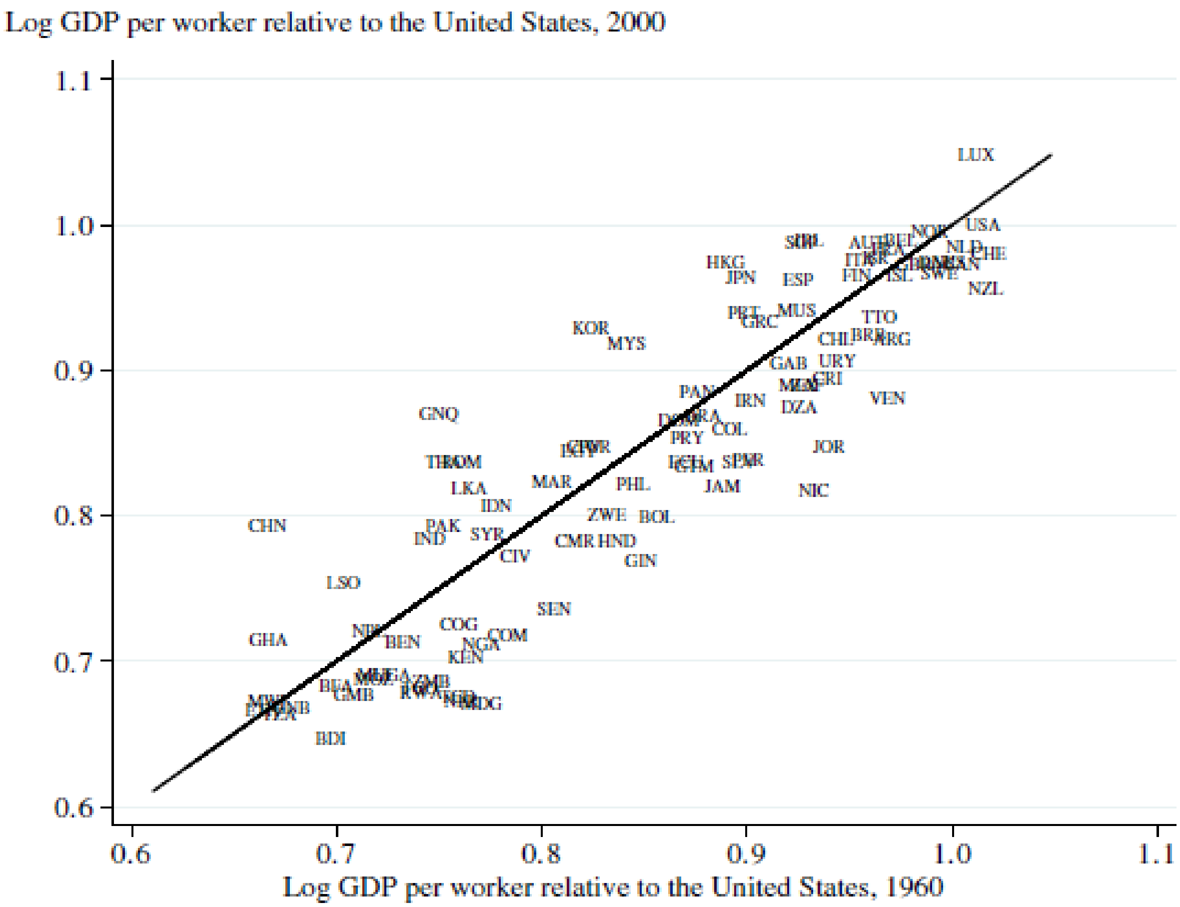
The figure below depicts the evolution of incomes over time for 10 countries.



- The figure suggests that the difference in per capita income that existed in 1960 exist today.
- There are a few exceptions:
    - India, Botswana, South Korea, and Singapore seem to be converging.
    - Nigeria and Guatemala seem to have remained stagnant.

The figure below depicts log GDP per worker relative to the United States for 1960 and 2000.

- The 45 degree line indicates that a country is the same *income* distance away from the U.S. in 1960 as the year 2000.
- Countries above (below) the 45 degree have increased (decreased) their GDP per worker relative to the U.S. and are converging to (diverging from) the U.S. GDP per worker.

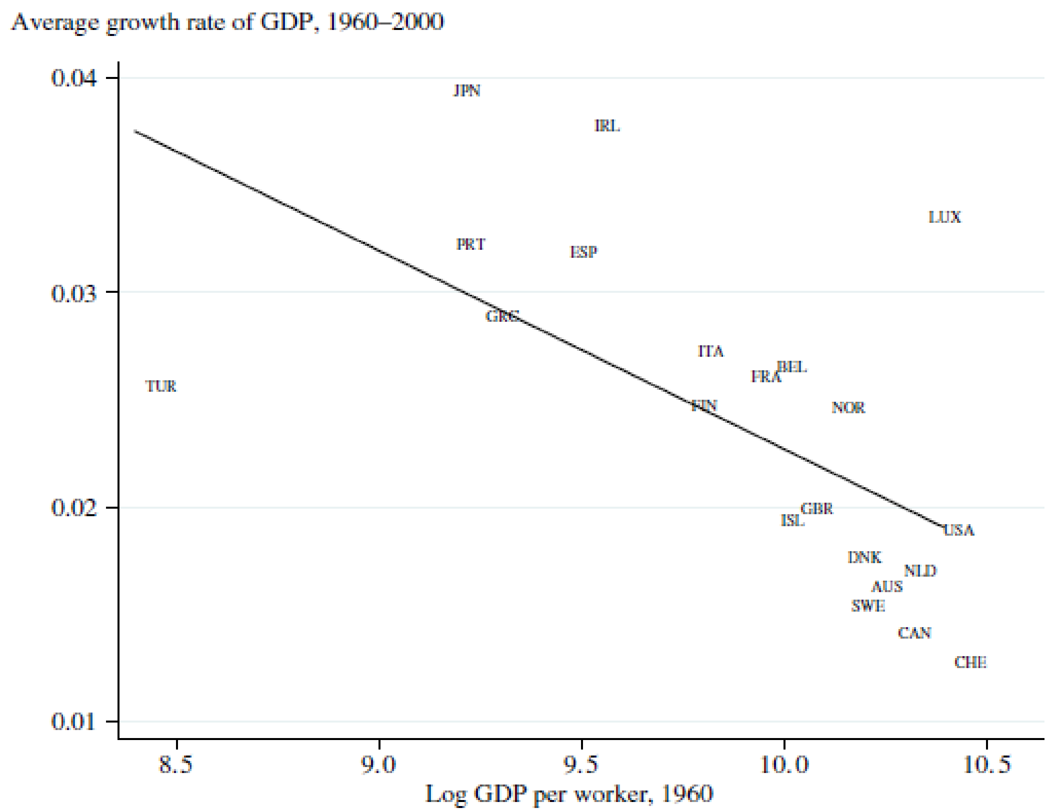


- Some countries are converging while others are diverging.

Another way to view convergence is grouping similar countries together and asking whether these countries are converging – conditional convergence.

The OECD countries are typically grouped: all have some form of democracy, similar levels of education, infrastructure, and banking systems.

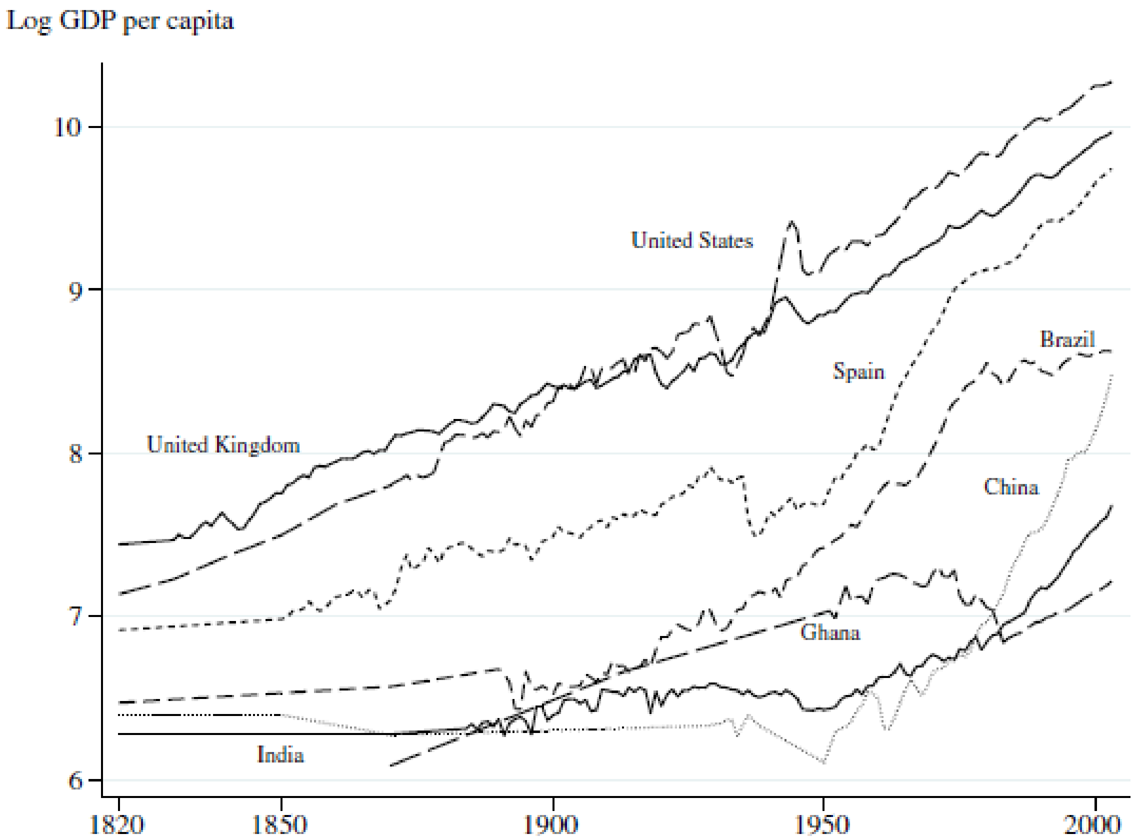
The figure below plots the annual growth rates of GDP per worker between 1960 and 2000 against log GDP per worker in 1960 for core OECD countries.



- The data suggest that there is some conditional convergence. That is, similar type countries are converging towards similar log GDP per worker.
  - High income countries have low growth rates.
  - Low income countries have high growth rates.
- The relationship between GDP and GDP growth holds broadly, but there are some exceptions.
  - Luxembourg has a high GDP and high growth rate.
  - Turkey has an extremely low GDP and lower than expected growth rate.

The above figures suggest that the gap in GDP occurred prior to 1960 and that some countries are converging to higher income countries while others are falling behind. If the growth gap occurred prior to the 1960s, when did this growth gap emerge?

The figure below displays the evolution of income per capita for various countries from 1820 through 2000.



- Small differences in growth rates sustained over a long time can make a big difference.
- Evidence suggests that differences in income per capita existed prior to 1820.
- Much of the divergence is identified during the late 19th - early twentieth century.
- Prior to the Industrial Revolution, there was very limited economic growth.
- While certain civilizations, including ancient Greece, Rome, China, and Venice, managed to grow, their growth was either not sustained or progressed only at a slow pace.

So, what causes growth?

- Even though some economists think that they have solved the mystery, the elusive quest for growth in many countries remains.

- There seems to be more correlation than causation. That is, necessary conditions but not sufficient conditions for growth.

- Many policies that developed nations have provided to lesser developed nations are ill advised and have done great harm to growth of their nations.

- An example: How many of you will buy a soccer ball knowing that it was made by a child in a LDC? The U.S. Fair Labor Standards Act was declared constitutional in 1941.

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Lecture 3: The Solow Growth Model

1. Introduction

The Solow growth model is developed for a Robinson Crusoe type economy. A representative household produces a single final good which is allocated between consumption and savings for future production. The dynamics of the model are examined and in particular, the

- Transition path
- Steady state.

The golden rule of capital accumulation is solved along with the dynamical path from technology shocks.

1.1. The Solow Growth Model

Even though the model is extremely basic – the model only focuses on four variables: output (Y), physical capital (K), labor (L), and "knowledge" or the "effectiveness of labor" (A) – the Solow growth model is a dynamic general equilibrium model.

1. For simplicity, all households are identical so that the economy trivially admits a *representative household* – meaning that representation is the result of the behavior of a single household.
    a. This household has no utility function.
    b. While households consume and save, this is dictated to them exogenously from the model.
    c. The savings rate is defined as the fraction of output, $s \in (0,1)$, to be used as an input in production next period.

2. Technology is a pure public good. A pure publicly good has two properties:
    a. Non-excludable
    b. Non-rivalous

Since technology is free, all production functions are identical. Thus, there is one aggregate production function.

3. The lack of optimization on the household side is the main difference between the Solow and the Neoclassical growth model (to be studied next). Abstracting from the utility framework permits an understand of the mechanics behind most macroeconomic models.

1.1.1. The Environment of the Basic Solow Model

The basic set up of the Solow Model is as follows:

- Time is discrete and is indexed by $t \in \{0, 1, 2, \ldots\}$. Time periods correspond to a year, a generation, or any other arbitrary length of time. For now, we do not need to specify a time scale.

- The economy is closed (an isolated island).

- There are no markets.

- The number of individuals in the economy grows at a constant rate n.

- Individuals provide labor inelastically.

- There is physical capital at the beginning of time, $t=0$, in the amount K_0. This is subject to:

    - Depreciation per period at the fixed rate $\delta \in (0,1)$.
    - Investment in the amount I_t in period t that can be used next period together with undepreciated capital.

- At the beginning of period t, aggregate physical capital stock is combined with labor to produce a final good, Y_t.

- Technological progress starts with an initial technology of A_0 and is assumed to grow at rate g.

- Individuals can either consume, c_t, or invest, i_t, in the physical capital stock, k_t.
    - Think of the final good as an agricultural product that can be used in consumption or as an input for the production next period.

Exercise 1: Assuming the population growth rate is fixed at rate n, find period t's population given the population in period 0, L_0.

Exercise 2: Given any L_t, find L_{t+1}. Label it (1).

Exercise 3: Given the technological growth rate and the level of technological progress for period t, A_t, find next period's technology level. Label it (2).

Exercise 4: Construct the law of motion for physical capital. Label it (3)

Exercise 5: Construct the *aggregate resource constraint*. That is, think of an equation with output on one side and how it is allocated on the other side. Label it (4).

1.1.2. Technology and Production

The neoclassical aggregate production function takes the form

$$Y_t = F(K_t, A_t L_t) : \mathbb{R}^3_+ \to \mathbb{R}_+ \tag{5}$$

- Time does not enter the production function directly. That is, output changes over time occur only if the inputs and/or technology change, not the function itself.

- Given a fixed quantity of capital and labor, the amount of output rises only if *A* rises.

- *A* and *L* enter multiplicatively. *AL* is referred to as *effective labor*, and technological progress that enters in this fashion is known as *labor-augmenting* or *Harrod-neutral*.[3]

- *A* is a number, but represents a more abstract concept of the effects of production organization and utilization of factors of production.

1.1.2.1. Neoclassical production function

The production function is *neoclassical* if the following properties hold:

1. **Constant returns to scale (CRS), or homogeneity of degree one in *K* and *L*.**[4]

$$F(\lambda K, \lambda L, A) = \lambda \cdot F(K, L, A) \quad \forall \lambda > 0 \tag{6}$$

2. **Positive and diminishing marginal products.**

$$\begin{aligned} F_K(K,L,A) &\equiv \frac{\partial F(K,L,A)}{\partial K} > 0, \quad F_L(K,L,A) \equiv \frac{\partial F(K,L,A)}{\partial L} > 0, \\ F_{KK}(K,L,A) &\equiv \frac{\partial F(K,L,A)}{\partial K^2} < 0, \quad F_{LL}(K,L,A) \equiv \frac{\partial F(K,L,A)}{\partial L^2} < 0. \end{aligned} \tag{7}$$

3. **Inada conditions hold.**

$$\begin{aligned} \lim_{K\to 0} F_K &= \lim_{L \to 0} F_L = \infty, \\ \lim_{K\to \infty} F_K &= \lim_{L \to \infty} F_L = 0. \end{aligned} \tag{8}$$

- These three properties imply *essentiality*. That is, each factor is essential in production.

$$F(0, L, A) = F(K, 0, A) = 0.$$

---

[3] *A* is also referred to as technology, the productivity parameter, or the shift parameter.
[4] If *F* is homogenous of degree 1 in *K* and *L*, then F_K and F_L are homogenous of degree 0. Thus the marginal products depend only on the ratio *K/L*.

4. **The function takes continuous nonnegative arguments and maps to continuous nonnegative levels of output.**

Given 1 - 4, F satisfies

$$Y = F(K,L,A) = F_K(K,L,A)K + F_L(K,L,A)L \tag{9}$$

or equivalently

$$1 = \varepsilon_K + \varepsilon_L$$

where

$$\varepsilon_K \equiv F_K \cdot K / Y \text{ and } \varepsilon_L \equiv F_L \cdot L / Y$$

- This is referred to as *Euler's Theorem*.

While not necessary for a neoclassical production function, the model assumes that capital and labor are complementary:

$$F_{KL} > 0 \tag{10}$$

The production function can be written in *intensive form* assuming CRS. Define per capita measures using corresponding lower-case variables: $k_t = K_t / L_t A_t$, $c_t = C_t / L_t A_t$, $y_t = Y_t / L_t A_t$, and $i_t = I_t / L_t A_t$.

By CRS, output per capita can be expressed as[5]

$$y_t = f(k_t) \equiv F(k_t, 1) \tag{11}$$

Notice that by Euler's Theorem:

$$\begin{aligned} F_K(K,L,A) &= f'(k) \quad \text{and} \\ F_L(K,L,A) &= f(k) - k \cdot f'(k). \end{aligned} \tag{12}$$

[5] Assume $\lambda = 1 / A_t L_t$

Exercise 6: Verify that the Cobb-Douglas production function, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ where $\alpha \in (0,1)$, is a neoclassical production function.

Exercise 7: Write the Cobb-Douglas production function, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ where $\alpha \in (0,1)$, in intensive form and verify equation (12).

1.1.2.2. Technological progress

There are two other main ways technology enters the production function, Hicks-neutral and Solow-neutral.

An aggregate production function with Hicks-neutral technological progress has a functional form specified by

$$Y_t = A_t F(K_t, L_t)$$

- The technology term A_t is simply a multiplicative constant in front of another production function *F*.
- An increase in Hicks-neutral technological progress shifts production function upwards.

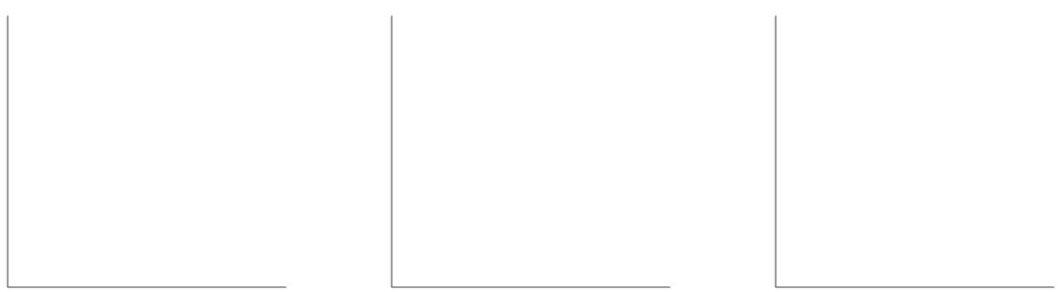
An aggregate production function with Solow-neutral technological progress has a functional form specified by

$$Y_t = F(A_t K_t, L_t)$$

- *A* also referred to as capital-augmenting technological progress because a higher A_t is equivalent to an economy having more capital.

To understand how these various forms of technological progress affect production, the isoquants can be plotted by combining different amounts of labor and physical capital for a given technology A_t such that the level of production is constant at $\bar{Y}_t$.

Exercise 8: Using isoquants with *K* and *L* as inputs, demonstrate the effect of increasing the three types of technological progress discussed, respectively, by doubling A_t.



- Hicks-neutral technological progress moves the isoquant outward linearly.
- Increases in Solow-neutral and Harrod-neutral technological progress change the shape of the isoquant.
  - In the Solow-neutral case, the isoquant becomes more steep
  - In the Harrod-neutral case, the isoquant becomes less steep.

Uzawa (1961) shows that only production functions with *Harrod-neutral* technological progress is consistent with balanced growth. This result is very surprising and compelling.

Exercise 9: Verify algebraically that the Cobb-Douglas production function is Hicks, Solow, and Harrod neutral. What is the algebraic representation of technological progress for each case?

Exercise 10: Discuss how to draw a set of isoquants for two different levels of A.

2.2. Dynamics of the model

In most growth models, the key to the analysis will be to derive a dynamic system that characterizes the evolution of consumption and capital in the economy; that is, a system of difference equations in *C* and *K* (or *c* and *k*). This system is very simple in the case of the Solow model.

2.2.1. The dynamics of capital and consumption

Because the economy may grow over time exogenously, it is simpler to focus on the capital stock per unit of effective labor rather than on aggregate capital. This is done by dividing the law of motion for physical capital by A_tL_t:

$$\frac{K_{t+1}}{A_tL_t} = \frac{(1-\delta)K_t + I_t}{A_tL_t} \tag{13}$$

The time indices on the LHS are not the same. The time indices on the LHS can be matched using the laws of motion for labor and technological progress.

<u>Exercise 11</u>: Write the law of motion for physical capital in intensive form. Label it (14).

Equation (14) is the discrete-time version of the intensive form physical capital stock. Its continuous-time equivalent can be written as

$$k_{t+1} \approx (1-n-g-\delta)k_t + i_t \tag{15}$$

- The sum $n+g+\delta$ can be interpreted as the effective depreciation rate of per capita (albeit effective labor, *AL*) physical capital.
  - As population grows, per capita physical capital decreases.
  - As technology improves, labor is enhanced (enlarged) which causes per capita capital to decrease.
- This approximation becomes arbitrarily good as the economy converges to its steady state.

Combining the continuous-time per capita law of motion, (15), and the aggregate resource constraint in per capita terms ($i_t = y_t - c_t$) yields a difference equation for intensive form physical capital:

$$k_{t+1} - k_t = f(k_t) - (n + g + \delta)k_t - c_t \quad (16)$$

- The change in the per capita capital is given by aggregate output, minus capital depreciation, minus aggregate consumption.

2.2.2. Feasible and consistent allocations

Definition 1. A *feasible allocation* is any sequence $\{c_t, k_t\}_{t=0}^{\infty}$ that satisfies the resource constraint

$$k_{t+1} \leq f(k_t) + (1 - n - g - \delta)k_t - c_t \quad (17)$$

- The feasible allocation for capital has a choice variable, c_t, on the RHS.
  - Must be a function of exogenous parameters and state varibles.
- A feasible allocation is defined as an entire path of allocations. A feasible allocation does not refer to a static object; it specifies the entire path of the economy.

The set of feasible allocations represents the choice set. Since households do not maximize utility, there is no objective to maximize. Rather, a savings rate is set exogenously.

Assume that there exists some savings rate, *s*, such that

$$c_t = (1 - s) f(k_t) \quad (18)$$

Definition 2. A *consistent allocation* is any feasible allocation that satisfies the resource constraint with equality and satisfies (18) for some $s \in (0,1)$.

2.2.3. The transitional dynamics

Using the savings rate, (18), in (17) yields *the fundamental difference equation of the Solow growth model*:

$$k_{t+1} - k_t \leq sf(k_t) - (n + g + \delta)k_t \tag{19}$$

- Key equation of the Solow model. It characterizes the dynamics of the model.
- It states that the rate of change of the capital stock per unit of effective labor is the difference between two terms.
    - *Actual investment*: $sf(k_t)$ is the amount of new investment per unit of effective labor.
    - *Breakeven investment*: $(n + g + \delta)k_t$ is the amount of investment that must occur to keep *k* at its existing level.
        - When actual investment exceeds the investment needed to break even, *k* rises.
        - When actual investment is less than the investment needed to break even, *k* falls.

Proposition 1. Given any initial point $k_0 > 0$, the dynamics of the physical capital per unit of effective labor is given by the path $\{k_t\}_{t=0}^{\infty}$ such that

$$k_{t+1} = G(k_t) \quad \forall t \geq 0 \tag{20}$$

where

$$G(k_t) \equiv sf(k_t) + (1 - n - g - \delta)k_t \tag{21}$$

- *G* is a function that takes the current state of the economy as given and tells what the state of the economy is next period. The state variables usually consist of stocks and shocks.
- Since $f(0) = 0$, actual investment and break-even investment are equal at *k=0*.
- The Inada conditions imply that:
    - When *k* is small, $f'(k)$ is large, and thus $G(k)$ is quite steep.
    - When *k* is large, $f'(k)$ is falls to 0, and thus $G(k)$ is quite flat.

This can be demonstrated graphically.

Exercise 12: Assume a Cobb-Douglas production function. Solve for the *fundamental difference equation*.

2.2.3.1. Violation of assumptions

The assumptions on the model are vital for the transition dynamics.

Exercise 13: Assume that one of the Inada conditions does not hold. Show how this changes the transition path graphically.

Exercise 14: Assume that diminishing returns is violated. Show how this changes the transition path graphically.

2.2.4. Steady state

The previous section provides the transition dynamics of the model for physical capital per effective unit of labor. We now turn our attention to the steady state.

The steady state is defined as the point where physical capita per effective unit of labor is constant. Mathematically, a steady-state corresponds to a stationary point of the difference equation (19) or a steady state is any fixed point *k** of *G* in (21).

*k** is a steady state if and only if it solves

$$0 = sf(k_t) - (n + g + \delta)k_t$$

Equivalently,

$$\phi(k^*) = \frac{f(k^*)}{k^*} = \frac{n + g + \delta}{s} \tag{24}$$

Proposition 2. Let $n + g + \delta \in (0,1)$ and $s \in (0,1)$. Then there exists a unique *steady state* where *k** satisfies (24) and per capita output is given by $y^* = f(k^*)$ and per capita consumption is given by $c^* = (1-s)f(k^*)$.

<u>Proof</u>

- Using the properties of *f* and l'Hôpital's rule imply that $\lim_{k\to 0} f(k)/k = \infty$, $\lim_{k\to\infty} f(k)/k = 0$ and

$$\phi'(k) = \frac{f'(k)k - f(k)}{k^2} = \frac{-F_L}{k^2} < 0 \tag{25}$$

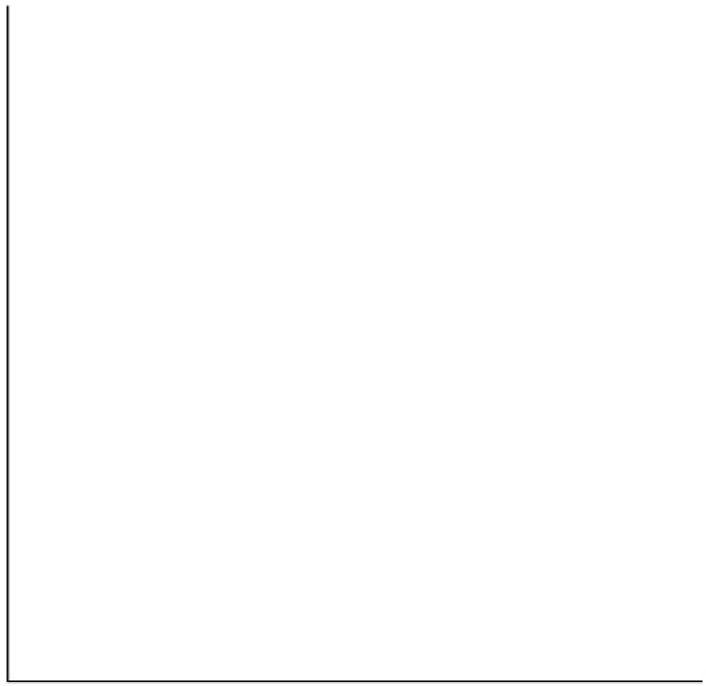
- By the Intermediate Value Thm.[6], there exists a unique *k** such that (24) is satisfied. ■

Proposition 2. implies that (24) has a unique steady state:

$$k^* = \phi^{-1}\left(\frac{n + g + \delta}{s}\right) \tag{26}$$

[6] If *f* is a real-valued continuous function on the interval [*a*, *b*], and *u* is a number between *f(a)* and *f(b)*, then there is a *c* ∈ [*a*, *b*] such that *f(c)* = *u*.

The allocations between consumption and physical capital can also be demonstrated graphically using equation (21).



- The amount of investment, $sf(k)$ is used to replenish the capital-labor ratio given depreciation, population growth, and technological progress.
    - The amount capital needed to be replenished is $(n+g+\delta)k$, the linear line.
    - The amount of capital actually invested is $sf(k)$.

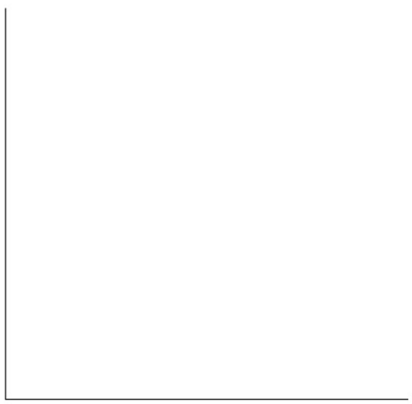
<u>Exercise 15</u>: Assume a Cobb-Douglas production function. Solve for steady state physical capital.

2.2.4.1. Comparative statics with steady state

The comparative statics of steady-state consumption and physical capital are easily obtained. Since $k^* = \phi^{-1}\left(\dfrac{n+g+\delta}{s}\right)$:

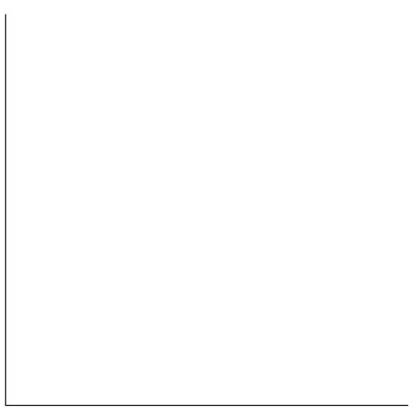
- k^* is a decreasing function of $n, g,$ and δ.
- k^* is increasing function of s.

<u>Exercise 16</u>: Suppose n increases. Draw the change in the dynamics graphically.



- c^* decreases with $n, g,$ and δ.
- c^* has non-monotonic effects with s. Recall $c^* = (1-s)f(k^*)$.

<u>Exercise 17</u>: Suppose s increases. Draw the change in the dynamics graphically.



Equivalently, the dynamics of the model can be stated in terms of the growth rate in physical capital per unit of effective labor

$$\gamma_t = \frac{k_{t+1} - k_t}{k_t} \tag{22}$$

Exercise 18: Using (19) and (22), solve for the growth rate, γ_t, as a function of the current state of the economy at time *t*. Label it (23).

2.2.5. Convergence to Steady State

Now that a steady state is established, a nature question is whether the economy will converge to this steady state if the economy is started away from it. Another way to view this is whether the economy will return to the steady state upon being perturbed by a shock.

The following proposition establishes convergence.

Proposition 2.3. Given any initial $k_0 \in (0, \infty)$, the economy converges asymptotically to the steady state. The transition is monotonic.

- The growth rate is positive and decreases over time towards 0 if $k_0 < k^*$.
- The growth rate is negative and increases over time towards 0 if $k_0 > k^*$.

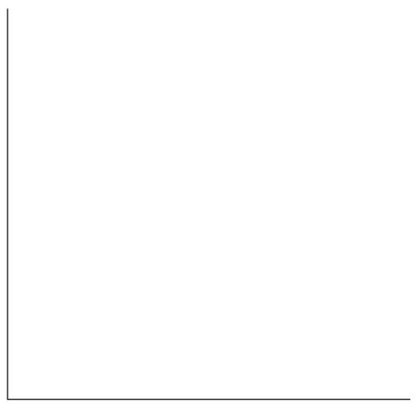
Proof: From the properties of *f*, $G'(k_t) = sf'(k_t) + (1 - n - g - \delta) > 0$ and $G''(k_t) = sf''(k_t) < 0$. That is, *G* is strictly increasing and strictly concave. Moreover, $G(0) = 0$ and $G(k^*) = k^*$. It follows that $G(k) > k$ for all $k < k^*$. It follows that $k_t < k_{t+1} < k^*$ whenever $k_t \in (0, k^*)$ and therefore the sequence $\{k_t\}_{t=0}^{\infty}$ is strictly increasing if $k_0 < k^*$. By monotonicity, k_t converges asymptotically to some $\hat{k} < k^*$. By continuity of *G*, $\hat{k}$ must satisfy $\hat{k} = G(\hat{k})$, that is $\hat{k}$ must be a fixed point of *G*. But we already proved that *G* has a unique fixed point, which proves that $\hat{k} = k^*$. A symmetric argument proves that, when $k_0 > k^*$, $\{k_t\}_{t=0}^{\infty}$ is strictly decreasing and again converges asymptotically to k^*.

Next, consider the growth rate of the capital stock. This is given by

$$\gamma_t = s\phi(k_t) - (n + g + \delta) \equiv \gamma(k_t).$$

Note that $\gamma(k_t) = 0$ iff $k=k^*$, $\gamma(k_t) > 0$ iff $k<k^*$, and $\gamma(k_t) < 0$ iff $k>k^*$. Moreover, by diminishing returns, $\gamma'(k_t) = s\phi'(k) < 0$. It follows that $\gamma(k_t) > \gamma(k_{t+1}) > \gamma(k^*)$ whenever $k_0 \in (0, k^*)$ and $\gamma(k_t) < \gamma(k_{t+1}) < \gamma(k^*)$ whenever $k_t \in (k^*, \infty)$. This proves that γ_t is positive and decreases toward zero if $k_0 < k^*$ and it is negative and increases toward zero if $k_0 > k^*$. ■

This proposition can be demonstrated graphically for various levels of *k*.



- The saving curve, $\phi(k)s$ is the savings line and is downward sloping (see Eq. 25).
- The depreciation curve, $(n + g + \delta)$, is a horizontal line.
- The vertical distance between these two curves in the growth rate.

To the left of the steady state, the $\phi(k)s$ curve lies above $(n + g + \delta)$. As *k* grows, the growth rate of capital per effective unit of labor declines. This is because there is diminishing returns to capital. An analogous description of dynamics can be easily made if *k* start above *k**.

The negative slope implies conditional convergence. Countries that are well below the steady state will grow faster than those that are closer to the steady state.

2.3. The Golden Rule

Up to this point, a consistent exogenous savings rate has been assumed and nothing has been said about *optimal* savings.

- For given values of g, n and δ, there is a unique steady-state value $k^*>0$ for each value of the saving rate s.

- One possible optimal steady state is to maximize the steady-state level of per capita consumption denoted by c_{gold} with corresponding savings rate s_{gold}. Since we are treating the savings rate as an exogenous parameter and have not specified the objective function of the household yet, we cannot say whether s_{gold} is better than some other savings rate.

- Since the savings rate is constant over time, each household in each period receives the same amount of consumption per capita. Thus, the biblical Golden Rule – do unto others as you would have others do unto you – is interpreted, in economic terms: if the same amount of consumption is provided to members of each current and future generation, then the maximum amount of per capita consumption is c_{gold}, i.e., the current generation does not provide less to future generations than to itself.

First write the steady-state relationship between c^* and s, (18), and suppress the other parameters.

$$c^* = (1-s)f(k^*(s))$$

Using the relationship $sf(k^*) = (n+g+\delta)k^*$, the above condition can be rewritten as

$$c^* = f(k^*(s)) - (n+g+\delta)k^*(s) \tag{27}$$

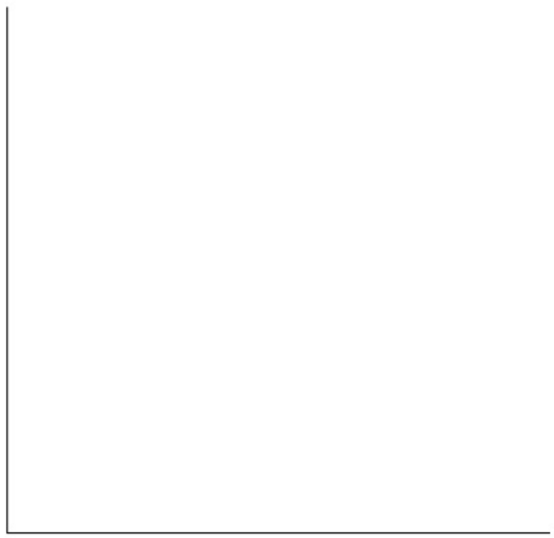
Now differentiating (27) with respect to s (again using the Implicit Function Thm.), yields

$$\frac{\partial c^*}{\partial s} = f'(k^*(s))\frac{\partial k^*}{\partial s} - (n+g+\delta)\frac{\partial k^*}{\partial s} \tag{28}$$

Defining the golden rule savings rate s_{gold} to be such that $\partial c^* / \partial s = 0$, the steady-state golden rule per capita capital is defined as k^*_{gold}. The condition that determines the value of k^*_{gold} is

$$f'(k_{gold}) = n+g+\delta \tag{29}$$

The relationship between consumption and the saving rate can be graphically demonstrated:



- When the economy is below k^*_{gold}, a higher savings rate will increase consumption.
- When the economy is above k^*_{gold}, a lower savings rate will increase consumption.
    - Households are investing too much and not consuming enough. This is the essence of a phenomenon called *dynamic inefficiency*. However, since we do not know how household discount the future without a utility function, little can be said if this is truly inefficient.

<u>Exercise 19</u>: Assume a Cobb-Douglas production function and solve for k_{gold}, c_{gold}, and s_{gold}.

2.4. Shocks

The Solow model can be interpreted as a primitive Real Business Cycle (RBC) model. RBC models attempt to understand how the business cycle affects the economy.

The model can be used to predict the response of the economy to productivity shocks for both temporary and permanent shocks.

2.4.1. Permanent Productivity Shocks

Suppose output is given by

$$Y_t = A_t F(K_t, L_t)$$

or $y_t = A_t f(k_t)$ where A_t denotes total factor productivity.

Consider a permanent negative shock to *A*. The *G(k)* function shifts down and slowly transitions to the new steady state.

<u>Exercise 20</u>: Assume a Cobb-Douglas production function and arbitrary parameters values. What determines the rate at which it reaches the new steady state?

2.4.1. Temporary Productivity Shocks

Now assume that the shock is only temporary which lasts for only one period. This shifts the *G(k)* function downward, but only temporarily. Initially, capital and output fall towards the low steady state but during the next period, productivity rises to the original level and output and capital start to proceed back to the original higher stead state.

2.5. Cross-Country Tests of the Solow Model

The Solow Growth model predicts:

1. Conditional convergence.
2. Differences in savings (capital accumulation) affect growth.

These predictions can be tested empirically.

2.5.1. Cross-Country Empirical Tests of the Solow Model's Conditional Convergence Prediction

The first test is conditional convergence: countries that are well away from steady state physical capital have higher growth rates than those that are closer to it.

This prediction can be easily verified using the following equation:

$$g_{i,t,t-1} = b0 + b1 \ln y_{i,t-1} + \varepsilon_{i,t} \tag{30}$$

where $g_{i,t,t-1}$ is the growth rate of country *i* between dates *t-1* and *t*, $\ln y_{i,t-1}$ is the initial (time *t-1*) log output per capita of this country, and $\varepsilon_{i,t}$ is a stochastic term capturing all omitted influences (in particularly, technology).

- The speed of convergence is capture by *b1*.
- The error term collects terms such as *n*, *g*, and δ .

Regressions of this form have been estimated by many individuals. The first was Solow (1957) who desired to know how much of growth of the U.S. economy can be attributed to increased labor and capital inputs, and how much of it is due to the residual, technological progress?

- Solow's conclusion was quite striking: a large part of the growth was due to technological progress.

- Regression done by Barro (1991) and Barro and Sala-i-Martin (1992) on the OECD countries have demonstrated that *b1* is indeed negative.
  - Countries that were relatively poor have had higher growth.

- However, recall from Lecture 2 that worldwide convergence has not happened. It may be too demanding so we may need to loosen the model by assuming that *b0* is country specific (measuring speed of convergence).

- The bad news is that even if there is convergence, there is no guarantee that *b1* is negative if one estimates (30) since economies with certain growth-reducing characteristics are likely to have both low steady-state output levels and low initial levels.

With these issues in mind, this model has been extended to permit cross-country differences to enter into the regression:

$$g_{i,t,t-1} = X_{i,t}^{T}\beta + b1\ln y_{i,t-1} + \varepsilon_{i,t} \tag{31}$$

where $X_{i,t}^{T}$ is a vector including the variables such as male and female schooling rate, the fertility rate, the investment rate, the government-consumption ratio, the inflation rate, changes in terms of trades, openness, and such institutional variables as rule of law and democracy.

- Regression similar to equation (31) have not only been used to support conditional convergence but also to estimate the determinant of economic growth. In particular, it may appear natural to presume that the estimates of the coefficient vector β will contain information about the causal effects of various variables on economic growth.

There are several criticism of such approach:

- Most, if not all the variables in $X_{i,t}^{T}$, as well as $\log y_{i,t}$ are endogenous in the sense that they are jointly determined.
- Investment-like variables on the right-hand side are difficult to link to theory.
- Solow growth is for a closed economy. This model must assume independence between observations to be valid to run the regression. This is quite problematic. One could potentially alleviate this by adding a full set of country fixed effects and a full set of year effects.

2.5.2. Cross-Country Empirical Tests of the Solow Model's Steady State Predictions

The Solow model implies that steady-state capital, productivity, and income are determined by *A*, *δ*, *g*, *s*, and *n*. Assuming that countries share the same technology up to a level difference, if the Solow model is correct, observed cross-country income and productivity difference should be 'explained' by observed cross-country differences in *s* and *n*.

If one assumes a Cobb-Douglas production function, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, per capita (effective units) steady state (intensive) capital is

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{1/(1-\alpha)} \tag{32}$$

- The level of productivity is not observed directly and capital stocks are difficult to measure.

Thus it is more helpful to solve for steady-state output per worker. This is done by solving for Y_t / L_t which is easily observable from data:

$$\frac{Y_t}{L_t} = A_t K_t^{\alpha} (A_t L_t)^{-\alpha} = A_t k_t^{\alpha} = A_t \left(\frac{s}{n+g+\delta} \right)^{\alpha/(1-\alpha)} \tag{33}$$

Taking the natural log of both sides of this equation yields

$$\ln \frac{Y_t}{L_t} = \ln A_0 + tg + \frac{\alpha}{(1-\alpha)} \ln s - \frac{\alpha}{(1-\alpha)} \ln(n+g+\delta) \tag{34}$$

using $A_t = (1+g)A_{t-1} = (1+g)^t A^0$ and the fact that $\log(1+x) \approx x$ for small x.

- LHS is now output levels.
- RHS contains growth.

Mankiw, Romer, and Weil (1992) (henceforth MRW) estimate this equation on a cross-section of 98 industrialized and developing countries, with each country taken as one observation.

They set:

1. Y_t / L_t equal to end-of-sample (1985) GDP per worker
2. 1960-85 average ratio of investment to GDP as a proxy for the saving rate *s*.
3. Population growth is set to the 1960-85 average rate of population growth for each country
4. g and δ are both assumed to be the same across all countries, with $g + \delta = .05$.
5. Any differences across countries in initial technology A_0 are assumed uncorrelated with the explanatory variables, and are absorbed into the cross-section error term.

Given these assumptions, MRW obtain the following estimates using OLS:

$$\ln \frac{Y_t}{L_t} = \underset{(1.59)}{5.48} + \underset{(.14)}{1.42} \ln s - \underset{(.56)}{1.97} \ln(n+g+\delta), \; R^2 = .59 \tag{35}$$

- The standard errors are in parenthesis.
- The cross-country variation in investment rate *s* and population growth rates *n* can account for a bit more than 50% of the observed cross-country variation in productivity levels.
- Interestingly, MRW fail to reject the hypothesis that the coefficients on ln *s* and ln($n + g + \delta$) are equal in absolute terms value but of opposite sign as the model predicts.
- The implied value of α ($\alpha(1-\alpha)^{-1}$=1.42) seems to be a bit high. Some developed nations have savings rates as high as .5, but .59 seems to be unrealistic.

Exercise 21: Why does the model estimate such a high value of α? *Hint*: See (33).

Exercise 22: How can the empirical model be extended to include what is missing?

Solving for the capital accumulation equations and steady state equations yields the revised estimating equation:

$$\ln\frac{Y_t}{L_t} = \ln A_0 + tg + \frac{\alpha}{(1-\alpha-\beta)}\ln s_K + \frac{\beta}{(1-\alpha-\beta)}\ln s_H - \frac{\alpha+\beta}{(1-\alpha-\beta)}\ln(n+g+\delta)$$

where s_K is defined as s from the parsimonious model and s_H is proxied by the fraction of the working-age population with secondary school education.

- Given these assumptions, MRW obtain an implied value of α and β of .3. β is now inconsistent with Mincer regression on micro data. It's too small.

- The augmented Solow growth model with human capital can account for 60% to 80% of the observed cross-country differences in productivity levels.

However, there are some important criticisms

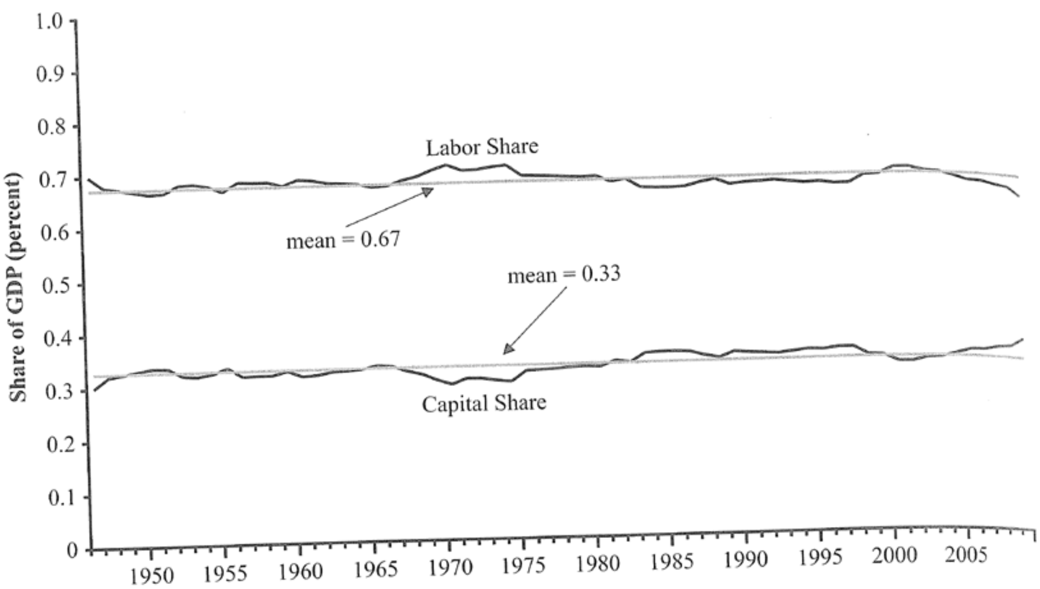
- Some or all of the right-hand side variables are endogenous.
- MRW use investment as a share of GDP to proxy for savings but investment is itself an endogenous variable. Thus it is difficult to believe that the growth rate of technological efficiency is exogenous and equal across countries.
- This model was estimated based on the *steady state* of the Solow model. There is no reason to presume that over much of the 1960-85 sample period countries were in a steady state.

4. Calibrating models

By using calibration exercises, researchers can test the functional forms for goodness of fit (among other applications). Often, a Cobb-Douglas production function is assumed. We can calibrate the model using the C-D production function and test its predictive power.

4.1. Parameterizing the model

The Cobb-Douglas requires a value for α, the capital share of GDP. This is easily identified using times series data of GDP.



- The implied value of α, the capital share of GDP, is .33.

Variable or parameter	1948	2008	Annual Rate of Growth
Y (Trillions of 2005 dollars)	1.854	13.312	3.34%
A	4.6	9.63	1.24%
L (Billions of worker hours)	112.4	263.1	1.43%
K (Trillions of 2005 dollars)	5.42	40.2	3.43%
α	.33	.33	0.00%

The second parameter required is *A*, the technological change variable. Using the C-D production function and U.S. economy data in 2008, the implied value of *A* is:

$$A = \frac{Y}{K^{\alpha}L^{1-\alpha}} = \frac{\$13.312T}{(\$40.2T)^{0.33}(263.1T \text{ worker-hours per year})^{0.67}} = 9.63.$$

Thus, the Cobb-Douglas production function is calibrated as

$$Y = 9.63K^{0.33}L^{0.67}$$

Exercise 23: If new laborers worked similar hours to existing workers and labor input grew by 5%, by how much would GDP increase?

Using historical data, we can now answer how much of growth in GDP between two years can be attributed to each factor of production and to technology.

Exercise 24: What would GDP have been in 1948 if the 2008 labor input had been available?

- Same methodology can be used to find counterfactual GDP using the capital input.

Exercise 25: What would GDP have been in 1948 if the 2008 labor and capital inputs had been available?

- Model is too parsimonious for accurate predictions. Add more stuff?

5. Conclusion

In closing, the words of North and Thomas, 1973, p. 2 seems fitting “The factors we have listed (innovation, economies of scale, education, capital accumulation, etc.) are not causes of growth; they are growth.”

6. In-class Exercises

1. Consider the Solow growth model from the lecture notes with an additional capital stock: human capital (this is a term we use to represent the stock of skills, education, competencies, and other productivity-enhanced characteristics embedded in labor).

   Suppose that the aggregate production function is modified to

$$Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta}$$

   where $0 < \alpha < 1$, $0 < \beta < 1$, and $\alpha + \beta < 1$. The law of motions for physical and human capital stocks are:

$$K_{t+1} = (1-\delta_k)K_t + I_t^k$$
$$H_{t+1} = (1-\delta_h)H_t + I_t^h$$

   where $\delta_k, \delta_h \in (0,1)$.

   - Population grows at rate n
   - Technology grows at rate g.
   - Savings rates are:
     - Physical capital: s_k share of output.
     - Human capital; s_h share of output.

   a) Write the laws of motion in per effective unit of labor (intensive form).

   b) Write out in intensive form the aggregate resource constraint and investment equations using shares of output.

   c) Modify the laws of motion so that they correspond to the continuous time equivalent.

c) Use the investment equations and production function to solve for the Solow Fundamental Laws of Motion.

d) Solve for per capita steady-state physical capital, human capital, production, and consumption.

e) What happens to k^*, h^*, and y^* when s_k increases?

f) What determines if s_k or s_h is more important in increasing per capita output?

2. Consider the basic Solow growth model with one exception:

- Non-renewable natural resources are used in the production process such that at the beginning of any period *t*, the total *remaining* stock of an exhaustible resource is R_t.
- A part of this stock is extracted and used as energy input, E_t, during period *t*.
- A resource depletion equation will state that the stock of oil will be reduced from one period to the next by exactly the amount used in production in the first of the periods.
- Constant extraction rate, $0 < s_E < 1$.

The production function is given as

$$Y_t = K_t^{\alpha}(A_t L_t)^{\beta} E_t^{\kappa}$$

where $\alpha > 0$, $\beta > 0$, $\kappa > 0$ and $\alpha + \beta + \kappa = 1$

<u>Other assumptions:</u>

- Population grows at rate *n*
- Technology grows at rate *g*.
- Savings rate of *s* of output that is invested in physical capital.

a. State the law of motion for physical capital.

b. Identify relationship between energy and the resource per period and then state the laws of motion for the natural resource and energy extraction.

c. Does E_t or any per capita measure of E_t go to a steady state? If not, provide a ratio of variables that does not go to 0 as the economy progresses.

d. Solve the Solow fundamental difference equation for the capital-output ratio and provide the steady state.

e. Explain intuitively what happens when s_e increases.

Lecture 4: Applications Using the Solow Growth Model

1. Introduction

The Solow growth model can be applied to model poverty traps. Poverty traps are difficult to obtain endogenously within a model and are typically formed exogenously from a model assumption such as a cost to superior technology.

- A poverty trap is a defined as self-perpetuating condition where an economy, caught in a vicious cycle, suffers from persistent underdevelopment (i.e., a steady state with lower capital and output).

2. The model

Consider the Solow growth model from the lecture notes but with two production technologies:

$$Y_t^A = AK_t^{\alpha} L_t^{1-\alpha}$$
$$Y_t^B = BK_t^{\alpha} L_t^{1-\alpha}$$

where $B > A$ and $0 < \alpha < 1$.

- Technology A is public and technology B is proprietary.
- Technology B requires a subscription fee of bL_t.
    - Who pays the fee is irrelevant.
- No technology growth.

Output per worker can be easily established:

$$y_t^A = Ak_t^{\alpha}$$
$$y_t^B = Bk_t^{\alpha} - b$$

Irrespective of the technology acquired, the law of motions for physical capital stock is

$$K_{t+1} = (1-\delta)K_t + I_t$$

where $\delta \in (0,1)$.

The question that this model would like to answer is not whether a poverty trap will form, but rather when will the subscription fee be paid to obtain access to the superior technology.

3. Solving the model

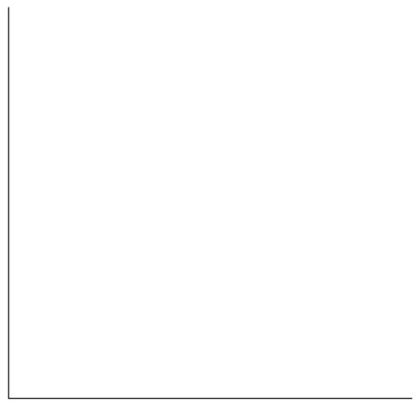
Exercise 1: When should an economy pay the subscription fee to gain access to the proprietary technology?

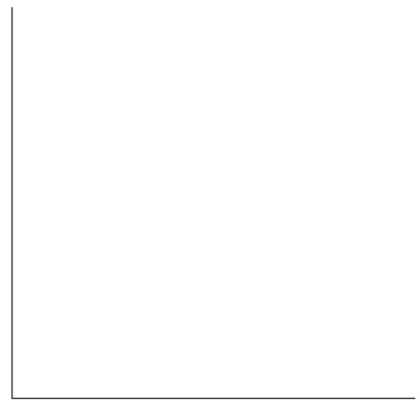
Exercise 2: What is output at this inflection point?

Exercise 3: Draw output as a function of physical capital.

Exercise 4: Solve for the continuous time Solow fundamental difference equation.

Exercise 5: Draw two possible phase diagrams using the fundamental difference equations.





What condition is requires to determine phase diagram?

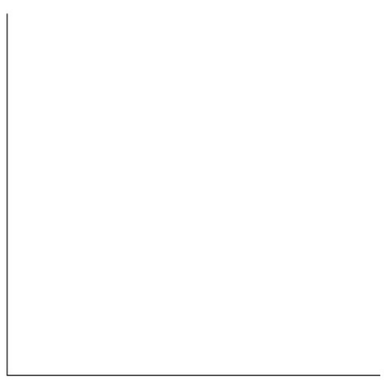
Unlike previous models, the initial conditions matter for certain parameter restrictions.

- If an economy starts off with $k_0 < \tilde{k}$, it will remain in this steady state.
- If the initial capital stock is high, it proceed to the high-capital steady state.

A key feature of this model is that any marginal improvement does not pull the economy away from the low-capital / low-output steady state. The classic poverty alleviating theory based on this model is the 'Big Push' proposed by Rosenstein-Rodan and Hirschman and formalized by Murphy, Shleifer, and Vishny.

- Proposed that the root cause of poverty trap is that industrialization requires a larger initial investments (larger than any one firm), and so firms only industrialize if others do.
- Positive production externality occurs from knowledge spillovers and infrastructure.
- If investment is coordinated, then the 'big push' can push the economy out of the trap.

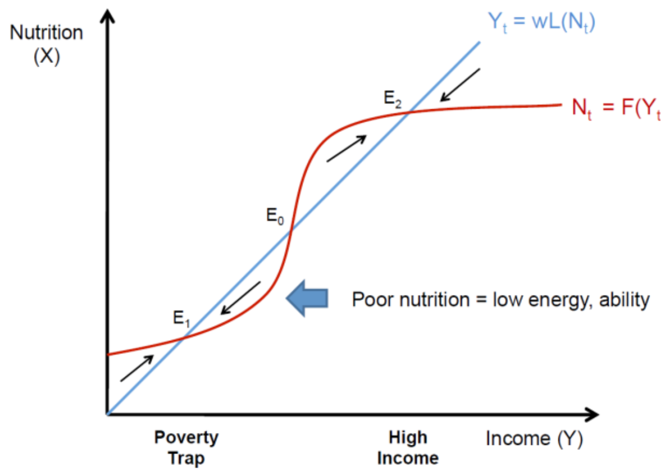
Exercise 6: Draw an increase in capital that 'pushes' the economy out of a poverty trap.



4. Extensions to the model

The idea of a poverty trap can be extended not just across time, but also across incomes.

For example, Banerjee and Duflo explain that the bi-directional causality relationship between nutrition and income can be explain in a poverty trap setting.



Another common example with more supporting evidence is the role of credit market failure.

- Normal market: Financial market and individuals behave well.
    - Some of the poor have high potential returns to investment.
    - Some investments may be lumpy.
    - Profitable to borrow if market interest rate is less than return on investment.
- Poverty trap market: There may be reasons why markets or individuals are not behaving well.
    - Credit market failure
        - Poor countries have weak, sparse banking sectors.
        - Information asymmetries are large.
        - The poor have little collateral (and debt contracts may be hard to enforce)
        - MFIs or moneylenders typically lend for short spans (2-3 months)
    - Other financial market failure
        - Many savings institutions do not allow saving for >2-3 months (e.g. ROSCAs).
        - High cost of saving
        - High inflation rate
        - Most long-term savings instruments (e.g. land, housing, livestock) are lumpy, illiquid, and may yield a low return
    - Other "failures"
        - Self-control problems over small amounts of money
        - Pressure to share with others in one's social network

5. Evidence to support poverty traps

5.1. Macro-level evidence

The above model suggests that only a large investment in the capital stock will push the economy out of the poverty trap.

- This is the *transformational* perspective on development.
- Suggests that developing nations will never catch-up to developed nations without external intervention (either by governments and external funds such as FDI).

The evidence suggests that macro-level poverty traps do not exist (Easterly, 2008).

- Poorest countries change all the time (few stay in "traps").
- Big increases in aid do not seem to result in big jumps in growth.

5.2. Micro-level evidence

There is also the single equilibrium model.

- The *marginal* perspective on development.
- Not economy-wide problem, but various 'micro-level' issues.

The evidence suggests that micro-level poverty traps do exist.

5.3. Empirical evidence

Poverty trap is often interpreted as an explanation for the cross-country income difference.

- Frequently viewed as an alternative to the models that attribute cross-country income difference to the cross-country difference in, say, TFP and/or the investment distortions.
- This is a misinterpretation.
    - The message of poverty trap models is the self-perpetuating nature of poverty.
    - Suggests that the long run performance of an economy could be much better if its initial condition were better.
    - It does not mean that the cross-country difference in the long run performance is due mostly to the difference in their initial conditions.

6. Conclusion

It isn't clear that poverty traps even exist. It may be a slow transition (over some range) that makes it appear that some economies are in a 'trap'. Empirically, rigidities are difficult to distinguish from traps.

Lastly, the economic development along the transition path may require self-discovery or a learning-by-doing externality. That is, when firms are inexperienced and unproductive, they cannot attract workers from other sectors (the spillover effect), hence they are not able to accumulate their experiences.

- A great show to see spill-over effects is "How It's Made' on Netflix. After watching 5 episodes, one sees spillovers between production processes.

Lecture 5: The Neoclassical Growth Model

1. Introduction

The neoclassical growth model is based on the Solow growth model augmented with households who maximize utility.

The model is solved two ways:

- o A social planner who chooses consumption and investment optimally so as to maximize social welfare. This leads to *optimal* growth.
- o Households choosing consumption and investment optimally so as to maximize their utility. This leads to *equilibrium* growth.

1.1. The Neoclassical growth model

- The Neoclassical growth model extends the Solow growth model by introducing household optimization which endogenizes savings.
- This enables a better understanding of the factors that affect savings decisions and also to discuss the optimality of equilibria.
- Can only analyze households being "better off" if some information on well-defined preference orderings exist.
- The standard neoclassical growth model is commonly referred to as the *Ramsey problem* (or Cass-Koopman model) after Frank Ramsey who posed the question of how much a nation should save.
  - o He answered this question using a model that is now the prototype for studying optimal intertemporal allocation of resources.
  - o The model present here is essentially that of Ramsey's.

1.1.1. The environment of the Neoclassical growth model

- Time is discrete and is indexed by $t \in \{0,1,2,\ldots\}$ and the economy remains closed.
- The amount of labor, $n_t \in [0,\bar{n}]$, is now endogenous.
  - o $\bar{n}$ is normalized to 1.
  - o Interpret n_t and l_t as the fraction of time that is devoted to production and leisure, respectively, during period *t*.

<u>Exercise 1</u>: Solve for the time constraint of an individual.

- Let K_t be the aggregate physical capital stock at the beginning of period t which is combined with aggregate labor N_t to produce a final good, Y_t.

- Abstract from population growth (n = 0) and exogenous technological change (g = 0).

1.1.2. Households and their preferences

There are an uncountable number of households with total measure of 1.

- A unit measure of households implies that averages and aggregates are equal.
- Households in the economy may be truly 'infinitely lived', or alternatively they may consist of generations with full (or partial) altruism linking each other within the household. Households are equated with individuals and thus ignore all possible sources of conflict or different preferences within the household.
- Assume that all households are identical so as the economy admits a representative household.

- Preferences are defined over streams of consumption and leisure, $x = \{x_t\}_{t=0}^{\infty}$, where $x_t = (c_t, l_t)$, and are represented by a utility function $U : \mathbb{R}_+ \to \mathbb{R}$, such that

$$U(x) = U(x_0, x_1, ...)$$

- Preferences are recursive if there is a function W such that, for all $x = \{x_t\}_{t=0}^{\infty}$,

$$U(x_0, x_1, ...) = W[x_0, U(x_1, x_2, ...)]$$

We can now represent preferences as a consumption-leisure stream $x = \{x_t\}_{t=0}^{\infty}$ induces a utility $\{U_t\}_{t=0}^{\infty}$ according to the recursion

$$U_t = W[x_t, U_{t+1}].$$

Preferences are additively (time) separable if there are functions υ_t such that

$$U(x) = \sum_{t=0}^{\infty} \upsilon_t(x_t)$$

- Time-separable preferences implies independence across utility from past or future dates.

Assuming that preferences are both *recursive* and *time-separable*:

$$U_t = U(x_t) + \beta U_{t+1}$$

Equivalently,

$$U_t(x) = \sum_{k=0}^{\infty} \beta^k U(x_{t+k})$$

- β is called the discount factor, with $\beta \in (0,1)$.

Assume that U is a neoclassical function. Recall that neoclassical means following properties are satisfied:

1. U is continuous and (although not always necessary) twice differentiable.
2. U is strictly increasing and strictly concave:

$$U_c(c,l) > 0, \quad U_l(c,l) > 0,$$
$$U_{cc}(c,l) < 0, \quad U_{ll}(c,l) < 0,$$
$$U_{cl} < U_{cc}U_{ll}.$$

3. U satisfies the Inada conditions

$$\lim_{c\to 0} U_c = \lim_{l\to 0} U_l = \infty,$$
$$\lim_{c\to \infty} U_c = \lim_{l\to \infty} U_l = 0.$$

Exercise 2: Provide an example of a neoclassical utility function.

Calculate the present value of lifetime utility for each of the following. Interpret β and σ.

a) *c1=2, c2=2, β = 0.75, σ=0.*

b) *c1=2, c2=2, β = 1.00, σ=0.*

c) *c1=1, c2=3, β = 0.75, σ=0.*

d) *c1=1, c2=3, β = 1.00, σ=1.*

e) *c1=1, c2=3, β = 1.00, σ=-1.*

1.2. Technology and production function

The neoclassical production function takes the form

$$y_t = F(k_t, n_t) = n_t f(\kappa_t)$$

where

$$\kappa_t = k_t / n_t$$

In the social planner's problem, there is no money. Saving is in the form of storing current period's output for next period's capital stock.

Exercise 3: Why is money not necessary for this model? Is this assumption valid?

- How is this type of savings similar to regular savings?

2. The Social Planner's problem

In this section, we analyze the neoclassical growth model by considering the optimal plan of a benevolent social planner, who chooses the static and intertemporal allocation of resources in the economy so as to maximize social welfare.

The social planner's problem is important as it provides optimal growth. That is, the social planner's maximization is Pareto optimal.

We will later show that the allocations that prevail in a decentralized competitive market environment coincide with the allocations dictated by the social planner.

2.1. The resource constraint, and the law of motion for physical capital

Existing capital depreciates over time at a fixed rate $\delta \in (0,1)$. The capital stock in the beginning of the next period is

$$k_{t+1} = (1-\delta)k_t + i_t .$$

Finally, we impose the following natural non-negative constraints

$$c_t \geq 0, n_t \geq 0, l_t \geq 0, k_t \geq 0.$$

Exercise 4: Solve for the resource constraint of this economy. Write out the constraints for periods t = 0, 1, 2, and 3 and then identify how physical capital links constraints between periods

The constraint is straightforward to understand: total output produced $F(k_t,n_t)$ with physical capital, k_t, and labor, n_t, together with the fraction $1-\delta$ of the capital that is not depreciated make up the total resources of the economy at date t. Out of these resources, c_t is spent as consumption and the rest becomes next period's capital stock, k_{t+1}.

2.2. The Ramsey problem

The social planner chooses a plan $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ so as to maximize lifetime utility subject to the resource constraint of the economy, taking the initial capital stock, $k_0>0$, as given. That is,

$$\max_{\{c_t,n_t,k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, 1-n_t)$$

subject to

$$c_t + k_{t+1} \le F(k_t, n_t) + (1-\delta)k_t,\ \forall t \ge 0,$$

with $k_t \ge 0$ and $n_t \in [0,1], \forall t \ge 0$, given k_0.

The optimal growth problem requires that the social planner choose an entire sequence of consumption and labor levels and the capital stocks, subject only the resource constraint. There are no additional equilibrium constraints.

2.2.1. Solving the model

There are several ways to solve the model. We will study:

1. Optimal control
2. Dynamic programming

Both yield the same solution, but provide different insights.

2.3. Optimal control

Let μ_t denote the Lagrange multiplier for the resource constraint. The Lagrangian of the social planner's problem is

$$L^0 = \sum_{t=0}^{\infty} \beta^t U(c_t, 1-n_t) + \sum_{t=0}^{\infty} \mu_t [F(k_t, n_t) + (1-\delta)k_t - c_t - k_{t+1}] \tag{1}$$

Let $\lambda_t \equiv \beta^t \mu_t$ and define the Hamiltonian as

$$H_t = H(k_t, k_{t+1}, c_t, n_t, \lambda_t) \equiv U(c_t, 1-n_t) + \lambda_t [F(k_t, n_t) + (1-\delta)k_t - c_t - k_{t+1}] \tag{2}$$

The Lagrangian can be rewritten as

$$L^1 = \sum_{t=0}^{\infty} \beta^t \{U(c_t, 1-n_t) + \lambda_t [F(k_t, n_t) + (1-\delta)k_t - c_t - k_{t+1}]\} = \sum_{t=0}^{\infty} \beta^t H_t \tag{3}$$

or, in recursive form, $L_t^1 = H_t + \beta L_{t+1}^1$.

Given that $k_t, c_t,$ and n_t enter only the period t utility and resource constraint, c_t and n_t only appears in H_t. Similarly, k_t enters in periods t and $t + 1$ utility and resource constraints; thus they appear in H_t and H_{t+1}.

Lemma 1 *If* $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ *is the optimum and* $\{\lambda_t\}_{t=0}^{\infty}$ *the associated multipliers, then*

$$(c_t, n_t) = \arg\max_{c,n} \overbrace{H(k_t, k_{t+1}, c, n, \lambda_t)}^{H_t}$$

taking (k_t, k_{t+1}) *as given, and*

$$k_{t+1} = \arg\max_{k'} \overbrace{H(k_t, k', c_t, n_t, \lambda_t) + \beta H_{t+1}(k', k_{t+2}, c_{t+1}, n_{t+1}, \lambda_{t+1})}^{H_t + \beta H_{t+1}}$$

taking (k_t, k_{t+2}) *as given.*

We henceforth assume an interior solution. As long as $k_0 > 0$, interior solution is indeed ensured by the Inada conditions on F and U.

2.3.1. The first-order conditions (FOCs)

The FOC with respect to c_t gives

$$\frac{\partial L^1}{\partial c_t} = \beta^t U_c(c_t, 1-n_t) - \beta^t \lambda_t = 0 \qquad (4)$$
$$\text{Simplified: } U_c(c_t, 1-n_t) = \lambda_t$$

The FOC with respect to n_t gives

$$\frac{\partial L^1}{\partial n_t} = -\beta^t U_l(c_t, 1-n_t) + \beta^t \lambda_t F_n(k_t, n_t) = 0 \qquad (5)$$
$$\text{Simplified: } U_l(c_t, 1-n_t) = \lambda_t F_n(k_t, n_t)$$

The FOC with respect to k_{t+1} gives

$$\frac{\partial L^1}{\partial k_{t+1}} = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1}[F_k(k_{t+1}, n_{t+1}) + 1 - \delta] = 0 \qquad (6)$$
$$\text{Simplified: } \lambda_t = \beta \lambda_{t+1}[F_k(k_{t+1}, n_{t+1}) + 1 - \delta] = 0$$

Notice that λ_t is the link between saving capital in period t and using capital in period $t+1$. λ_t is referred to as the shadow price (of capital) because it dictates how much capital will be transferred between periods.

2.3.2. The Euler equation

Combining (4) with (5) yields

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = F_n(k_t, n_t) \qquad (7)$$

and combining (4) and (6) yields

$$\frac{U_c(c_t, l_t)}{\beta U_c(c_{t+1}, l_{t+1})} = 1 - \delta + F_k(k_{t+1}, n_{t+1}) \qquad (8)$$

The first condition indicates the marginal rate of substitution between consumption and leisure is equal to the marginal product of labor.

The second condition indicates that the marginal rate of intertemporal substitution in consumption equals the marginal rate of capital net of depreciation (plus one).

- At a utility maximum, the consumer cannot gain from feasible shifts of consumption between periods.
- Called the *Euler Equation*.

Exercise 5: Why are equations (7) and (8) important?

Exercise 6: What impacts the decision to transfer resources between periods?

2.3.3. The transversality condition (TVC)

A transversality condition describes what must be satisfied at the end of the time horizon, i.e., the terminal condition as we transverse beyond the planning horizon. The nature of the transversality condition (TVC) depends greatly on the statement of the problem. For example

1. The state variable x must equal zero at the terminal time T, i.e., $x_T=0$.
2. The state variable x must be less than some function of t, $x_T \leq \phi\ (T)$.
3. The state variable x must be equal to zero or some function of t in the limit, $T\rightarrow\infty$.

The central idea behind all TVCs is that if there is any flexibility at the end of the time horizon, then the marginal benefit from taking advantage of that flexibility must be zero at the optimum. You can apply this general principal to problems with more than one variable, to problems with constraints, and to problems with a salvage value.

Suppose for a moment that the horizon was finite in the Ramsey problem, i.e., $T< \infty$. Then, the Lagrangian would be $L^0 = \sum_{t=0}^{T} \beta^t H_t$ and the Kuhn-Tucker condition with respect to k_{T+1} would give

$$\frac{\partial L^0}{\partial k_{T+1}} = \beta^T \frac{\partial \mathrm{H}_\mathrm{T}}{\partial k_{T+1}} \geq 0 \text{ and } k_{T+1} \geq 0 \text{ with complementary slackness.}$$

Equivalently,

$$\lambda_\mathrm{T} = \beta^\mathrm{T} \mu_\mathrm{T} \geq 0 \text{ and } k_{T+1} \geq 0, \text{ with } \beta^\mathrm{T} \mu_\mathrm{T} k_{T+1} = 0.$$

The latter means that either k_{T+1}=0, or otherwise the shadow value of k_{T+1} =0 must be zero.

When $T = \infty$, the terminal condition $\beta^{T}\mu_{T}k_{T+1} = 0$ is replaced with

$$\lim_{t\to\infty} \beta^{t}\mu_{t}k_{t+1} = 0$$

This means that the (discounted) shadow value of capital converges to zero. Equivalently,

$$\lim_{t\to\infty} \beta^{t}U_{c}(c_{t},l_{t})k_{t+1} = 0 \tag{8}$$

- Note that if all constraints hold with inequality, the K-T conditions are Lagrange conditions.

<u>Exercise 7</u>: Explain (8).

The TVC is a condition for optimization. In most economic models, it plays the role of ruling out solutions in which wealth grows rapidly forever but is never used to provide consumption or dividends.

Proposition 1 *The plan* $\{c_{t},n_{t},k_{t+1}\}_{t=0}^{\infty}$ *is a solution to the social planner's problem if and only if*

$$\frac{U_{l}(c_{t},l_{t})}{U_{c}(c_{t},l_{t})} = F_{n}(k_{t},n_{t})$$

$$\frac{U_{c}(c_{t},l_{t})}{\beta U_{c}(c_{t+1},l_{t+1})} = 1-\delta+F_{k}(k_{t+1},n_{t+1})$$

$$c_{t}+k_{t+1} \le F(k_{t},n_{t})+(1-\delta)k_{t}$$

for all $t \ge 0$*, and* $k_0>0$ *is given, and*

$$\lim_{t\to\infty} \beta^{t}U_{c}(c_{t},l_{t})k_{t+1} = 0$$

Proof: See Acemoglu (2007).

- The first equation can be solved for $n_t = n(c_t,k_t)$, which can then be substituted into the second and third equations.
- Left with a system of two difference equations in two variables, namely c_t and k_{t+1}.
- The initial condition and the TVC give the boundary conditions for this system.

2.4. Dynamic Programming (not covered in class)

We will now redo the optimization problem and solve it using dynamic programming (DP). Consider again the social planner's problem. For any *k> 0*, define

$$V(k) \equiv \max \sum_{t=0}^{\infty} \beta^t U(c_t, 1-n_t)$$

subject to

$$c_t + k_{t+1} \leq F(k_t, n_t) + (1-\delta)k_t, \; \forall t \geq 0,$$

$$c_t, n_{t,} 1-n_t, k_{t+1} \geq 0, \; \forall t \geq 0$$

- V is called the *value function*.

The Bellman equation for this problem is

$$V(k) \equiv \max U(c, 1-n) + \beta V(k')$$

subject to

$$c + k' \leq F(k, n) + (1-\delta)k$$

$$c \in [0, F(k,1)], n \in [0,1], k' \geq 0, l \in [0,1]$$

- Let

$$[c(k), n(k), G(k)] = \arg\max\{...\}.$$

These are the policy rules. The key policy rule is G, which gives the dynamics of capital. The other rules are static.

Define $\bar{k}$ by the unique solution to

$$\bar{k} = (1-\delta)\bar{k} + F(\bar{k},\ 1)$$

and note that $\bar{k}$ represents an upper bound on the level of capital that can be sustained in any steady state.

Without serious loss of generality, we will henceforth restrict $k_t \in [0, \bar{k}]$.

<u>Exercise 8</u>: Why is having the solution being bound above and below important?

Once the solution is bounded, these bounds can be used to create a state space to find a solution. Unless the space is bounded, it is difficult to "find" the equilibrium.

<u>Exercise 9</u>: Do these bound pose any constraint on the solution?

The bound pose no constraint on the solution. It simply defines where feasible equilibrium are.

Let B be the set of continuous and bounded functions $v:[0,\bar{k}] \to \mathbb{R}$ and consider the mapping $T : B \to B$ defined as follows:

$$Tv(k) \equiv \max U(c, 1-n) + \beta v(k')$$

subject to

$$c + k' \leq F(k,n) + (1-\delta)k$$

$$c \in [0, F(k,1)], n \in [0,1], k' \in [0,\bar{k}], l \in [0,1]$$

- The conditions we have imposed on U and F imply that T is a contraction mapping.
- It follows that T has a unique fixed point $V = T\,V$ and this fixed point gives the solution.

The Lagrangian for the DP problem is

$$L = U(c, 1-n) + \beta V(k') + \lambda[F(k,n) + (1-\delta)k - c - k']$$

The FOC with respect to c gives

$$\frac{\partial L}{\partial c} = 0 \Leftrightarrow U_c(c, 1-n) = \lambda \tag{10}$$

The FOC with respect to n gives

$$\frac{\partial L}{\partial n} = 0 \Leftrightarrow U_l(c, 1-n) = \lambda F_n(k,n) \tag{11}$$

The FOC with respect to k' gives

$$\frac{\partial L}{\partial k'} = 0 \Leftrightarrow \lambda_t = \beta V_k(k') \tag{12}$$

The envelope condition is

$$V_k(k) = \frac{\partial L}{\partial k} = \lambda[1 - \delta + F_k(k,n)] \tag{13}$$

Combining (10) with (11) yields

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = F_n(k_t, n_t) \tag{14}$$

Combining (10), (12), and (13) updated one period yields

$$\frac{U_c(c_t, l_t)}{\beta U_c(c_{t+1}, l_{t+1})} = 1 - \delta + F_k(k_{t+1}, n_{t+1}) \tag{15}$$

- (14) and (15) are the same conditions derived with optimal control.
- Note that we can state the Euler equation, (15), as

$$\frac{V_k(k_t)}{V_k(k_{t+1})} = \beta[1 - \delta + F_k(k_t, n_t)].$$

Alternatively, the Bellman equation for this problem can be written as

$$V(k) \equiv \max U(F(k,n)+(1-\delta)k-k',1-n)+\beta V(k')$$

The FOC with respect to *n* gives

$$U_c(c,1-n)F_n(k,n)-U_l(c,1-n)=0 \tag{16}$$

The FOC with respect to k' gives

$$-U_c(c,1-n)+\beta V_k(k')=0 \tag{17}$$

The envelope condition is

$$V_k(k)=U_c(c,1-n)[1-\delta+F_k(k,n)] \tag{18}$$

- These FOCs can be combined to yields (14) and (15).

2.5. Definition 1. *A feasible allocation* $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ *for the economy is Pareto optimal if there exists no other feasible allocation* $\{\hat{c}_t, \hat{n}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}$ *such that*

$$\hat{c}_t + \hat{k}_{t+1} \le F(\hat{k}_t, \hat{n}_t) + (1-\delta)\hat{k}_t,\ \forall t \ge 0,$$

and

$$\sum_{t=0}^{\infty} \beta^t U(\hat{c}_t, 1-\hat{n}_t) \ge \sum_{t=0}^{\infty} \beta^t U(c_t, 1-n_t)$$

with at least one strict inequality in the preceding relationship.

Exercise 10: Why does at least one strict inequality need to hold in Definition 1?

2.5.1 Proposition 2. *The optimal allocation derived from the social planner's problem is Pareto optimal.*[7]

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[7] This specification presumes that the social planner gives the same weights across households. Naturally there also exist Pareto optimal allocations with unequal distribution of consumption across households, though these are less natural and less interesting in the context of economies with a normative representative household.

3. Decentralized Competitive Equilibrium

Our main interest is not optimal growth (the solution to the social planner problem) but equilibrium growth (the solution to the competitive equilibrium). A detailed analysis of competitive equilibrium growth is present in what follows.

We will find that the competitive equilibrium can be obtained from the optimal growth problem.

- The *second welfare theorem* implies that the optimal growth path -- characterized in what follows -- corresponds to the equilibrium growth path (it can be decentralized as a competitive equilibrium).

3.1. Households

The preferences of household *i* are given by

$$U_0^i = \sum_{t=0}^{\infty} \beta^t U(c_t^i, l_t^i)$$

In recursive form, $U_0^i = U(c_t^i, l_t^i) + \beta U_{t+1}^i$

The time constraint for household *i* can be written as

$$l_t^i = 1 - n_t^i$$

Households can save via:

1. Capital: i_t^i with return r_t. Total capital holds in period *t* is k_t^i .
2. Bonds: x_t^i with return R_t. Total bond holds in period *t* is b_t^i .

Households are also owners of all firms with α_t^i equal to the share of each firm held by household *i* in firm *p* during period *t*. Firm *p* earns profits π_t^p in period *t*.

<u>Exercise 11</u>: Construct the household's budget constraint by putting the expenditures on the LHS and income on the RHS.

In equilibrium, firm profits are zero because of CRS. It follows that $\pi_t^p = 0$.

Household i accumulates capital according to

$$k_{t+1}^i = (1-\delta)k_t^i + i_t^i$$

and bonds according to

$$b_{t+1}^i = b_t^i + x_t^i$$

<u>Exercise 12</u>: Combine the above equations to rewrite the household budget.

The natural non-negativity constraint imposed on capital holdings is

$$k_{t+1}^i \geq 0$$

Household can either lend or borrow in risk-free bonds. We only impose the following *natural borrowing constraint*:

$$-(1+R_{t+1})b_{t+1}^i \leq (1-\delta + r_{t+1})k_{t+1}^i + \sum_{s=1}^{\infty}\left\{\left(\prod_{j=1}^{s}\frac{1}{1+R_{t+j}}\right)(1+R_{t+1})w_{t+s}\right\}.$$

This constraint simply requires that the net debt position of the household (the LHS) does not exceed the present value of the labor income the household can attain by full-time working (set $l_t = 0$), consuming nothing, along with current assets.

This constraint is found by solving the flow budget constraint forward. That is, update the flow budget constraint one period, solve for b_{t+1}, and plug it into the period t constraint. Through this iterative process, one will easily see how future wages are discounted to period t.

Exercise 13: Derive the natural borrowing constraint.

To obtain maximum amount of borrowing, assume:

1. *All current and future consumption is 0,* $c_t^i = 0$
2. *No leisure time,* $n_t^i = 0$

Update household budget constraint to period t+1:

$$k_{t+2}^i + b_{t+2}^i \leq (1-\delta + r_{t+1})k_{t+1}^i + (1+R_{t+1})b_{t+1}^i + w_{t+1} \qquad (a)$$

Set future capital to zero: $k_{t+2}^i = 0$.

$$b_{t+2}^i \leq (1-\delta + r_{t+1})k_{t+1}^i + (1+R_{t+1})b_{t+1}^i + w_{t+1} \qquad (b)$$

Update (b) to period t+2:

$$b_{t+3}^i \leq (1+R_{t+2})b_{t+2}^i + w_{t+2} \qquad (c)$$

Plug (c) into (b) at b_{t+2}^i:

$$\frac{b_{t+3}^i}{(1+R_{t+2})} \leq (1-\delta + r_{t+1})k_{t+1}^i + (1+R_{t+1})b_{t+1}^i + w_{t+1} + \frac{w_{t+2}}{(1+R_{t+2})} \qquad (d)$$

Update (b) to period t+3:

$$b_{t+4}^i \leq (1+R_{t+3})b_{t+3}^i + w_{t+3} \qquad (e)$$

Plug (e) into (d) at b_{t+3}^i:

$$\frac{b_{t+4}^i}{(1+R_{t+2})(1+R_{t+3})} \leq (1-\delta + r_{t+1})k_{t+1}^i + (1+R_{t+1})b_{t+1}^i + w_{t+1} + \frac{w_{t+2}}{(1+R_{t+2})} + \frac{w_{t+3}}{(1+R_{t+2})(1+R_{t+3})} \qquad (f)$$

Continue to update (b) to next period and plug in until:

$$\underbrace{\lim_{s\to\infty}\left(b_{t+s}^i \prod_{s=2}^{\infty}\frac{1}{1+R_{t+s}}\right)}_{=0} - (1+R_{t+1})b_{t+1}^i \leq (1-\delta + r_{t+1})k_{t+1}^i + \sum_{s=1}^{\infty}\left\{\left(\prod_{j=1}^{s}\frac{1}{1+R_{t+j}}\right)(1+R_{t+1})w_{t+s}\right\}.$$

Arbitrage between bonds and capital implies that, in any equilibrium, the interest rate on riskless bonds must equal the rental rate of capital net of depreciation:

$$R_t = r_t - \delta$$

- o If $R_t < r_t - \delta$, all individuals would like to short-sell bonds and there would be excess supply of bonds.
- o If $R_t > r_t - \delta$, nobody in the economy would invest in capital.

- Households are then indifferent between bonds and capital.

Exercise 14: What are short-sales.

Letting $a_t^i = k_t^i + b_t^i$ denote total assets, the budget constraint reduces to

$$c_t^i + a_{t+1}^i \leq (1+R_t)a_t^i + w_t n_t^i. \tag{19}$$

and the natural borrowing constraint becomes

$$-a_{t+1} \leq \sum_{s=1}^{\infty}\left\{\left(\prod_{j=1}^{s}\frac{1}{1+r_{t+j}}\right)(1+R_{t+1})w_{t+s}\right\} < M < \infty \tag{20}$$

3.1.1. Household *i*'s Maximization Problem

Given a price sequence $\{R_t, w_t\}_{t=0}^{\infty}$, household *i* chooses a plan $\{c_t, n_t, a_{t+1}\}_{t=0}^{\infty}$ so as to maximize lifetime utility subject to its flow (or per period) budget constraints

$$\max U_0^i = \sum_{t=0}^{\infty}\beta^t U(c_t^i, l_t^i)$$

subject to

$$c_t^i + a_{t+1}^i \leq (1+R_t)a_t^i + w_t n_t^i$$

$$c_t^i \geq 0, n_t^i \in [0,1], a_{t+1}^i < M \quad \forall t$$

Let $\mu_t^j = \beta^t \lambda_t^j$ be the Lagrange multiplier for the budget constraint, we can write the Lagrangian as

$$L_0^i = \sum_{t=0}^{\infty}\beta^t\{U(c_t^i, 1-n_t^i) + \lambda_t^i[(1+R_t)a_t^i + w_t n_t^i - a_{t+1}^i - c_t^i]\} = \sum_{t=0}^{\infty}\beta^t H_t^i \tag{21}$$

where

$$H_t^i = U(c_t^i, 1-n_t^i) + \lambda_t^i[(1+R_t)a_t^i + w_t n_t^i - a_{t+1}^i - c_t^i] \tag{22}$$

The FOC with respect to c_t^i gives

$$\frac{\partial L_0^i}{\partial c_t^i} = \beta^t U_c(c_t^i, 1-n_t^i) - \beta^t \lambda_t^i = 0$$

Simplified:

$$U_c(c_t^i, 1-n_t^i) = \lambda_t^i \tag{23}$$

The FOC with respect to n_t^i gives

$$\frac{\partial L_0^i}{\partial n_t^i} = -\beta^t U_l(c_t^i, 1-n_t^i) + \beta^t \lambda_t^i w_t = 0$$

Simplified:

$$U_l(c_t^i, 1-n_t^i) = \lambda_t^i w_t \tag{24}$$

The Kuhn-Tucker condition with respect to a_{t+1}^i gives

$$\frac{\partial L_0^i}{\partial a_{t+1}^i} = -\beta^t \lambda_t^i + \beta^{t+1}[1+R_{t+1}]\lambda_{t+1}^i, \leq 0 \tag{25}$$

Simplified:

$$\lambda_t^i \geq \beta[1+R_{t+1}]\lambda_{t+1}^i \tag{25}$$

with equality whenever $-a_{t+1} < M$. That is, the complementary slackness condition is

$$\lambda_t^i - \beta[1+R_{t+1}][a_{t+1}^i - M] = 0$$

Combining (23) with (24) yields

$$\frac{U_l(c_t^i, l_t^i)}{U_c(c_t^i, l_t^i)} = w_t \tag{26}$$

Households equate their marginal rate of substitution between consumption and leisure with the (common) wage rate.

Combining (23) and (25) yields

$$U_c(c_t^i, l_t^i) \geq \beta(1+R_{t+1})U_c(c_{t+1}^i, l_{t+1}^i) \tag{27}$$

with equality whenever $-a_{t+1} < M$.

- As long as the borrowing constraint does not bind, households equate their marginal rate of intertemporal substitution with the (common) return on capital.
- If the borrowing constraint is binding, the marginal utility of consumption today may exceed the marginal benefit of savings: the household would like to borrow, but it can't.
- If $-a_0 < M$, then $-a_t < M$ for all t dates and the Euler equation will be satisfied with equality.

Moreover, if the borrowing constraint never binds, backdating (25) one period, $\lambda_t^i = \beta^{-1}[1+R_t]^{-1}\lambda_{t-1}^i$, and iterating backwards implies

$$\begin{aligned}\lambda_t^i &= \beta^{-1}[1+R_t]^{-1}\lambda_{t-1}^i \\ &= \beta^{-2}[1+R_t]^{-1}[1+R_{t-1}]^{-1}\lambda_{t-2}^i \\ &= \beta^{-3}[1+R_t]^{-1}[1+R_{t-1}]^{-1}[1+R_{t-2}]^{-1}\lambda_{t-3}^i \\ &\vdots \\ &= \beta^{-t}\prod_{s=0}^{t}[1+R_s]^{-1}\lambda_0^i\end{aligned}$$

or

$$\beta^t\lambda_t^i = \prod_{s=0}^{t}[1+R_s]^{-1}\lambda_0^i$$

Using complementary slackness and the above equation, we can therefore rewrite the terminal condition as

$$\lim_{t\to\infty}\beta^t\lambda_t^i a_{t+1}^i = \lim_{t\to\infty}\beta^t\lambda_t^i M = \lambda_0^i \lim_{t\to\infty}\prod_{s=0}^{t}[1+R_s]^{-1}M$$

Note that

$$\prod_{j=0}^{t}[1+R_j]^{-1}M = \sum_{s=t}^{\infty}\prod_{j=0}^{s}[1+R_j]^{-1}w_s$$

and the natural borrowing constraint $\sum_{s=1}^{\infty}\left\{\left(\prod_{j=1}^{s}\frac{1}{1+r_{t+j}}\right)(1+R_{t+1})w_{t+s}\right\} < M < \infty$ implies that

$$\lim_{t\to\infty}\sum_{j=t}^{\infty}\prod_{s=0}^{j}[1+R_s]^{-1}w_j = 0$$

- This arrives at the more familiar version of the transversality condition:

$$\lim_{t\to\infty}\beta^t\lambda_t^i a_{t+1}^i = 0$$

or, equivalently using (23),

$$\lim_{t\to\infty}\beta^t U_c(c_t^i, 1-n_t^i)a_{t+1}^i = 0 \tag{28}$$

3.2. Firms

There is an arbitrary number P_t of firms in period t, indexed by $p \in [0,P_t]$.

> Firms employ labor and rent capital in competitive labor and capital markets, have access to the same neoclassical technology, and produce a homogeneous good that they sell competitively to the households in the economy.

Let K_t^p and L_t^p denote the amount of capital and labor that firm p employs in period t. Then, the profits of that firm in period t are given by

$$\Pi_t^p = F(K_t^p, L_t^p) - r_t K_t^p - w_t L_t^p$$

The firms seek to maximize profits. The FOCs for an interior solution require

$$\begin{aligned} r_t &= F_K(K_t^p, L_t^p) \\ w_t &= F_L(K_t^p, L_t^p) \end{aligned} \tag{29}$$

- Under CRS, an interior solution to the firms' problem to exist if and only if r_t and w_t imply the same K_t^p / L_t^p.
- The FOCs pin down the capital-labor ratio for each firm K_t^p / L_t^p, but not the size of the firm, L_t^p.
- Because all firms have access to the same technology, they use exactly the same capital-labor ratio.

3.3. Market Clearing

Each market must clear each period. Thus, below is per period market clearing and all markets in all periods must clear. This means that quantity demanded is equal to quantity supplied in each period so that there exists a market equilibrium price in each market.

- The bond market clears if and only if

$$\int_0^1 b_t^i di = 0$$

Exercise 15: Why does the bond market have to sum to 0? Does this mean that there are no bonds in the economy?

- The capital market clears if and only if

$$\int_0^{P_t} K_t^p dp = \int_0^1 k_t^i di$$

Equivalently, $\int_0^{P_t} K_t^p dp = k_t$, where $k_t = K_t = \int_0^1 k_t^i di$ is the per capita and aggregate physical capital stock.

- The labor market, on the other hand, clears if and only if

$$\int_0^{P_t} L_t^p dp = \int_0^1 n_t^i di$$

Equivalently, $\int_0^{P_t} L_t^p dp = n_t$, where $n_t = L_t = \int_0^1 n_t^i di$ is the per head and aggregate labor supply.

3.4. General Equilibrium

Definition 2. *A competitive equilibrium for the economy consists of paths of allocation* $\{(c_t^i, n_t^i, k_{t+1}^i, b_{t+1}^i)_{i\in[0,L_t]}, (K_t^p, L_t^p)_{p\in[0,P_t]}\}_{t=0}^{\infty}$ *and a price system* $\{R_t, r_t, w_t\}_{t=0}^{\infty}$ *such that*

(i) Given $\{R_t, r_t, w_t\}_{t=0}^{\infty}$*, the path* $\{c_t^i, n_t^i, k_{t+1}^i, b_{t+1}^i\}_{t=0}^{\infty}$ *maximizes the utility of household i, for every i.*

(ii) Given $\{r_t, w_t\}_{t=0}^{\infty}$*, the pair* (K_t^p, L_t^p) *maximizes firm profits, for every firm p and period t. That is, factor prices are given by (29).*

(iii) The bond, capital and labor markets clear in every period.

(iv) The government's budget constraint is satisfied (not in baseline model though).

In what follows, since there is equivalence between the social planner's problem and the competitive equilibrium, it matters not which one we use. That is, the steady state and transitional dynamics are identical between the social planner's problem and the competitive equilibrium.

4. Steady State

Proposition 4. *There exists a unique (positive) steady state* $(c^*, n^*, k^*) > 0$.

- *The steady-state values of k, n, y, c, w, r, and R are all independent of the utility function.*
- *Determined by the production function F, δ, and β.*
- *In particular, the capital-labor ratio* $\kappa^* \equiv k^*/n^*$ *equates the net-of-depreciation MPK with the discount rate,*

$$f'(\kappa^*) = \beta^{-1} - 1 + \delta,$$

and is a decreasing function of $\beta^{-1} - 1 + \delta$.

We can also identify all other relevant prices and quantities in the economy:

$$R^* = \beta^{-1} - 1, \; r^* = \beta^{-1} - 1 + \delta$$
$$w^* = F_L(\kappa^*, 1) = \frac{U_l(c^*, 1-n^*)}{U_c(c^*, 1-n^*)},$$
$$y^* = f(\kappa^*), \quad c^* = y^* - \delta k^*$$

where $f(\kappa^*, 1) = F(\kappa^*, 1)$ and $\phi(\kappa^*) = f(\kappa^*)/\kappa$.

5. Transitional Dynamics

As in the Solow growth model, starting from any initial value $k_0>0$, the economy asymptotically goes to the steady state. The transition is monotonic.

Consider the condition that determined labor supply in the Neoclassical model:

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = F_n(k_t, n_t)$$

We can solve this for n_t as a function of contemporaneous consumption and capital: $n_t = n(c_t, k_t)$. Substituting this into the Euler condition and the resource constraint, we conclude:

$$\frac{U_c(c_t, l_t)}{U_c(c_{t+1}, l_{t+1})} = \beta[1-\delta + F_k(k_{t+1}, n_{t+1}(c_{t+1}, k_{t+1}))]$$

$$k_{t+1} = F(k_t, n_t(c_t, k_t)) + (1-\delta)k_t - c_t$$

This is a system of two first-order difference equation in c_t and k_t. Together with the initial condition (k_0 given) and the transversality condition, this system pins down the path of $\{c_t, k_t\}_{t=0}^{\infty}$.

3.4.1. Welfare Theorems

There are two fundamental welfare theorems in economics:

1. The *first welfare theorem* states that the competitive equilibrium of the economy is Pareto optimal.

2. The *second welfare theorem* states that there exists a price system that can support any Pareto optimal allocation as a competitive equilibrium.

It can be shown in the Ramsey model that the decentralized market economy and the centralized social planner economy are equivalent. The following proposition combines the first and second fundamental welfare theorems, as applied to the Ramsey model.

Proposition 3 *The set of competitive equilibrium allocations for the market economy coincide with the set of Pareto allocations for the social planner.*

Proof:

Part A: First consider how the solution to the social planner's problem can be implemented as a competitive equilibrium (the 2nd Welfare Thm.).

Let $\kappa_t = k_t / n_t$ and choose the price path $\{R_t, r_t, w_t\}_{t=0}^{\infty}$ given by

$$\begin{aligned} R_t &= r_t - \delta \\ r_t &= F_K(k_t, n_t) = f'(\kappa_t) \\ w_t &= F_N(k_t, n_t) = f(\kappa_t) - f'(\kappa_t)\kappa_t \end{aligned}$$

Trivially, these prices ensure that the FOCs are satisfied for every household and every firm if we set $c_t^i = c_t, n_t^i = n_t$ and $K_t^p / L_t^p = k_t$ for all *i* and *p*.

Next, we need to verify that the proposed allocation satisfies the budget constraint of the households. From the resource constraint,

$$c_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

From CRS and the FOCs for the firms, $F(k_t, n_t) = r_t k_t + w_t n_t$. Combining, we get

$$c_t + k_{t+1} = (1-\delta + r_t)k_t + w_t n_t$$

The budget constraint of household i is given by

$$b_{t+1}^i + c_t^i + k_{t+1}^i = (1-\delta + r_t)k_t^i + w_t n_t^i + (1+R_t)b_t^i$$

- For this to be satisfied at the proposed prices with $c_t^i = c_t$ and $n_t^i = n_t$, it is necessary and sufficient that $k_t^i + b_t^i = a_t$ for all *i* and *t*.

- Finally, it is trivial to check the bond, capital, and labor markets clear.

<u>Part B</u>: We next consider the converse, how a <u>competitive equilibrium coincides with the Pareto solution (1st Welfare Thm)</u>.

Because agents have the same preferences, face the same prices, and are endowed with identical level of initial wealth, and because the solution to the individual's problem is essentially unique (where essentially means unique with respect to c_t^i, n_t^i, and $a_t^i = k_t^i + b_t^i$ but indeterminate with respect to the portfolio choice between k_t^i and b_t^i), we have that $c_t^i = c_t, n_t^i = n_t$ and $a_t^i = a_t$ for all *i* and *t*.

By the FOCs to the individual's problem, it follows that $\{c_t, n_t, a_t\}_{t=0}^{\infty}$ satisfies

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = w_t$$

$$\frac{U_c(c_t, l_t)}{U_c(c_{t+1}, l_{t+1})} = \beta(1-\delta + r_{t+1})$$

$$c_t + a_{t+1} \leq (1+R_t)a_t + w_t n_t$$

for all $t \geq 0$*, and* $a_0 > 0$ *is given, and*

$$\lim_{t\to\infty} \beta^t U_c(c_t, l_t)a_{t+1} = 0$$

Combining the above with the FOCs of the firm and the budget constraint gives

$$c_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

for all $t \geq 0$, which is simply the resource constraint of the economy. Finally, and $\lim_{t\to\infty} \beta^t U_c(c_t^i, 1-n_t^i)a_{t+1}^i = 0$ with $a_{t+1} = k_{t+1}$ implies the social planner's transversality condition, while $a_0 = k_0$ gives the initial condition. ■

6. Exogenous Labor Supply and Log Preferences

We now solve the Social Planner's Problem using functional forms.

- Suppose that population size is 1.
- Suppose that n_t =1 for all t.
    - Leisure is not valued, or
    - Labor supply is exogenously fixed to 1.
- Suppose that capital fully depreciates, $\delta = 1$.
- Preferences are given as:

$$u(c_t) = \ln c_t$$

- The *per capita* production function takes the form:

$$f(k_t, 1) = Ak_t^{\alpha}$$

- The social planner solves

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to

$$c_t + k_{t+1} = Ak_t^{\alpha}$$

<u>Exercise 16</u>: Write out the Lagrangian for this problem and solve for the first-order conditions.

<u>Exercise 17</u>: Solve for the Euler equation.

Exercise 18: Use the Euler equation to solve for the steady state capital. Then solve for steady state output. Now use the budget constraint to solve for steady state consumption.

Exercise 19: We now solve for the transition path. Move k_{t+1} to the LHS of the Euler Equation.

Adding 1 to both sides:

Substituting the budget constraint to the LHS:

(30)

Iterating (30) forward yields:

(31)

We now use the transversality condition to show that the above limit expression goes to zero.

$$\lim_{T\to\infty}\frac{(\alpha\beta)^T y_T}{c_T}=\lim_{T\to\infty}(\alpha\beta)^T\left(\frac{y_T}{c_T}\right)=\lim_{T\to\infty}(\alpha\beta)^T\left(1+\frac{k_{T+1}}{c_T}\right)$$

Note that $\alpha\beta<1$ which implies that the first term goes to zero in the limit.

However, the second term is simply the transversality condition multiplied by α^T:

$$\lim_{T\to\infty}\frac{(\alpha\beta)^T k_{T+1}}{c_T}=\lim_{T\to\infty}\alpha^T\beta^T k_{T+1}u'(c_T) \tag{32}$$

Given that the transversality condition holds (i.e., $\lim_{T\to\infty}\beta^T k_{T+1}u'(c_T)=0$), then (32) holds as well since $\lim_{T\to\infty}\alpha^T=0$.

Thus,

$$\lim_{T\to\infty}(\alpha\beta)^T\left(\frac{y_T}{c_T}\right)=0 \tag{33}$$

Using (33) in (31) as T goes to infinity and recognizing that this is a geometric series yields:

$$\frac{c_t}{y_t}=(1-\alpha\beta) \tag{34}$$

Exercise 20: Solve for c_t as a function of k_t.

Exercise 21: Use the resource constraint ($y_t = k_{t+1} + c_t$) in the equation solved in Exercise 20 to solve for k_{t+1} as functions of k_t.

Given any $k_0>0$, the above expressions map out the entire equilibrium allocation path (the transition path to the steady state).

Exercise 22: Solve for the output growth rate.

Notice that as the capital stock increases, the growth rate decreases (i.e., the model exhibits diminishing returns to capital).

- What happens to growth in steady state?

7. References

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8. In-class exercise

8.1 The donut eating problem (a.k.a. cake eating problem)

We review a series of similar models but permit variations based on:

1. Vary the terminal condition by length of periods and amount of state variable remaining. That is, we will analyze models with a finite number of periods, i.e., $T < \infty$ and infinite number of periods, i.e., $T = \infty$.
2. Varying the rate of discounting of the future from no discounting, $\beta = 1$ and some discounting, $0 < \beta < 1$.

<u>Model setup</u>.

- $n_t = 1$ for all t.
- $\delta = 1$
- Preferences are given as:

$$u(c_t) = \ln c_t$$

- The production function takes the form:

$$F(K_t, 1) = AK_t \quad \text{where } A = 1$$

- ❖ Do the first and second welfare theorems hold? If so, what does this mean?

a. Finite cake eating problem ($T < \infty$ and $\beta = 1$).

Set up the social planner's problem.

Solve the social planner's problem.

b. Finite cake eating problem ($T < \infty$ and $0 < \beta < 1$).

Set up the social planner's problem.

Solve the social planner's problem.

c. Finite cake eating problem with leftovers ($T < \infty$, $0 < \beta < 1$, and $x_T \geq b$).

Set up the social planner's problem.

Solve the social planner's problem.

d. Infinite cake eating problem ($T = \infty$ and $0 < \beta < 1$).

Set up the social planner's problem.

Solve the social planner's problem.

e. Infinite cake eating problem ($T = \infty$ and $\beta = 1$).

Set up the social planner's problem.

Solve the social planner's problem.

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Lecture 6: Applications Using the Neoclassical Growth Model

1. Introduction

The neoclassical growth model is the workhorse of growth theory. This model has been extended to study human capital externalities among many other applications. In particular, the literature has extended exogenous growth models to endogenous growth models. Exogenous growth models have unexplained technical progress whereas endogenous growth theory holds that investment in human capital, innovation, and knowledge are significant contributors to economic growth. The theory also focuses on positive externalities and spillover effects of a knowledge-based economy which will lead to economic development.

There are three applications which we will analyze.

1. Endogenous growth with only reproducible input.
2. Endogenous growth even with non-reproducible input.
3. Endogenous growth through innovation.

2. The donut (cake) eating problem with production.

This exercise introduce the concept of endogenous growth. That is, output grows without having technological or population growth pushing it along.

Model setup

- The cake can grow if it is stored: $y_t = Ak_t$.
    - A is constant.
- Infinite number of periods, i.e., $T = \infty$.
- $0 < \beta < 1$.
- $n_t = 1$ for all t.
- $\delta = 1$

The CRRA utility function is given by:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \text{ for } \sigma>0,\ \sigma \neq 1,$$
$$= \ln c_t \quad \text{for } \sigma = 1$$

where $1/\sigma$ is the intertemporal substitution elasticity between consumption in any two periods, i.e., it measures the willingness to substitute consumption between different periods. The smaller σ, the more willing is the household to substitute consumption over time. Since the coefficient of relative risk aversion is constant, this utility is known as the constant relative risk aversion (CRRA) utility.

There are three other properties that are important:

1. The CRRA utility function is increasing in $c^{1-\sigma}$if $\sigma < 1$, but decreasing if $\sigma > 1$.
   Dividing through by $1-\sigma$ assure the marginal utility is always positive.
2. If $\sigma \to 1$, the utility converges to $\ln c_t$.
3. $u'''(c) > 0$ implies a positive motive for precautionary savings.

Show that the above utility function's relative risk aversion is constant.

What is the difference between the discount rate and the intertemporal substitution elasticity?

Set up the social planner's problem.

Solve for the Euler equation.

Find an equation to iterate forward (*hint*: modify the Euler equation).

Iterate forward to through time.

What inequality must be satisfied to assure that the TVC holds?

This equation is sometimes referred to as the existence requirement. For any solution to exist, this equation must be satisfied.

Simplify this condition as much as possible.

This expression is the critical discount factor.

If the critical discount factor does not hold, then society is saving too much and the TVC is violated. Recall that this implies that individuals have an incentive to save even in the limit. When societies save too much, consumption (and utility) could be higher if they saved less. This type of 'saving too much' and accumulating savings even in the limit is called dynamic inefficiency.

Solve for the transition path if the critical discount factor holds.

All variables grow at same rate. This is called balanced growth.

In the previous exercises, we saw that growth was bounded, i.e., $\gamma_t = 0$ as the economy approached the steady state.

What condition determines whether growth will be bounded or whether unbounded growth occurs?

- Unbounded (or positive) growth is referred to as endogenous growth.
- This model:
    - Simplest endogenous growth model.
    - Yields a constant rate of growth even if $\gamma_t>0$.
    - Uses the assumption that the production function does not exhibit diminishing returns to scale to lead to endogenous growth.

Compare the *critical discount factor* to the *growth criteria*. Is there boundaries on β?

Another way of analyzing this is to compare the growth rate to the return on capital. One can think of expanding output through either growing faster or amassing capital.

What does this tell you about savings?

3. Introduction to non-reproducible inputs endogenous growth model (Rebelo 1991)

The first in-class exercises in the previous section obtained endogenous growth but required all inputs (physical capital) to be reproducible. One question is whether endogenous growth occurs if one factor is non-reproducible, such as labor.

3.1. The model

This model simply extends the neoclassical growth model studied in the main lecture packet by permitting leisure in the utility function. For convenience, it is assumed that there exists a physical capital sector which takes physical capital as an input and produces more physical capital.

The per capital utility function is

$$\sum_{t=0}^{\infty} \frac{\beta^t c_t^{1-\sigma} l_t^{\eta(1-\sigma)}}{1-\sigma}$$

where l_t is the amount of leisure time enjoyed by the individual.

The per capita aggregate resource constraint and the law of motion of physical capital are:

$$c_t = y_t = Ak_{1t}^{\alpha} n_t^{1-\alpha} \tag{1}$$

$$k_{t+1} = (1-\delta)k_t + Bk_{2t} \tag{2}$$

where the second term in (2) in the production function for physical capital.

For completeness, the total capital and labor in the economy are

$$k_t = k_{1t} + k_{2t} \tag{3}$$

$$n_t + l_t = 1 \tag{4}$$

2.2. Solving the model

<u>Exercise 1</u>: Do the 1st and 2nd welfare theorems hold?

Exercise 2: Use equations (1)–(4) along with the utility function to form the Lagrangian for the Social Planner's problem:

$$L:\sum_{t=0}^{\infty}\left\{\frac{\beta^t c_t^{1-\sigma}(1-n_t)^{\eta(1-\sigma)}}{1-\sigma}+\lambda_t(Ak_{1t}^{\alpha}n_t^{1-\alpha}-c_t)+\mu_t((1-\delta)k_t+B(k_t-k_{1t})-k_{t+1})\right\}$$

Exercise 3: Provide the four first-order conditions (c_t, n_t, k_{t+1}, and k_{t1+1}) and label them (5) – (8).

c_t: $$\beta^t c_t^{-\sigma}(1-n_t)^{\eta(1-\sigma)}=\lambda_t \tag{5}$$

n_t: $$\eta\beta^t c_t^{1-\sigma}(1-n_t)^{\eta(1-\sigma)-1}=\lambda_t(1-\alpha)y_t n_t^{-1} \tag{6}$$

k_{t+1}: $$\mu_t=\mu_{t+1}((1-\delta)+B) \tag{7}$$

k_{1t+1}: $$\mu_{t+1}B=\lambda_{t+1}\alpha y_{t+1}k_{1t+1}^{-1} \tag{8}$$

The growth rates can now be solved for using the first-order conditions.

Exercise 4: Use (5) and (6) to solve for the labor decision.

- ❖ Does the labor share fall to zero in any period?

Exercise 5: Use the labor decision to solve for the relationship between the growth rate in consumption (and output) and capital.

-

Exercise 6: Backdate (8) to period t and plugging this and (8) into (7).

Exercise 7: Use the above expression along with (1) and (5) to yield the relationship between consumption and capital and output.

- Growth rates are not equal.

We now have two growth expressions and two unknown growth rates. We can use this system to solve for the solution.

Exercise 8: Solve for the growth rate in consumption and physical capital in the goods sector.

❖ What condition must be satisfied for growth to occur in this model?

2.3. Analyzing the model

<u>Exercise 9</u>: What affects growth in output and consumption?

2.4. Conclusion

The functional form of the production function seems to effect the conclusions of the model.

For example, if the production function exhibits increasing returns to scale in the renewable factor:

$$y_t = Ak_{1t}^{\alpha} n_t^{1-\alpha} + Bk_{1t}$$

Then the labor share falls to zero as t goes to infinity (everyone enjoys leisure forever, yeah!) and there still is endogenous growth.

If the production function exhibits increasing returns to scale in the non-renewable factor:

$$y_t = Ak_{1t}^{\alpha} n_t^{1-\alpha} + Bn_{1t}$$

Then the labor share does not fall to zero and there is no growth.

3. Introduction to endogenous growth through innovation (Romer 1990)

The *AK* models are of theoretical interest, but empirical evidence do not support the view that an economy can sustain indefinite growth in per capita income through physical and human capital deepening alone.

This application of the neoclassical model will provide an alternative endogenous growth model in which invention is a purposeful economic activity that requires real resources. This is the 'micro' side of new growth theory.

By explicitly modeling the research and development process, one can gain important insights into the effects of both government policy and international integration on growth.

The trickiest problem in introducing a R&D sector is deciding how to deal with the fact that ideas are *non-excludable*. If firms are to have an incentive to innovate, there must exist some type of institutional mechanism that allows an inventor to appropriate rents from his discovery. In an application of the Solow growth model, a poverty trap occurred due to proprietary technology requiring a subscription fee for use. This same idea will be used here.

There are many ways to model how innovation affects production.

- Expand the variety (or quality) of products.
- Increase efficiency.

3.1. The model

To maintain consistency across models, inventions are modeled as leading to the development of new intermediate goods (capital) that enhances the productivity of labor in the production of a single final good.

Think of the economy as having three sectors:

1. New technology (A) using a share of labor.
2. Intermediate capital goods using final output to produce capital for *next* period.
    a. A firm creates either existing capital or new capital, K_j.
    b. Types of capital are index by j.
3. Final goods (Y) using a share of labor and intermediate capital goods.

The utility function is

$$\sum_{t=0}^{\infty} \frac{\beta^t c_t^{1-\sigma}}{1-\sigma}$$

The total amount of labor in the economy is fixed:

$$N = N_y + N_A$$

The household is quite trivial in this model and is dragged along. As such, the model assumes an exogenous savings rate, *r*. The household's maximization is simple as they only maximize consumption of final goods across time. The standard Euler equation yields the growth rate in consumption:

$$\gamma_c = \frac{c_{t+1}}{c_t} = [\beta(1+r)]^{-\sigma}$$

3.1.1. Final goods production

The final good production function is given by

$$Y_t = N_{yt}^{1-\alpha}(K_{1,t}^{\alpha} + K_{2,t}^{\alpha} + K_{3,t}^{\alpha} + \ldots + K_{A_t,t}^{\alpha}) = N_{yt}^{1-\alpha}\sum_{j=1}^{A_t} K_{j,t}^{\alpha} \tag{1}$$

- $j = \{1, 2, 3, 4, \ldots, A_t\}$ indexes the types of capital goods K_j that can be used in production.
- A_t is the number of types of capital that have been invented as of date t.
- N_{yt} is the labor used in final goods production.

There are some important aspects of this production function:

1) This production exhibits constant returns to scale in all inputs taken jointly.
2) This production function is additively separable. This implies that increases in K_j has no effect on the marginal productivity of K_j, $i \neq j$. That is,

$$\frac{\partial Y_t}{\partial K_j} = \alpha N_y^{1-\alpha} K_j^{\alpha-1}$$

   a) Independent of the level of all other capital goods *i*.
   b) Decreasing returns to investment in any capital already in use.
3) If the range of capital (A_t) was fixed, the model would provide qualitatively similar results to previous models with the economy spreading more resources across different types of capital.
4) The marginal product of *new* capital is

$$\frac{\partial Y_t}{\partial K_j} = \alpha N_y^{1-\alpha} K_j^{\alpha-1}\Big|_{K_j=0} = \infty$$

Given the production function, equation (1), and the assumption that there is 100% depreciation of intermediate capital goods, the final good firm's objective is to maximize profits:

$$\max_{\{K_j\}} \Pi_t = N_{yt}^{1-\alpha}\sum_{j=1}^{A_t} K_{j,t}^{\alpha} - \sum_{j=1}^{A_t} p_j K_{j,t}^{\alpha} \tag{2}$$

where p_j is the price of capital good K_j in terms of final goods.

3.1.2. Technology sector (producing new capital technology A)

Production in the R&D sector is assumed to depend on the amount of labor employed, $N_{A,t}$, and the current technology, captured by the range of existing technology for capital goods, *A*:

$$A_{t+1} = \theta A_t N_{A,t} + A_t \quad (3)$$

- θ is the productivity shift parameter.
- There is no depreciation of technology.
- The price of technology is P_A.

There are some important aspects of this technology production:

1. Greater the body of existing knowledge, A_t, the lower the labor cost of generating new knowledge.

   Even if inventors can ration the use of their creations in final goods production, they can't prevent other inventors from using their ideas in new R&D.

2. Embodies the assumption that there are constant returns to scale in *A* taken alone.

3.1.3. Intermediate capital goods

The intermediate capital goods sector buys technology from R&D firms and produces intermediate capital goods. Once an intermediate goods producer buys the technology to produce capital good *j*, it becomes the monopoly supplier of that type of capital to the final goods sector.

Once developed, the technology shows how to combine raw material to produce quantities of the new capital good. That is, in period *t*, one unit of final goods can be converted into one unit of $K_{j,t+1}$.

The price for intermediate goods can be found by maximizing (2).

<u>Exercise 10</u>: Maximize (2) with respect to K_j (one type of capital) and label it (4).

Each producers faces a constant-price-elasticity demand curve.

- A 1% increase in price leads to $1/(1-\alpha)$ a percent fall in demand.

Each intermediate goods firm maximizes profits by producing K_j to maximize

$$\Pi_j = \frac{p_j K_j}{1+r} - K_j$$

where r is the market interest rate and reflects the fact that capital sold in period t must be produced in the previous period.

Using (4):

$$\Pi_j = \frac{\alpha N_y^{1-\alpha} K_j^{\alpha}}{1+r} - K_j \tag{5}$$

<u>Exercise 11</u>: Maximize intermediate good producers' profits by choosing K_j and label it (6).

$$\frac{\partial \Pi_j}{\partial K_j} = \frac{\alpha^2 N_y^{1-\alpha} K_j^{\alpha-1}}{1+r} - 1 = 0$$

$$K_j^* = \left(\frac{\alpha^2}{1+r}\right)^{1/(1-\alpha)} N_y \tag{6}$$

<u>Exercise 12</u>: Use the profit-maximized amount of capital to simplify the price, (4). Label it (7).

Given that cost of producing the capital good is $1+r$ (in terms of the final consumption goods), (8) implies that a price that is a constant markup over cost.

Exercise 13: Eliminate the price and capital variables in the intermediate good producers' profit function, equation (5), by substituting (6) and (7). Label it equation (8).

$$\Pi^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2}{1+r}\right)^{1/(1-\alpha)} N_y \tag{8}$$

We now turn our attention to the creation of technology. Since there is free entry and exit in the intermediate goods production, the value of a technology must be equal to the entire present discounted value of the profit stream an intermediate goods producer will earn after purchasing it:

$$p_A^* = \sum_{s=t}^{\infty} \frac{\Pi^*}{(1+r)^{s-t}} = \frac{(1+r)\Pi^*}{r} \tag{9}$$

3.2. Solving the model

We are now able to solve for the two important aspects of the model:

1. Percent of labor share devoted to R&D.
2. How final output is divided between investment and consumption.

Exercise 14: Assuming that N_A is constant over time, use (2) to solve for the growth rate of A. Label it (10).

In steady state, the number of capital good types grows at rate γ_A, whereas the quantity of each type of capital good remains constant at K^*.

Exercise 15: To obtain the share of labor between the R&D sector and the final goods sector, equate the marginal product of labor in the two sectors. Label it (11).

$$MPL_A = \frac{\partial(p_A^*\theta AN_A)}{\partial N_A} = p_A^*\theta A$$

$$MPL_Y = \frac{\partial Y}{\partial N_Y} = (1-\alpha)N_y^{-\alpha}AK^{*\alpha}$$

$$p_A^*A\theta = (1-\alpha)N_y^{-\alpha}AK^{*\alpha} \tag{11}$$

Exercise 16: Solve (11) for N_y by taking (11) and substituting in (6), (7), and (9). Label it (12).

Exercise 17: Using (12) and $N = N_y + N_A$ in (10), solve for the growth rate of A solely as a function of exogenous parameters. Label it (13).

3.3. Analyzing the model

A straightforward corollary of the Romer model is that as the country's population grows, its rate of per capita income growth will increase.

This seems not so satisfying as Nigeria has enormous population growth and are dirt poor while tiny Luxembourg as a per capita income more than ten times that of India. If we think of world population instead of a particular country, this seems satisfactory.

Kremer (1993) uses a Romer-style model to show that as world population increases, the rate of innovation increases significantly. However, the empirical evidence is sketchy at best.

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Lecture 7: The Overlapping Generations Model

1. Introduction

The Diamond overlapping generations model (OLG) is introduced. The suboptimality of the OLG model is discussed.

The key difference between the OLG model and the neoclassical model is that there is turnover in the population.

- New individuals are continually born
- Old individuals are continually dying.

The arrival of new individuals in the economy is not only a realistic feature, but it also introduces a range of new economic interactions.

- Decisions made by older generations that affect the prices and quantities of goods faced by younger generations.
- These economic interactions have no counterpart in the neoclassical growth model.

The First Welfare Theorem cannot be applied in OLG models. The pareto suboptimality of the competitive equilibrium is closely connected to potential inefficiencies in the OLG model. We will explore these inefficiencies.

The overlapping-generations model was first introduced by Paul Samuelson and later by Peter Diamond.

With death of individuals during each period, it turns out to be simpler to assume that time is discrete rather than continuous.

1.1. The basic environment of the Overlapping-generations model

Time is discrete and is indexed by $t \in \{0, 1, 2, \ldots\}$ and the economy remains closed.

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period of their lives. All individuals born at time *t* live for dates *t* and *t+1*.

Labor and capital markets are perfect.

1. Labor supply is equal to labor demand and thus, the wage rate is equal to the marginal productivity of labor.
2. Capital supply equals capital demand and thus, the interest rate is equal to the marginal productivity of capital.

Similar to the neoclassical model, constant labor supply is assumed. We abstract from population growth and exogenous technological change.

1.2. Individuals

The preferences of the representative individual are given by

$$U = u(c_t^y) + \beta u(c_{t+1}^o)$$

The budget constraints are

$$c_t^y + s_t = w_t$$
$$c_{t+1}^o = R_{t+1} s_t$$

where R_{t+1} denotes the rental rate of capital in period $t+1$ earned on savings from period t and w_t denotes the wage rate. Note that labor is supplied inelastic so $n_t = 1$, e.g., wage rate is equal to lifetime labor income ($I_t = w_t n_t = w_t$).

Combining the above budget constraints yields the individual's lifetime budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} = w_t.$$

- Similar to previous lifetime budget constraints analyzed except only two periods.

1.2.1. Individual *i*'s maximization problem

Given a price sequence $\{R_{t+1}, w_t\}_{t=0}^{\infty}$, the individual chooses a plan $\{c_t^y, c_{t+1}^o, s_t\}_{t=0}^{\infty}$ so as to maximize lifetime utility subject to its flow (or per period) budget constraints

$$\max U$$

subject to

$$c_t^y + s_t = w_t$$
$$c_{t+1}^o = R_{t+1} s_t$$

Exercise 1: Write out the Lagrangian. Label it (1).

Exercise 2: Maximize the individual's utility. Label the FOCs (2) and (3).

Exercise 3: Combine (2) and (3) to yield the Euler equation.

- Individuals equate their marginal rate of intertemporal substitution with the (common) return on capital.

Exercise 4: Assume a CRRA utility function. Solve for the savings function.

1.2. Firms

The firm's profit maximization problem is identical to that of the standard neoclassical growth model. However, for most applications, this is dragged along while household and individual nuances are modeled.

Let K_t and L_t denote the amount of capital and labor that firm *m* employs in period *t*. Then, the profits of that firm in period *t* are given by

$$\Pi_t = p_t F(K_t, L_t) - (r_t + \delta)K_t - w_t L_t$$

where p_t is the market price for the consumption good and δ is the capital depreciation rate.

The firm seeks to maximize profits. Setting output price as the numeraire (i.e., divide all prices by p_t so that $p_t = 1$), the FOCs for an interior solution require

$$\begin{aligned} r_t &= F_K(K_t, L_t) - \delta \\ w_t &= F_L(K_t, L_t) \end{aligned} \tag{5}$$

1.3. Market Clearing

The capital market clears if and only if

$$K_{t+1} = (1-\delta)K_t + i_t.$$

The labor market clears if and only if

$$L_t = n_t = 1$$

The goods market clears if and only if

$$y_t + (1-\delta)K_t = c_t^0 + c_t^y + s_t$$

Using the capital market and goods market clearing condition implies that:

$$K_{t+1} = N_t s_t$$

1.5. General Equilibrium

Definition 3.2. *Given an initial capital K_0, a competitive equilibrium is a sequence of private savings $\{s_t\}_{t=0}^{\infty}$, a sequence of capital $\{K_t\}_{t=0}^{\infty}$, sequences of factor prices $\{R_t, w_t\}_{t=0}^{\infty}$ such that*

(i) Given $\{R_t, w_t\}_{t=0}^{\infty}$, the individual solves their maximization problem.

(ii) Given $\{R_t, w_t\}_{t=0}^{\infty}$, the firm maximizes profits.
That is, factor prices are given by (5).

(iii) The markets clear in every period.

2. Suboptimality of OLG

Gale (1973) shows that the standard OLG model presented above is suboptimal. That is, the competitive equilibrium does not correspond to the Pareto optimal solution. The main reason is because markets are not complete (young want to save for retire, but young not born to enter contract). This section explores why the OLG model is suboptimal for a simplified model.

The assumptions on the model are as follows:

- No production.
- Individuals receive a helicopter drop endowment in each the period when young or when old. This argument is easily extended to drops in both when young and old.
  - Individuals receive their endowment in the first period: $w^y = 1$ and $w^o = 0$.
  - Individuals have too much consumption in the first period.
  - Not enough in the second period for one's preferences.
- No storage capabilities, i.e., full depreciation.
- No population growth.
- The budget constraints are as follows $c^y = w^y$ and $c^o = w^o$

<u>Exercise 6</u>: Identify a social planner's reallocation plan to make a Pareto improvement.

<u>Exercise 7</u>: Suppose that there is no government. Is the competitive equilibrium Pareto optimal?

In reality, there are multiple generations which the older generations can enter into some contracts. However, during the last period of life, there is no such contract. Many individuals do have this sense that they could have spent more and saved less when there life is running out and they have excess funds. Since uncertainty of death is not modeled, we cannot speak to of these effects.

Exercise 8: Suppose the model was extended and savings was incorporated. Would this fix the suboptimality of competitive equilibrium OLG models?

3. In-class exercises

3.1. Dynamic Inefficiency of OLG model

Gale (1973) demonstrated that the standard OLG model is suboptimal and that this is not due to the inability to save. In fact, even with a savings function, the competitive equilibrium still displays *dynamic inefficiency* – an over accumulation of savings. This was further explored by Peter Diamond and is his extended work is called the *Diamond model*.

Suppose that:

- Time is discrete and runs to infinity.
- Individuals born at time t live for periods t and $t+1$.
- Population grows at rate n.
- Individuals can only work in the first period and supply one unit of labor inelastically, earning w_t.
- Utility of the individual of generation t is

$$U(c_t^y, c_{t+1}^o) = \ln(c_t^y) + \beta \ln(c_{t+1}^o)$$

- Cobb-Douglas production function.
- Full depreciation.

3.1.2. The decentralized competitive equilibrium

Exercise 9: Setup the individual's maximization problem and solve for the Euler equation. Label it (1).

❖ Why doesn't the interest rate affect savings?

Exercise 10: Setup the firm's profit maximization problem and solve the first-order conditions. Label them (2) and (3).

Define market clearing in the goods, labor, and capital market and a competitive equilibrium (see notes above).

Exercise 11: Combine the Euler equation with the capital market clearing condition and firm's first order condition to yield the law of motion for physical capital solely as a function of exogenous parameters and state variables. Label it (4).

Exercise 12: Solve for the competitive equilibrium steady state capital stock. Label it (5).

Equilibrium dynamics are identical to those of the basic Solow model and monotonically converge to k^*.

Proposition 1: In the canonical overlapping generations model with log preferences and Cobb-Douglas technology, there exists a unique steady state, with capital-labor ratio k^* given by (5). Starting with any k_0, equilibrium dynamics are such that k_t converges to k^*.

3.1.3. The Social Planner's Problem

Compare the overlapping-generations equilibrium to the choice of a social planner wishing to maximize a weighted average of all generations' utilities.

Suppose that the social planner maximizes

$$\beta_s^0 \beta u(\mathrm{c}_1^o) + \sum_{t=1}^{\infty} \beta_S^t U(c_t^y, c_{t+1}^o)$$

where $\beta_s^t < 1$ is the discount factor of the social planner, which reflects how the planner values the utilities of different generations. One condition imposed on the discount factor is $\sum_{t=0}^{\infty} \beta_S^t < \infty$.

Exercise 13: Write out the planner's feasibility constraint and setup the social planner's maximization problem. Label it (6).

Exercise 14: Solve for the FOCs, the static optimality condition, and the Euler equation. Label the Euler equation (7).

- This is identical to (4) – just in a different form.
- Allocate individuals' consumption exactly as the individual himself would do.
  - No market failures in the over-time allocation of consumption at given prices.
- Allocations across generations may differ from the competitive equilibrium: planner is giving different weights to different generations.

3.1.4. Steady State

In steady state, $c_t^y = c_{t+1}^y = c^y$ and $c_t^o = c_{t+1}^o = c^o$. The Euler Equation must be modified using the static optimality condition.

Imposing steady state yields:

$$(1+n) = \beta_S(\alpha A k^{*\alpha-1} + 1 - \delta)$$

This is the *Modified Golden Rule* (MGR). The golden rule sets β_S = 1.

Exercise 15: Solve for steady state *Golden Rule* physical capital stock.

Exercise 16: Compare the physical capital steady states to find a condition when the decentralized equilibrium has a higher capital stock than optimum.

What is the general condition that causes dynamic inefficiency?

Dynamic inefficiency arises due to old individuals' sole ability to consume during old age through the capital stock. The social planner, on the other hand, allocates consumption between young and old using current output. Thus, if the returns on savings and returns to output differ substantially, than consumption will differ as well.

- Abel, Mankiw, Summers, and Zeckhauser (1989) develop a criterion for addressing dynamic efficiency and apply this model to the United States and other OECD countries, suggesting that these countries are indeed dynamically efficient.

3.1.5. Understanding dynamic inefficiency

In the baseline overlapping-generations economy, the competitive equilibrium is not necessarily Pareto optimal. Specifically, the economy is dynamically inefficient whenever $r^* < n$. It is possible to reduce the capital stock starting from the competitive steady state and increase the consumption level of all generations. This is demonstrated below.

Now, if $k^* > k_{gold}$, then $\frac{\partial c^*}{\partial k^*} < 0$. This implies that reducing savings can increase total consumption for everybody. Dynamic inefficiency arises because of the heterogeneity inherent in the overlapping generations model which removes the transversality condition.

- When $\beta_S < 1$, $k_{MGR} < k_{GR}$ and the MGR capital stock is dynamically efficient.

Exercise 17: Show that there exists a Pareto improving redistribution.

- The first term reflects the direct increase in consumption due to the decrease in savings.
- The second term is the additional consumption in future periods from not oversaving.

The increase in consumption for each generation can be allocated equally during the two periods of their lives, thus necessarily increasing the utility of all generations.

- Pareto inefficiency of the competitive equilibrium is intimately linked with dynamic inefficiency.

Intuition

1. Dynamic inefficiency arises from over-accumulation.
2. Results from current young generation need to save for old age.
3. The more they save, the lower is the rate of return and may encourage to save even more t compensate for lower future income.
4. Effect on future rate of return to capital is a pecuniary externality on next generation.
5. If alternative ways of providing consumption to individuals in old age were introduced, over-accumulation could be eliminated.

If somehow this excess consumption could be moved to the second period of life, during old age, then the individual would be better off.

There are a few ways of solving dynamic inefficiency

- Introduce fiat currency.
- Tax young's endowment and transfer to old.

3.2. OLG model with fiat money

Fiat money is an intrinsically worthless commodity (piece of paper) that derives its value from the fact that it allows individuals to facilitate transactions.

- Money can act as a storage of wealth from one period to the next.
- In private transactions individuals are free to choose the currency.
- Governments can impose the obligation that individuals accept money in court settlements (legal tender).
- The amount of fiat money is determined by an external agent (central bank, government, etc.). Only possible if not (easily) counterfeited.
- Fiat money tends to be neutral if prices are perfectly flexible.
    - Money is said to be *neutral* if a one-time increase in the level of the money supply does not affect any of the relevant equilibrium quantities of consumption, capital, and output.

<u>Exercise 18</u>: Why do individuals typically hold fiat money?

In the OLG model, there is a natural scope for trading frictions.

- With perishable goods the young of generation t would like to trade with the unborn young of $t + 1$.
- The introduction of pure fiat money may alleviate this trading friction and make all generations better off.

OLG without fiat money: As we have already seen (Gale, 1973), in the OLG model with perishable goods, the decentralized equilibrium *without* money leads to dynamic inefficiency.

- The young consume their entire endowment and they starve when old.

OLG with fiat money: The outcome may be efficient in the model with money.

- The decentralized consumption levels coincide with the golden rule consumption levels.
- Money is neutral.
- The monetary equilibrium is extremely fragile. All generations must be convinced that all future generations will accept the money.

For simplicity, assume that there is **no population growth**. Introducing money brings only a few additional complexities to the baseline OLG model.

In period t the government introduces an amount M of pure fiat money by transferring an amount M/N of money to each old individual of generation $t - 1$.

1. Money is perfectly divisible and cannot be counterfeited.
2. Let P_t denote the nominal price of a single unit of the good in period t.
3. One unit of money buys $v_t = 1/P_t$ goods in t and so money loses value as the price level increases.
4. The return on money is $v_{t+1}/v_t = P_t / P_{t+1} = 1 / (1 + \pi_t)$ or approximately the real interest rate on money is $r_m = -\pi_t$.
5. The return on money in t depends on the expectations of individuals about the entire sequence $\{v_{t+j}\}_{j=1}^{\infty}$.
6. Money is worthless today if agents expect money to lose value in the (distant) future.

Suppose that individuals expect that in some period $v_{t+j} = 0$ with probability 1. In that case individuals will reject money as payment in period $t + j - 1$ because money will not have any value next period. But anticipating this, young individuals in $t + j - 2$ will reject money as it will be worthless in $t + j - 1$. This unravels all the way back to the current period.

- Assume that agents have perfect knowledge about the entire sequence of future prices.

The individual's problem.

Consider the problem of an individual household of generation t who decides to bring a quantity m_{t+1} of money to the next period.

$$\begin{aligned}
&\max \ln c_t^y + \beta \ln \mathrm{c}_{t+1}^o \\
&s.t. \\
&P_t c_t^y + m_{t+1} = P_t w_t \\
&m_{t+1} = P_{t+1} c_{t+1}^o
\end{aligned}$$

Exercise 19: Construct the Lagrangian using lifetime budget constraint.

Exercise 20: Solve for the Euler equation.

Exercise 21: Solve for the consumption sequence as a function of exogenous parameters and the endowment.

The dynamic optimality condition constrains prices (as discussed above).

$$w_t = c_t^y + c_{t+1}^o \rightarrow w_t = \frac{w_t}{1+\beta} + \frac{\beta}{1+\beta}\frac{P_t}{P_{t+1}} w_t \rightarrow 1 = \frac{1}{1+\beta} + \frac{\beta}{1+\beta}\frac{P_t}{P_{t+1}}$$

$$1+\beta = 1+\beta\frac{P_t}{P_{t+1}} \rightarrow \beta = \beta\frac{P_t}{P_{t+1}}$$

$$\frac{P_t}{P_{t+1}} = 1$$

That is, the price level must be constant over time for the decentralized competitive equilibrium to be Pareto optimal.

To solve for the equilibrium price level, the market clearing condition in the period introducing money is evaluated. In the period introducing money, the old can buy $c_t^o = m_t / P_t$ units of endowment from the young. Thus, static optimality requires:

$$c_t^y + c_t^o = w_t$$

$$\underbrace{\frac{w_t}{1+\beta}}_{c_t^y} + \underbrace{\frac{m_t}{P_t}}_{c_t^o} = w_1 \rightarrow \frac{m_t}{P_t} = \frac{\beta w_t}{1+\beta} \rightarrow P_t = \frac{1+\beta}{\beta}\cdot\frac{m_t}{w_t}$$

Notice that money is neutral: Doubling *M* increases the price level, while all else remains the same.

Discussion: Due to price stability, the maximization problem of the individual reduces to

$$\max \ln c_t^y + \beta \ln c_{t+1}^o$$
$$s.t.$$
$$c_t^y + c_{t+1}^o = w_t$$

- The individuals maximize their expected lifetime utility subject to the feasibility constraint for stationary allocations.
- This problem coincides with the maximization problem of a social planner whose objective is to maximize utility of all future generations and who is constrained to treat all generations alike (*The Golden Rule*).
- A monetary equilibrium with perfect foresight and a constant money supply decentralizes the Golden Rule.

Lecture 8: Applications Using the Overlapping Generations Model

1. Introduction

The overlapping generations model is the workhorse of public policy theory. This model has been extended to study human capital accumulation (with parental inputs), transfers, and health decisions, among many other applications. This lecture will introduce:

1. Government entitlement program, social security.
2. Human capital development.

2. Introduction to government entitlement program, social security.

Prior to studying social security, we will first study transfers. In an effort to reduce the burden on the old-aged individuals, governments propose transfers from younger workers. Governments also initiate transfers to the poor from higher income individuals. We will explore the effect of transfers to the old on the steady state capital stock.

Recall that when there is no population growth, the competitive equilibrium steady state physical capital in OLG models may exceed the optimal level chosen by a social planner. In the last lecture, we saw that introducing fiat money was a Pareto improvement – fiat money moved the equilibrium to the Golden Rule allocation.

We now consider whether government transfers can be a Pareto improvement. We also extend the model to permit population growth to occur. Under a model with fiat currency, with population growth, the value of each dollar increases over time since the quantity of goods increases with a fixed money supply. Thus, the dynamic are much more complicated.

2.1. The model

Consider an OLG economy where all individuals live two periods, supply labor inelastically in period 1 and are retired in period 2. Their preferences are given by

$$\ln c_t^y + \beta \ln c_{t+1}^o$$

The aggregate production function is given by

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha}$$

There is population growth between generations at rate *n*.

2.2.3. Social security.

The previous section demonstrated that the decentralized competitive equilibrium physical capital stock may be too large relative to the optimal physical capital stock. We will now study whether introducing social security can be Pareto improving.

The government taxes labor income at the constant and uniform rate τ. All tax revenue, TR_t, is rebated to old generation as lump sum transfers. This is referred to as pay-as-you-go (PAYG) social security and is considered an intergenerational transfer. That is, the government must satisfy its budget constraint:

$$\underbrace{N_t \tau w_t}_{TR_t} = \underbrace{N_{t-1} T_t}_{G_t} \Rightarrow$$

$$\tau w_t = \frac{N_{t-1} T_t^{SS}}{N_t} = (1+n)^{-1} T_t$$

- In a pay-as-you-go pension system, the pensions of the old are financed with the social security contributions of the young.
- The system works well as long as the population is growing and a large proportion of the young are employed.
- The system breaks down with increased life expectancy, relatively high ratio between pension and last salary.

The fully funded social security is when the government to sets aside funds from workers to provide to them during retirement. This can be considered an inter-period transfer or forced savings. That is,

$$\underbrace{R_{t+1} \tau w_t}_{TR_t} = \underbrace{T_{t+1}}_{G_t}$$

It should be apparent that in the canonical OLG study so far, fully funded social security has no effect on the capital stock while the PAYG reduces the capital stock if labor is supplied inelastically.

2.2.4. Solving the model

The firm's profit maximization problem remains the same while the household's utility maximization problem needs modification. The government is not allowed to run a deficit.

Exercise 1: Define the household's problem.

Exercise 2: Solve the first-order conditions and the Euler equation.

- The rate of savings is smaller than under the decentralized economy without social security.
- This provides the possibility of reducing the per capita steady state physical capital stock to the optimum. However, we must also check that this is welfare improving!

Exercise 3: Solve for the per capita physical capital stock.

- The per capita physical capital stock is always smaller under social security.

Exercise 4: Solve for steady state per capita physical capital.

- If the tax rate is 0, then this corresponds with the steady state per capita physical capital stock in the decentralized economy.

Exercise 5: Construct the lifetime budget constraint for an individual using the government's budget constraint.

Exercise 6: What happens to consumption when young and old under the decentralized economy with social security?

Exercise 7: How is it guaranteed that consumption will increase?

2.3. Extensions

There are several extensions which are important when studying social security. There are some which address the distributional effects of social security while other study the interaction between social security and other government funded programs such as public education and Medicare.

While the models are too technical for this first year graduate course, I provide a detailed summary of these various extensions.

a) Education and Social Security

This work was pioneered by Kaganovich and Zilcha (1999). The model studies the allocation of tax revenue between public education and social security. In the baseline model, the representative individual's utility function is:

$$U_t = \alpha_1 \ln c_t^y + \alpha_2 \ln c_{t+1}^o + \alpha_3 \ln h_{t+1}$$

where h_{t+1} is the human capital of the individual's child and is a function of inherited human capital from their parent, public education and private education expenditures.

They show that for some parameter restrictions, if the government raises the SHARE of tax revenue to public education, then everyone is made better off – even the old. Why? As the amount towards public education increases, this makes the young have higher human capital and thus earn more. Even though the SHARE for social security is smaller, the total tax revenue (the pie) is bigger, implying that everyone can be made better-off.

b) Glomm and Kaganovich (2005, 2008) – Extended KZ (1999) to understand the distributional effects of social security. Suppose that individuals are heterogenous with respect to their human capital and income. If one invests in public education, then this will decrease inequality. They show that if the government increases social security benefits, then this may accelerate growth and reduce income inequality.

The dynamics of this model are as follows. If poor people know that they will receive income in retirement, then they can save less and spend more on the human capital of their children. Increasing their children's human capital through private investment is complementary to public education – thus increasing their children's human capital. Higher human capital individuals view social security as a drain on them as they have to pay more into social security then they get. Thus, increasing social security SHARE requires them to save more and reduce the amount spent on human capital investments for their children. If the decrease is not as large as the increase of the lesser-able individuals, then this will raise growth but also reduce inequality (and vice versa)

c) Christophe Hachon (2008) shows that when life expectancy are differential and a function of one's income, social security programs which are progressive can actually be regressive, i.e., poor provide for the rich. One way to understand this is by augmenting the transfer payment to reflect how long one lives. The pension individual ω of generation *t-1* receives during retirement is given by

$$P_t(\omega) = v_t w_t H_t T(h_{t-1}(\omega))$$

where v_t is the benefit replacement rate in period *t* which is set to balance the government's pension budget.

d) Another extension is on fertility and the effect of PAYG social security on fertility. In Sinn (1998), he shows that PAYG acts as insurance for parents to have children and force children with low altruism towards parents to pay for them.

e) Phillpson and Becker asked how social security and Medicare effect the optimal quantity and quality of life of old people. What they find is that when there is Medicare and social security, individuals opt for higher quantity over quality of life. They live longer since social security is annuity based and Medicare is the vehicle to achieve longer life.

f) Bethencourt and Galasso (2000) show that there is political complementarity between health care and social security. Public health care increase the political constituency in favor of social security. The old make a coalition with the young poor in favor of a large welfare state with social security and Medicare.

3. Human capital in OLG models

3.1. Introduction

Human capital in a OLG model is quite natural considering the inter-generational decision making and that most children attend school when young. To model these, we extend a two period model with 'young' and 'old' to a three period model with 'youth', 'middle-aged' and 'old age'. Most often, the 'youth' period is a time of economic inactivity where parents make all decision in their behalf.

Human capital in an OLG model is the basic framework for many public economics questions that address the intertemporal transfers to one's offspring when young. Examples include the decision of where to live (Tiebout sorting), the decision for children to attend public vs. private school, and how much to invest in children's health.

3.2. The model

The human capital in an OLG model typically uses standard preferences and production function with the extension of human capital of children being in the utility function:

$$\ln c_{i,t}^{m} + \beta \ln c_{i,t+1}^{o} + \rho \ln h_{i,t+1}$$

where $h_{i,t+1}$ is the human capital of *i*'s child. This is a form of altruism with the parameter ρ dictating how much an individual cares about their children's human capital compared to their own consumption when middle-aged and old.

The human capital production function can be specified many different ways depending on the question the research is attempting to answer. Most human capital production functions permit:

1. Random draw of ability.
2. Human capital of the parent
    a. Proxy for genetics, home environment, etc.
3. Education quality of the child's school (public or private).
4. Private education input.
    a. Piano lessons, private tutors, and going to the zoo.

The basic human capital production function permits complementary between inputs:

$$h_{i,t+1} = BA_{i.t} h_{i,t}^{\gamma} E_{t}^{\eta} e_{i,t}^{\nu}$$

where E_t is per capita public education provided in period *t* and e_t is private education.

The income of individual *i* is a function of their human capital. That is:

$$I_{i,t} = w_t h_{i,t}$$

where w_t is the wage rate per unit of human capital and $I_{i,t}$ is the individuals' lifetime income.

There are several alternative education production functions that will change the income dynamics between generations. For example, in Gilpin and Kaganovich (2012), the education production function is two-stage (both occurring in the first period of life). The first stage is:

$$l_{i,t+1} = Ca_{i,t}E_t$$

where E_t is elementary and secondary public education and $a_{i,t}$ is a random innate ability draw.

The second stage (college) is:

$$h_{i,t+1} = bl_{i,t} + B[l_{i,t} - h^*]$$

where h^* in the college human capital requirement. Notice that this second stage indicates non-linear gains to college. That is, the gain depends on the extent to which the individual's pre-college human capital attainment $l_{i,t}$ exceeds the threshold h^*.

Exercise 8: Using the income formula above, graph the income dynamics of individuals in generation t if ability is a random draw on [0,1].

Exercise 9: On the same graph, show what happens if E_t increases for the same set of individuals.

3.2.1 Public Education

The human capital production function includes a publically provisioned education input. The model may also include an education production function which indicates how education is constructed. The most simple formulation is that the labor income tax is transformed one-for-one into education quality. That is, the government's budget constraint and public education production function are the same:

$$N_t E_t = \tau w_t \int_{i \in G_{i,t}} h_{i,t} = \tau w_t H_t$$

where H_t is the aggregate stock of human capital of generation t. The right-hand side is the total revenue that the government receives to finance education from adult laborers.

Exercise 10: Setup the individual's maximization problem.

Exercise 11: Solve the FOCs.

Exercise 12: Solve for the rate of savings and private investment in children's human capital. Label them (1) and (2).

Exercise 13: Normalize the population to 1 ($N_t = 1$) and use private education investments to solve for the human capital law of motion of an individual's child.

Exercise 14: Interpret the law of motion for human capital.

4.0 Applications

There are many applications of this model to answer some very important questions.

Glomm and Ravikumar (1992): This paper shows that privately funded education results in higher growth than publicly funded education with heterogeneous individuals. This was a critical piece of the literature as it demonstrated that public education is more of an ‘equalizer’ than a ‘growth’ promoter. There have been a few papers which demonstrate that this result can be overturned, but this paper is quite profound.

Eckstein and Zilcha (2002): This paper shows that education has a positive externality that parents do not consider when they purchase ‘private education’ for their children. Government intervention providing compulsory schooling increases economic growth while the intragenerational income distribution becomes more equal.

5.0. Galor and Moav, QJE 2000 - Ability-biased Technological Transition, Wage inequality, and economic growth

5.1. Introduction

In the last two decades the United States, and other advanced countries, has experienced rapid technological progress along with fundamental changes in the pattern of wage inequality.

1. The wage differential between skilled and unskilled labor has risen significantly despite the increase in the supply of skilled labor.
2. The wage dispersion within the groups of skilled and unskilled labor has widened, and real wages of unskilled labor have declined.

This is evidence by Autor, Katz, and Krueger [1998] - although the demand for college graduates in the United States has increased from the 1970s faster than previously, the college wage premium declined because of the large increase in supply of college graduates. In contrast, wage inequality within groups had increased over this period [Katz and Murphy 1992].

The phenomenon of rapid technological change along with a rise in the skill premium, despite an increase in the supply of skilled labor, has recurred in several time intervals in the last century. Over the period as a whole, however, wage inequality has evolved in a cyclical fashion

This paper develops a growth model, characterized by ability-biased technological transition, which accounts for the fundamental elements that have characterized the evolution of technology, education, and wage inequality in the United States and other advanced countries over the past several decades.

The study argues that an increase in the rate of technological change:

1) Raises the return to ability.
2) Raises wage inequality between and within groups of skilled and unskilled workers.
3) Raises the average wage of skilled workers.
4) Causes a temporary decline in the average wage of unskilled workers.
5) Raises education attainment.
6) Causes a potential coexistence of rising rates of technological progress and a transitory productivity slowdown that may have been observed in earlier parts of recent decades.

5.2. The model

Consider a small, open, overlapping-generations economy that operates in a perfectly competitive world where international capital movements are unrestricted and economic activity extends over infinite discrete time.

5.2.1. The firm

In every period the economy produces a single homogeneous good that can be used for either consumption or investment.

The good is produced by physical capital and a composite labor input (measured in efficiency units) that consists of skilled labor and unskilled labor.

The supply of all factors of production is endogenously determined.

The number of efficiency units of skilled and unskilled labor in every period is determined by occupational choices of the fixed number of individuals within a generation, as well as by the state of technology.

The stock of physical capital in every period is given by the sum of the economy's aggregate saving net of international lending.

Production occurs according to a neoclassical, constant-returns-to-scale, production technology that is subject to technological progress. The output produced at time *t*, Y_t, is:

$$Y_t = F(K_t, A_t H_t) \tag{1}$$

where K_t and H_t are the quantities of physical capital and efficiency units of the composite labor input employed in production at time *t*, and A_t is the technological level at time *t*.

The demand for factors of production is:

$$\begin{aligned} \bar{r} &= r_t = f'(k_t) \\ w_t &= A_t w(k_t) \end{aligned} \tag{2}$$

5.2.2. The internal structure of the composite labor input

The composite labor input H_t is a weighted sum of the number of efficiency units of skilled labor, h_t, and unskilled labor l_t employed in production at time *t*:

$$H_t = Bh_t + l_t(1 - bg_t) \tag{3}$$

where $B > 1$ and $0 < bg_t < 1$ is the reduction in the aggregate weight given to efficiency units of unskilled labor due to the rate of technological progress from period *t-1* to period *t*, $g_t = (A_t - A_{t-1}) / A_{t-1}$.

Exercise 1: What fundamental property is inherent in the above formula of labor?

5.2.3. Factor prices

Suppose that the world rental-rate is stationary at a level $\overline{r}$.

Exercise 2: What does this imply about the ratio of capital to efficiency units of labor?

Exercise 3: Find the ratio of capital to efficiency units of labor?

Exercise 4: Find the wage rate per efficiency unit of the composite labor input.

Exercise 5: Find the unskilled and skilled wage rate per unit of labor input. Label them (4).

5.2.4. Individuals

In each period a generation is born. It consists of a continuum of individuals of measure 1. Individuals, within as well as across generations, are identical in their non-altruistic preferences. They may differ, however, in their cognitive ability and thus in their education and their level of human capital. Ability is distributed uniformly over the unit interval.

Individuals live for two periods and have preferences defined over consumption in both periods.

First period: Consumption and saving

Each individual faces an occupational choice to become skilled or remain unskilled.

Skilled: Acquire education and work.
Education requires a fraction $0 < \eta < 1$ of time to form human capital.
Supply the remaining $1 - \eta$ to skilled labor earning the competitive skilled wage.
Unskilled: Join the labor force directly and earn the competitive unskilled wage.

Second period: Individuals retire consuming their entire savings.

5.2.4.1. Level of human capital and income

The level of human capital of skilled and unskilled workers is determined by their ability as well as by the technological environment.

- Technological progress changes the nature of occupations and reduces the adaptability of existing human capital for the new technological environment.

The efficiency units of labor of a member *i* of generation *t* depends:

- Positively on individual *i*'s ability, at a_t^i
- Negatively on the rate of technological change, g_t
- Positively on the complementarity of the rate of technological change and ability

The number of efficiency units of labor that member *i* of generation *t* supplies as an unskilled worker is

$$l_t^i = 1 - \underbrace{(1 - a_t^i) g_t}_{Learning} \tag{5}$$

Learning: Because of technological progress, the unskilled must devote some time to learn the new technology on the job. Thus, the efficiency units are less.

- This is a product of the composition labor input (see equation (3)).

The number of efficiency units of labor that member *i* of generation *t* supplies as a skilled worker is

$$h_t^i = (1-\eta)[a_t^i - \underbrace{(1-a_t^i)g_t}_{Learning}] \tag{6}$$

- Captures the direct positive effect of ability.
- Captures the cost of schooling.
- Notice that ability increases efficiency units of skilled but not unskilled.

Exercise 6: How are individuals rewarded for become skilled?

Exercise 7: What is a sufficient condition to assure some people become skilled?

Exercise 8: Provide the income of an unskilled and skilled individual from working assuming that the inequality in Exercise 7 holds with equality. Label them (7).

Exercise 9: Are incomes increasing in ability?

Exercise 10: Draw the income dynamics across the ability distribution.

5.2.5. Solving the model

Since individuals work only in the first period of their lives, maximization of first-period income is a necessary condition for maximization of utility.

Exercise 11: Find the ability threshold for individuals to go to college. Label it (8).

Exercise 12: Using the ability threshold for college, find the average income of skilled and unskilled.

Exercise 13: Define between skilled and unskilled income inequality as the ratio of average skilled income to average unskilled income.

5.2.6. Exogenous technological progress

We can now study how exogenous technological progress affects incomes, the number of skilled, and the income inequality within and between the groups of skilled and unskilled.

Technological change affects the average income of skilled and unskilled workers via three channels.

1. For a given composition of the labor force, the rise in the level of technology affects income of each group positively.
2. For a given composition of the labor force, the rate of technological change affects the income of each group negatively via the erosion effect.
3. The rate of technological progress alters the composition of ability within each of the groups.

Exercise 14: What happens to the ability threshold to attend college with respect to changes in the technology?

Exercise 15: Suppose the level of technology increases, but not the rate. What happens to the ability threshold and income? Draw a graph with the new income dynamics across the ability distribution.

Exercise 16: Suppose the rate of technological progress increases. What happens to the ability threshold and incomes? Draw a graph with the new income dynamics across the ability distribution.

5.2.7. Endogenous technological progress

Up to this point, technological change occurred exogenously. The rate of growth of technological progress is more likely a function of the number or concentration of skilled workers.

Suppose that g_{t+1} is a positive linear function of the number of skilled workers in period *t*:

$$g_{t+1} = \gamma(1 - a_t^*) = \frac{\gamma b g_t}{1 + b g_t^2} \tag{8}$$

Exercise 17: Find parameter restrictions required to obtain a steady state rate of technological growth.

What is interesting about endogenizing the path of technological progress is that we can see the dynamics as the rate of technological progress goes towards steady state.

If $g_0 < g^*$, then along the transition path:

1) Wage inequality within and between skilled and unskilled workers increases.
2) The average wage of skilled workers increases despite the increase in their relative supply.
3) The average wage of unskilled workers may decline, despite the decline in their relative supply

5.3.0. Conclusion

This paper demonstrated that ability-biased technological transition has caused increased inequality over the last half-century. That is, even though more people are becoming educated, those at the top of the human capital distribution benefit more than those at the bottom. Thus, we can have higher education population and increased inequality. This goes against the ‘great equalizer’ theory of Glomm and Ravikumar (1992).

6.0. Gilpin & Kaganovich, J. Pub E. - The Quantity and Quality of Teachers: Dynamics of the Trade-off

6.1. Introduction

A reduction of class sizes in K-12 schooling has been one of the main education policy priorities in the United States over the last several decades.

- The pupil-teacher ratio has decline through the contemporaneous ups and downs of the enrollment dynamics.
- The aptitude of teachers relative to other educated workers has also declined. Hoxby and Leigh (2004) estimate that in 1963 41% of all teachers were of the "middle" aptitude relative to their educated peers, with 17% above and 42% below the average; by comparison, in 2000, 28% of all teachers were of the "middle" aptitude with 5% above and 67% below average.
- Research, however, has shown that students' test scores have not risen despite increased individualized instruction.

This has compelled policy makers and researchers to question the role of the quantity of teachers vs. their quality as factors in student performance, particularly in light of a possible trade-off between the two.

Year	Pupil / teacher ratio[1]	Expenditures to GDP[2,b]	Relative Teacher Salary[3,b,d]
1955	26.9	3.3[c]	
1960	25.8	3.6	43.0
1965	24.7	3.9	
1970	22.3	4.6	44.0
1975	20.4	4.6	
1980	18.7	4.0	41.0
1985	17.9	3.8	
1990	17.2	4.3	35.0
1995	17.3	4.3	
2000	16.0	4.5	36.5
2005	15.7	4.6	

Source:
[1] Digest of Education Statistics 2007, Table 61.
[2] Digest of Education Statistics 2007, Table 25.
[3] Hanushek & Rivkin (2003).
Notes:
[b] In Percent.
[e] Deflated by CPI and in 2006 dollars.

This paper explores the following factors that may affect this trade-off:

1. The opportunity cost of teachers in terms of potential earnings in the alternative occupations in the production economy, and its evolution in the process of economic growth;
2. Collective bargaining driven by teacher unions as an institutional factor leading to wage compression among teachers;
3. The rise of skill premium resulting from technological change, which affects the opportunity cost of teachers.

6.2. The Model

An economy is populated by overlapping generations of individuals whose life-span consists of two periods: childhood and adulthood. Population size is constant and normalized to 1 for each generation.

First period

- Children make no decisions of their own and receive basic (or first stage) education which is provided publicly.

Second period: Individuals derive income from work. They spend it on consumption.

Adults are endowed with a unit of time and face an occupational choice:

1) Devote a fixed fraction *n* of it to acquiring advanced education and then the balance of time not spent on education is inelastically devoted to work
2) The individuals without college education will work for the full unit of time in the "unskilled" production workforce.

Post-tax income is $(1-\tau)I_t(\omega)$ where $I_t(\omega)$ is the individual's wage income derived from human capital and τ is the uniform rate of labor income tax collected by the government whose only outlay is funding basic public education.

6.2.1. Production

The production sector of the economy consists of perfectly competitive firms producing a homogeneous consumption good using unskilled and skilled human capital.

The aggregate production function is given by

$$Y_t = w\left(L_t + \theta H_t^y\right), \tag{1}$$

- L_t is the aggregate supply of unskilled human capital
- H_t^y is the aggregate supply of skilled human capital employed in the production sector in period t.
- $w > 0$ is a fixed productivity parameter per unit of unskilled labor.
- θ characterizes the net productivity augmentation of skilled human capital.

6.2.3. Income

Individuals working in the production of goods receive the wage at competitive rates w and θw, respectively, per unit of their unskilled or skilled human capital, whichever applies.

Income of individual ω who receives only basic education and attains the level of unskilled human capital $l_t(\omega)$ will be

$$I_t(\omega) = wl_t(\omega) \tag{2}$$

The individual ω who obtains college education, attains the level of skilled human capital $h_t(\omega)$ and is employed in the production sector, will receive income

$$I_t(\omega) = \theta wh_t(\omega) \tag{3}$$

Assume that income maximization is the sole motive to become educated.

6.2.3. Human Capital Formation

The first (basic) stage of education is produced in period $t-1$ by combining children's random innate ability (uniformly distributed on [0,1]) with public education according to

$$l_t(\omega) = Ca(\omega)E_{t-1} \tag{4}$$

- where C is a positive constant
- E_{t-1} is a uniform quality of public schooling received by each child in period $t-1$.
- $a(\omega)$ is the child's innate ability.

<u>Exercise 1</u>: What fundamental assumptions are embedded into (4).

The gains from college education depend on prior preparation:

$$h_t(\omega) = bl_t(\omega) + B[l_t(\omega) - h^*] \qquad (5)$$

- College education has a pre-requisite human capital threshold h^*.

<u>Exercise 2</u>: Interpret h^*.

<u>Exercise 3</u>: What condition must be imposed to assure that those ‘unqualified’ for college do not gain from going?

<u>Exercise 4</u>: What is wrong with equation (5).

Exercise 5: Provide the unskilled and skilled incomes using (4) and (5) for an individual and find an expression (or rule) for an individual to follow to choose to become educated.

6.2.4. Quality of Basic Education

The *per student* basic education quality, E_t, is a function of the quality and quantity of teachers chosen by a public education agency.

- Only college educated individuals are eligible to be employed as teachers.
- Σ_t is the set of individuals ω in generation t employed as teachers.
- z_t is the total (and fraction) of generation *t* that are teachers.
- The *teacher-student ratio* for generation $t+1$ students.

The aggregate teacher quality as the aggregate human capital of teachers

$$q_t = \int_{\omega \in \Sigma_t} h_t(\omega) d\mu_t(\omega) \tag{7}$$

The per student quality of basic education is

$$E_t = z_t^{\gamma} q_t^{\nu} \tag{8}$$

- $\gamma > 0$ implies a distinct effect of the class size.

6.2.5. Administration of Basic Education

The goal of the government is to maximize public education quality, E_t, subject to the budget constraint given by the revenue from a uniform labor income tax at a flat rate τ .

- The agency must decide on the number of teachers to be hired z_t .
- Union sets teacher salaries which determines the quality of the teachers hired (the segment Σ_t of the distribution of college-educated individuals).

6.3. The Teacher Collective Bargaining Regime: Recursive Dynamic Equilibrium

Collective bargaining equalizes teachers' salaries, I_t^{CB} .

- Given the alternative occupation available to the college educated individuals in the production sector, the teacher salary figure I_t^{CB} will uniquely determine the highest level of human capital attainment $\bar{h}_t^{CB}$ among the individuals who will choose to become teachers.

According to (3) it should satisfy the equation

$$\theta w \bar{h}_t^{CB} = I_t^{CB} \tag{16}$$

- All the college graduates with human capital level $h_t(\omega)$ at or below $\bar{h}_t^{CB}$ will be obviously motivated to accept the employment as a teacher rather than to work in the production sector.

- The education agency's goal to maximize the overall education quality may dictate a minimum qualification requirement $\underline{h}_t^{CB}$ for the human capital of teachers, such that the optimal set Σ_t^{CB} of teachers to be hired will consist of all individuals whose level of human capital $h_t(\omega)$ attained in college falls into the interval $[\underline{h}_t^{CB}, \bar{h}_t^{CB}]$.

- The number of teachers to be hired can be expressed as the measure of the above interval:

$$z_t^{CB} = \mu\left(\omega \mid \underline{h}_t^{CB} \le h_t(\omega) \le \bar{h}_t^{CB}\right) \tag{17}$$

Using (4) and (5), the cut-off innate ability levels $\underline{a}_t^{CB}$ and $\overline{a}_t^{CB}$ characterize the teachers who possess human capital $\underline{h}_t^{CB}$ and $\overline{h}_t^{CB}$:

$$\underline{a}_t^{CB} = \frac{\underline{h}_t^{CB} + Bh^*}{(b+B)CE_{t-1}} \quad \text{and} \quad \overline{a}_t^{CB} = \frac{\overline{h}_t^{CB} + Bh^*}{(b+B)CE_{t-1}} \tag{18}$$

The number of teachers is $z_t^{CB} = \overline{a}_t^{CB} - \underline{a}_t^{CB}$ (19)

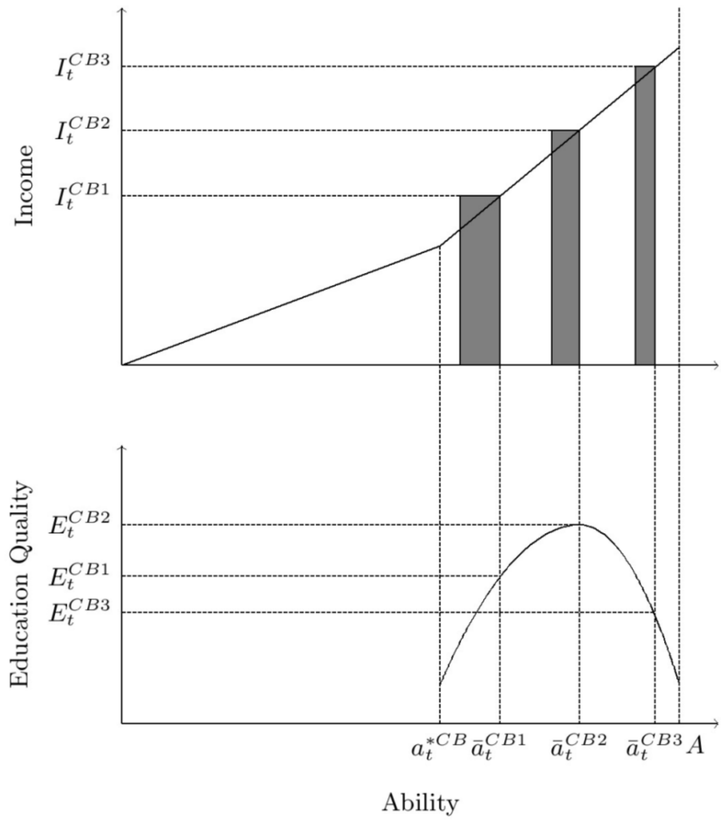
Using (8), the basic education quality optimization problem can be stated as

$$\begin{aligned} &\max_{z_t^{CB}, \overline{h}_t^{CB}} \quad E_t \\ &\text{subject to (23),} \\ &z_t^{CB}\theta w\overline{h}_t^{CB} = T_t^{CB} \text{ and} \\ &\underline{a}_t \geq a_t^* \end{aligned} \tag{20}$$

Tax revenue in period t is

$$T_t^{CB} = \tau\left(w_t L_t + \theta w_t H_t^y + z_t^{CB} I_t^{CB}\right) \tag{21}$$

The figure below offers an illustration for this problem.



- The vertical bars illustrate several feasible education policy combinations of the number of teachers to be hired and teacher salaries satisfying the education budget constraint: $\left(z_t^{CB1}, I_t^{CB1}\right), \left(z_t^{CB2}, I_t^{CB2}\right), \left(z_t^{CB3}, I_t^{CB3}\right)$.
- The width of a bar corresponds to the number of teachers z_t^{CB}
- Its height is teacher salary I_t^{CB}.

<u>Exercise 6</u>: Explain the difference between hiring strategies 1, 2, and 3 in the figure.

Recursively define the <u>optimal education policy</u> in period t based on the above general equilibrium construct. Namely, the education agency chooses teacher salaries I_t^{CB} and the number of teachers z_t^{CB} for period t by solving the optimization problem (20) where the top teacher quality $\overline{h}_t^{CB}$ is determined by equation (16), while taking as given the economy's general equilibrium values of the aggregate tax revenue T_t^{CB} and the distribution of skilled human capital attainment $h_t^{CB}(\omega)$ by generation G_t individuals.

Proposition 2 (RDE solutions under the teachers' collective bargaining regime).

(i) In each period $t = 0,1,\ldots$, *given the prior period's education quality* E_{t-1}^{CB} *and individual college attendance decisions, the basic education quality optimization problem (20) with the basic education budget* T_t^{CB} *defined by the relationship (21), has <u>a unique solution</u> characterized by the following expressions, respectively, for the optimal quantity and the absolute quality (i.e. the aggregate human capital) of teachers*

$$z_t^{CB} = \left(\left(\frac{\gamma}{2\nu+\gamma}\right)\left(\frac{\tau}{1-\tau}\right)\right)^{1/2}\left(J_t^{CB}\right)^{1/2}, \qquad q_t^{CB} = \frac{\nu\tau\left(b+B\right)ACE_{t-1}^{CB}}{2\nu+\gamma}J_t^{CB} \tag{22}$$

where

$$J_t^{CB} = \left(1 - \frac{2Bh^*}{\left(b+B\right)ACE_{t-1}^{CB}} + \frac{\theta\left(Bh^*\right)^2}{\left(\theta\left(b+B\right)-1\right)\left(b+B\right)\left(ACE_{t-1}^{CB}\right)^2}\right), \tag{23}$$

as well as the upper and lower ability cut-offs for teachers to be hired:

$$\overline{a}_t^{CB} = \frac{Bh^*}{\left(b+B\right)CE_{t-1}^{CB}} + z_t^{CB}\left(\frac{\nu\left(1-\tau\right)}{\gamma}+\frac{1}{2}\right), \qquad \underline{a}_t^{CB} = \frac{Bh^*}{\left(b+B\right)CE_{t-1}^{CB}} + z_t^{CB}\left(\frac{\nu(1-\tau)}{\gamma}-\frac{1}{2}\right) \tag{24}$$

(ii) This optimal solution for period t education policy uniquely determines period t basic education quality E_t^{CB} *and as a consequence, the college attendance decisions by generation* t *individuals. This <u>recursion defines the unique recursive dynamic equilibrium</u> in the model under the teachers' collective bargaining regime for all* $t = 0,1,\ldots$ *given the initial generation's basic education quality* E_{-1}.

6.5. The Dynamics of the Quantity and Quality of Teachers under Collective Bargaining

The main dynamics results for the teachers' collective bargaining regime are established.

Lemma 1 (Growth of Basic Education Quality). *The recursive equilibrium dynamics under the collective bargaining regime for teachers exhibits sustained growth of the quality of per student basic education. Specifically, there is a factor* $g > 1$ *such that* $E_t^{CB} > gE_{t-1}^{CB}$ *for all* $t = 0,1,...$

The result of Lemma 1 provides for an underlying endogenous growth feature of our model where the rise of the human capital attainments at the basic level is the "engine of growth".

Lemma 2 (The Interiority Property under the Collective Bargaining). *In the recursive dynamic equilibrium, the ability of the least qualified teacher exceeds the college attendance cut-off ability in all time periods, i.e.* $\underline{a}_t^{CB} > a_t^*$ *is true for* $t = 0,1,...$. *Thereby the <u>human capital of the least qualified teacher will not be the lowest among his contemporary college graduates</u>.*

Lemma 3 *The ability cut-off for college attendance* a_t^* *satisfies equality (6), i.e.,*

$$a_t^* = \frac{1}{CE_{t-1}^{CB}} \frac{\theta Bh^*}{\theta\left(b+B\right)-1}$$

which means that an individual will choose to attend college if and only if his resulting skilled human capital given by formula (5) adjusted for the net productivity augmentation θ *will exceed his unskilled human capital derived from the basic stage of education according to its production function (4).*

Proposition 3 (The Rising Talent Premium). *The recursive equilibrium dynamics exhibits growing inequality within the group of skilled individuals, as well as the rise in inequality between this group and the unskilled.*

Lemma 4. *The effects of increased quality of basic education* E_{t-1}^{CB} *in period* $t-1$ *are expanded college attendance (lower college attendance ability cut-off) in the next generation, larger quantity of teachers, and their lower quality given by their lower ability cut-off values. Specifically, the following is true for all* $t = 0,1,\dots$ *in the recursive dynamic equilibrium:*

$$\frac{\partial a_t^*}{\partial E_{t-1}^{CB}} < 0, \quad \frac{\partial z_t^{CB}}{\partial E_{t-1}^{CB}} > 0, \quad \frac{\partial \overline{a}_t^{CB}}{\partial E_{t-1}^{CB}} < 0, \quad \frac{\partial \underline{a}_t^{CB}}{\partial E_{t-1}^{CB}} < 0$$

Combining the last two facts, $\frac{\partial \overline{a}_t^{CB}}{\partial E_{t-1}^{CB}} < 0, \quad \frac{\partial \underline{a}_t^{CB}}{\partial E_{t-1}^{CB}} < 0$ for $t = 0,1,\dots$, with Lemma 1, which shows that education quality E_{t-1}^{CB} does in fact grow over time, yields one of our central results.

Theorem 1 (Dynamics of the Quantity and Quality of Teachers under Collective Bargaining). *The recursive dynamic equilibrium (RDE) exhibits the following evolution of education policy variables:*

- *the quantity of teachers* z_t^{CB} *grows over time;*
- *the relative quality of teachers characterized by the range of their innate abilities falls: both the upper and the lower cut-offs* $\overline{a}_t^{CB}$, $\underline{a}_t^{CB}$ *decrease over time;*
- *the college attendance ability cut-off* a_t^* *also drops over time (thus the college-bound population expands);* a_t^* *remains consistently below the lower ability cut-off for teachers* $\underline{a}_t^{CB}$ *(according to Lemma 2).*

Corollary. *While the relative quality of teachers falls over time in the RDE under the collective bargaining regime (according to Theorem 1), the absolute quality of teachers characterized by their human capital attainment grows: both the human capital of the top teacher and the least qualified one,* $\overline{h}_t^{CB}$, $\underline{h}_t^{CB}$*, rise over time.*

Discussion. The intuition for the above results derives from the mechanics of economic growth in our model.

- Rising per student quality of basic education increase returns to higher education.
- More students attend college (those with relatively low ability).
- Human capital attainment of highly able students increases disproportionately relative to their less able peers due to increasing returns to ability
- Economic growth drives the rise of income inequality within the group of college graduates.
- The opportunity cost of highly able college graduates measured by their increasing relative earnings in the production sector rises over time.
- Hiring high ability individuals as teachers becomes a relatively more expensive option
- Pushes the quality-quantity trade-off in favor of quantity over quality.

In other words, hiring a larger number of teachers of relatively lower quality is becoming an increasingly more cost-effective policy.

6.6. Conclusions

The debate over how technological progress has affected education inputs still remains today. There is much to be learned by the trade-off between teachers and technology as well as how outside wages influence the pool of teachers.

My own research has studied aspects of education production function:

1) Whether work environment or salaries have a greater impact on teacher attrition.
2) Whether the quality of teachers is affected by uniform salaries across teaching subjects (humanities vs. math).
3) Whether for-profit vs. not-for-profit colleges perform equally.
4) Whether hiring more teachers of the same quality or higher quality teachers (but holding the quantity of teachers constant) has similar effect on education production.

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Lecture 9: Fiscal and Monetary Policies

1. Introduction

This lecture provides an introduction to fiscal and monetary policy analysis. This topic is extremely large and could encompass several courses on their own from monetary theory to public economics, developmental economics, and growth theory. As such, only a few models are explored:

1. Fiscal and monetary theories of inflation.
2. The ten money doctrines.
3. Ricardian equivalence.
4. Time inconsistency with regards to government's optimal policy.
5. Government input in the production function.

2. Fiscal and monetary theories of inflation

Issues in monetary theory that address the necessary coordination of monetary and fiscal policies are introduced.

A transaction technology with shopping time and real money balances as inputs is imposed. The important feature about the transaction technology is that the demand for currency is a decreasing function of the rate of return on currency.

The model is used to illustrate doctrines in monetary economics.

2.1 Shopping time monetary economy

Consider an endowment economy with no uncertainty. A representative household has one unit of time. There is a single good of constant amount $y > 0$ in each period t. The good can be divided between private consumption $\{c_t\}_{t=0}^{\infty}$ and government purchases $\{g_t\}_{t=0}^{\infty}$, subject to

$$c_t + g_t = y \tag{1}$$

The preferences of the household are

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\delta}}{1-\delta} + \frac{l_t^{1-\alpha}}{1-\alpha} \right) \tag{2}$$

where $\beta \in (0,1)$, $c_t \geq 0$ and $l_t \geq 0$ are consumption and leisure at time t, respectively.

With one unit of time per period, the household's time constraint is

$$1 = l_t + s_t \tag{3}$$

To acquire the consumption good, the household allocates time to shopping.

The amount of shopping time s_t needed to purchase a particular level of consumption c_t is negatively related to the household's holding of real money balances m_{t+1}/p_t.

- The shopping or transaction technology is

$$s_t = H\left(c_t, \frac{m_{t+1}}{p_t}\right) = \frac{c_t \varepsilon}{1 + m_{t+1}/p_t} \tag{4}$$

where ε is positive.

A few comments on the technology:

2.1. Households

The household maximizes (2) subject to its time constraint (3), the transaction technology (4), and the sequence of budget constraints:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = y - \tau_t + b_t + \frac{m_t}{p_t} \tag{5}$$

- m_{t+1} is the nominal money balances held between t and $t+1$. Notice that households must decide on money holdings for next period and forgo interest to hold this money. This implies that bonds can be bought and sold between periods, not within period.
- p_t is the price level.
- b_t is the real value of one-period government bond holdings that mature at the beginning of time period t, denominated in units of time t consumption.
    - Similar to zero-coupon bond where it is bought/sold for less than face value.
- τ_t is a lump-sum tax at t.
- R_t is the real gross rate of return on one-period bonds between t and $t+1$.

Maximization is subject to $m_{t+1} \geq 0$ for all $t \geq 0$, and given initial stocks m_0 and b_0.

<u>Must ensure conditions for no arbitrage between money and bonds.</u>

One concern in the model is that individuals may attempt to arbitrage by choosing large money holdings financed by issuing bonds. The condition that restricts arbitrage is provided below.

<u>Exercise 1</u>: Update equation (5) to $t+1$. Label it (5').

<u>Exercise 2</u>: Substitute (5') into (5) through b_{t+1} and collect like terms. Label it (5'').

- $R_{mt} = p_t / p_{t+1}$ is the <u>real gross return on money</u> held between t to $t+1$ and is the <u>inverse</u> of $1+\pi$ (the gross rate of inflation).
    - The return on money increases if next period's price level is lower than the current price level.
    - Zero inflation implies that $R_m = 1$.

<u>Exercise 3</u>: Using the above equation, what is the condition to ensure that arbitrage is not profitable?

Exercise 4: Relate the expression in Exercise 3 back to the rate of inflation and nominal interest rate.

where the last link is due to the Fisher equation:

In other words, the net nominal interest rate (i_t) cannot be negative.

Exercise 5: State the household's maximization problem.

Exercise 6: Provide the four first-order conditions. Label them (a) – (d).

Exercise 7: Combine (a) and (b) and interpret the Lagrange multiplier on the budget constraint. Label it (6).

The Lagrange multiplier on the budget constraint is equal to the marginal utility of consumption reduced by the marginal disutility of having to shop for that increment of consumption.

Exercise 8: Combine (6) into (c) to obtain the real interest rate. Label it (7).

- Notice that the real interest rate is simply equal to the discounted intertemporal rate substitution between consumption and leisure. That is, the rate of return on holding a bond is equal to the loss in utility by not being able to use the money for shopping in one period relative to the gain in utility by having more money in the next period.

Exercise 9: Combine (c) and (d). Label it (8).

Exercise 10: Combine (b), (8), and (6) to obtain the benefit of the marginal unit of real money balances held from *t* to *t+1* expressed in time *t* utility. Label it (9)

- The first term on the LHS is the lost interest earnings (R_t - R_{mt}) discounted at the rate R_t.
- The second term on the LHS is the marginal rate of substitution between consumption and leisure minus the cost of shopping more in units of consumption.
- The third term is the benefit of an additional unit of real money balances while shopping.

Implicitly, (9) defines the money demand function.

Exercise 11: Define the money demand function. Label it (9').

- Increases in consumption.
- Increases in the return for money.
- Decreases in the return for bonds.

2.2. Government

The government finances the purchase of the stream $\{g_t\}_{t=0}^{\infty}$ subject to the sequence of budget constraints

(10)

- B_t is government debt to the private sector denominated goods.
- M_t is the stock of government issued currency.
    - Earns government revenue by printing new money. Notice that inflation erodes the value of seigniorage obtained.
- B_0 and M_0 are given.

2.3 Equilibrium

Exercise 12: Define a competitive equilibrium.

2.3. Analyzing government policy

We now turn our attention to studying how government policy affects the economy. Similar to previous analyses, we will make distinct the short-run and the long-run. To do so, we assume a stationary equilibrium for $t > 0$ and allow the economy to start at different initial conditions (at $t = 0$). To simplify the analysis, we restrict much government policy:

$$g_t = g, \quad \forall t$$
$$\tau_t = \tau, \quad \forall t > 0$$
$$B_t = B, \quad \forall t > 0.$$

There are still sufficient policy variables free to discuss how government policies affect the economy:

1) Fiscal policy: changes in tax revenue.
2) Monetary policy: an open market operation.

2.3.1. Stationary equilibrium ($t > 0$)

We seek an equilibrium for which

$$p_t / p_{t+1} = R_m$$
$$R_t = R$$
$$c_t = c$$
$$s_t = s$$

Exercise 13: Substituting these into (7) and (9').

- The real return on bonds is simply the inverse of the discount rate.

Using the above conditions in the government's budget constraint (10) and that $M_t = m_t$ gives:

$$g - \tau + \frac{B(R-1)}{R} = f(R_m)(1-R_m) \qquad (10')$$

where the RHS is the rate of seigniorage revenue from printing currency.

$$f(R_m)(1-R_m) = \frac{M_{t+1}}{p_t} - \frac{M_t}{p_{t-1}}\frac{p_{t-1}}{p_t} = \frac{M_{t+1} - M_t}{p_t}$$

- $g - \tau$ is the net of interest deficit, or operational deficit.
- $g - \tau + \frac{B(R-1)}{R}$ is the gross of interest government deficit.
- $1 - R_m$ is the inflation tax.
- $f(R_m)$ is the quantity of real money balances and the base of the inflation tax.

Exercise 14: How does government policy determine the stationary rate of return on currency, R_m?

Exercise 15: Graph the RHS of equation (10') for any given level of g, τ, and B.

- This figure is referred to as the *Laffer curve* for the inflation tax rate.
    - Similar to the standard *Laffer curve*, there is a 'good' and 'bad' side.

Exercise 16: Which side of the Laffer curve is 'good' and which is 'bad'?

Exercise 17: Is it possible for the return on seigniorage to be negative? If so, how?

Exercise 18: What determines the rate of seigniorage graphically?

2.3.2. Short-run equilibrium (*t = 0*)

Since the return on money is determined in the stationary equilibrium, this implies that equation (10) at $t = 0$ can be written as:

$$\frac{M_0}{p_0} = f(R_m) - (g + B_0 - \tau_0) + \frac{B}{R} \tag{11}$$

Notice that (10') and (11) create a recursive system of equations. First, equation (10') identifies R_m and then equation (11) determines p_0 (since M_0 is given).

Exercise 19: Graph the RHS of equation (11) for any given level of g, τ, B, and R_M.

3.0. The ten money doctrines

We now use equations (10') and (11) to explain some important doctrines about money and government finance. Recall that:

$$g-\tau+\frac{B(R-1)}{R}=f(R_m)(1-R_m) \tag{10'}$$

$$\frac{M_0}{p_0}=f(R_m)-(g+B_0-\tau_0)+\frac{B}{R} \tag{11}$$

3.1. Quantity Theory of Money (QTM)

The classical quantity theory of money experiment is to increase M_0 by some factor (a helicopter drop of money) and leave all other government policies fixed.

Exercise 20: What happens in equation (10')?

Exercise 21: What happens in equation (11)?

- The standard quantity theory of money ($MV=PY \Rightarrow M\overline{V}=P\overline{Y} \Rightarrow \uparrow M \rightarrow \uparrow P$) has been heavily criticized by Keynes as false and by von Mises as lacking a demand for money function.
    - In the standard quantity theory of money, V is the demand and is constant.

3.2. Sustained deficits cause inflation

Suppose the government is operating on the ‘good’ side of the Laffer curve and seeks to increase its deficits from period $t > 0$.

Exercise 22: Which equation does this affect?

Exercise 23: Will increase deficits necessarily be inflationary?

3.3. Fiscal prerequisites of zero inflation policy

Suppose the government desires to have a non-inflationary equilibrium ($R_m = 1$).

Exercise 24: Which equation does this affect?

Exercise 25: What government policy restriction will be necessary and sufficient to sustain the equilibrium?

3.4. Some unpleasant monetarist arithmetic (Sargent and Wallace)

One concern of monetary policymakers is that they have no control over current or future tax rates when they conduct monetary policy. Suppose the central bank conducts an open market sale and fiscal policymakers set taxes.

Exercise 26: How is an open market sale conducted?

Exercise 27: Using the Laffer curve for the inflation tax, what necessarily happens to long-term inflation?

Exercise 28: Using equation (11), what happens to inflation in the short-run?

Conclusion: Without fiscal policy cooperation, monetary policy will lead to sustained long-run inflation with a chance of temporarily driving down price in the short-run.

3.5. An open-market operation delivering neutrality

Suppose that monetary and fiscal policymakers coordinated their efforts.

<u>Exercise 29</u>: Explain in words (or equations) what fiscal policymakers can do to accommodate an open market sale.

3.6. The optimal quantity of money

Freidman's (1969) ideas about the optimum quantity of money can be represented by equations (10') and (11). Friedman noted that, given the stationary levels of g and B, the household prefers stationary equilibria with higher rates of return on currency.

<u>Exercise 30</u>: Why would households prefer higher returns on currency?

<u>Exercise 31</u>: What is the highest rate of return on money achievable in the model?

Exercise 32: How can the government achieve the highest rate of return on money?

Exercise 33: In this model, what is the optimal quantity of money?

3.7. Legal restrictions to boost demand for currency

Suppose that the government can somehow force households to increase their real money balance.

Exercise 34: How can the government force households to increase money holdings?

Exercise 35: If individuals increase their money holdings, how does this affect the model?

3.8. One big open market operation

Lucas (1986) and Wallace (1989) describe a large open market purchase of private indebtednesses at time 0. The purpose of the operation is to provide the government with a portfolio of interest-earning claims on the private sector, one that is sufficient to permit it to run a gross of interest surplus.

Exercise 36: How does this relate to the recent financial crisis?

Exercise 37: How could this be optimal?

3.9. The fiscal theory of inflation

Assume that the government sets g, τ, τ_0, and B, that B_0 and M_0 are inherited from the past, and that the model then determines R_m and p_0 via equations (10') and (11).

Exercise 38: Does monetary policy or fiscal policy determine the price level? Is this intuitive?

3.10. The fiscal theory of the price level

Woodford (1995) and Sims (1994) converted the FT of inflation to the FT of the price level by altering the assumptions about the variables that the government sets.

Woodford assumes that B is endogenous and that instead the government sets in advance a present value of seigniorage $\frac{f(R_m)(1-R_m)}{(R-1)}$. This assumption is equivalent to the government commit to fix either the nominal interest rate or the gross rate of inflation R_m^{-1}.

Rearranging (10') to obtain:

$$\frac{B}{R} = \frac{1}{R-1}[\tau - g + f(R_m)(1-R_m)] \tag{12}$$

which can be substituted into (11) to obtain

$$M_0/p_0 = \tau_0 + \frac{\tau}{R-1} - \frac{gR}{R-1} - B_0 + \frac{(R-R_m)f(R_m)}{R-1} \tag{13}$$

That is, the present value of government surplus (taxes minus government spending) plus the present value of future seigniorage yields the price level.

- p_0 is set to equate the real value of total initial government indebtedness is equivalent to the net of interest government surplus, including seigniorage revenues.

4. Introduction to Ricardian Equivalence

Ricardian equivalence, (also known as the Barro-Ricardo equivalence proposition) is an economic theory that suggests consumers internalize the government's budget constraint and thus the timing of any tax change does not affect their change in spending.

Consequently, Ricardian equivalence suggests that it does not matter whether a government finances its spending with debt or a tax increase, the effect on total level of demand in an economy being the same.

4.1. The model

4.1.1. The household

Consider an infinite horizon economy similar to the Ramsey-Cass-Koopman model. The representative consumer maximize time-separable utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. The corresponding budget constraint for the consumer is given as:

$$c_t + b_{t+1} = w_t(1-l_t) - \tau_t + R_t b_t \tag{2}$$

- b_{t+1} is the quantity of bonds purchased by the consumer in period t, which come due in period $t+1$.
- Lump sum tax (head tax) is defined as τ.

The representative consumer is permitted to issue private bonds (to save) which are perfect substitutes for government bonds.

The representative consumer takes lump-sum taxes, the wage rate and the one-period gross interest rate as given and maximizes (1) subject to (2).

This yields the following first-order conditions:

$$\beta^t u_c(c_t, l_t) = \lambda_t \tag{3}$$

$$\beta^t u_l(c_t, l_t) = \lambda_t w_t \tag{4}$$

$$\lambda_t = \lambda_{t+1} R_{t+1} \tag{5}$$

Combining (3) and (5) yields the standard Euler Equation:

$$\frac{u_c(c_t, l_t)}{u_c(c_{t+1}, l_{t+1})} = \beta R_{t+1}$$

- The ratio of consumption between periods (synonymous to the savings rate) is dictated by the discount factor and the interest rate on savings.

The trade-off between consumption and leisure is found by combining (3) and (4):

$$u_l(c_t, l_t) = w_t u_c(c_t, l_t)$$

- The discount factor and taxation play no role in the trade-off between consumption and leisure.
- The trade-off is between the ratio of prices ($w_t/1$) and the marginal rate of substitution between consumption and leisure.
    - The price of consumption is always normalized to 1. Why is this possible?

4.1.2. The government

Government purchases g_t units of consumption goods in period t and these purchases are consumed without utility (this is a simplifying assumption). Government purchases are financed through lump-sum taxation as specified in the consumer's problem and by issuing one-period government bonds.

The per period government budget constraint is

$$g_t + R_t B_t = \tau_t + B_{t+1} \tag{6}$$

- B_t is the number of one-period bonds issued by the government in period $t-1$.
- $B_0=0$: no government debt or surplus at the beginning of time.
- A bond issued in period t is a claim to R_{t+1} units of consumption in period $t+1$.
- R_{t+1} is the one-period (gross) interest rate.

4.2. Solving the model

To understand the Ricardian Equivalence proposition, we first obtain the lifetime budget constraints of households and the government.

Exercise 1: Using (2) in period 0, iterate forward to obtain the household's lifetime budget constraint.

τ

(7)

Exercise 2: What is the TVC for this model?

Exercise 3: Using (6) in period 0, iterate forward to obtain the government's lifetime budget constraint.

(8)

Exercise 4: Combine the two lifetime budget constraints, (7) and (8).

(9)

Exercise 5: What do we know about the sequence of taxes, $\{\tau_t\}_{t=0}^{\infty}$?

Exercise 6: What parameters and state variables are not present in (9)?

Exercise 7: What happens if the government spends more?

4.3. Conclusion

There are some critical assumptions for the Ricardian Equivalence result to hold:

5. Time inconsistency

Time inconsistency arises in monetary and fiscal policy due to the government's ability to change its policies at a later date (to undue previous policies). Households and firms can also be time inconsistent with their choices, but typically do not have a macroeconomic effect on the economy.

Simple examples of time inconsistency are:

1. Policy to not negotiate with terrorists.
2. Policy to not tax income from capital.
3. A parent announces that he or she will punish a child whenever the child breaks a rule. When the child misbehaves, parent is tempted to forget to punish as it maybe unpleasant for the parent.
4. Professor announces that course will have cumulative final exam. At the time of the final, the professor is tempted to cancel the exam so that he or she won't have to grade it.

Policymakers may want to announce in advance the policy they will follow to influence the expectations of private decisionmakers. After the private decisionmakers have acted, these policymakers may be tempted to renege on their announcement. Understanding that policymakers may be inconsistent over time, private decisionmakers are led to distrust policy announcements. In this situation, to make their announcements credible, policymakers may want to make a commitment to a fixed policy rule.

Time consistency of a policy plan can be defined as a sequence of state contingent functions $\{g_t(h_t)\}_{t=0}^{\infty}$ where g_t is government policy. History of all events up to date t is denoted by h_t. A time t continuation of a plan is the part of the plan from time t onward. A plan is time consistent with respect to some objective if

1. that plan maximizes the objective, and
2. for all periods t, the time t continuation maximizes the objective from time t onward.

The definition says that if the government plans to do something, but is free at some later date to change its plans, it will not change its plans. If a proposed future policy function is optimal, it will remain optimal up to and including the date when the policy function will actually have to be implemented.

A plan is *time inconsistent* if it is not time consistent, i.e., if the government wishes to change its plans as time passes. To repeat, a "change" in plans is not simply a matter of doing something different at a new date. It is changing what was intended for that date, conditional upon the circumstances that arise; the state-contingent action is changed.

$t = 0: p_0^0, p_1^0, p_2^0, p_3^0, p_4^0, p_5^0, \ldots$

$t = 1: p_1^1, p_2^1, p_3^1, p_4^1, p_5^1, \ldots$

Time consistencies implies that at *t=1*, $p_1^0 = p_1^1$.

5.1. The model (Kydland and Prescott, 1977)

The model illustrates time inconsistency using the Phillips curve. The Phillips curve can be written in the form

$$u = u^{*} - d\left(\pi - E\left[\pi\right]\right) \tag{1}$$

where u is the current rate of unemployment, and u^{*} is the natural rate of unemployment, d is a constant, π is the rate of inflation, and $E[\pi]$ is the forecasted or expected inflation rate at the beginning of the period.

The social welfare function is the <u>negative</u> of

$$S = -u - c\pi^{2} \tag{2}$$

where c is a constant.

- Unemployment is bad and very high or very low inflation is bad. Ideal inflation in this setup is zero, although the problem could be re-worked so that ideal inflation is some positive or negative number.

<u>Exercise 1</u>: Substitute (1) into (2) at u to obtain the social welfare function. Label it (3).

5.2.1. Discretion vs. commitment

The government has two options:

1. The government commits publicly to a rate of inflation before expectations are formed, which implies that expected inflation is the committed rate of inflation.
2. The government does not commit and expected inflation is the rate that people (correctly) believe the government will choose in the following period, given what is optimal at that time.

The ***commitment equilibrium*** occurs if the government chooses a rate of inflation initially and is not allowed to deviate from it in the following periods. To maximize social welfare, inflation should be set to the optimal rate. Given that individuals are rational, expected inflation is equal to inflation.

$$E[\pi] = \pi = 0 \tag{4}$$

No matter what inflation rate is chosen, unemployment remains at u^*.

Exercise 2: Find the optimal social welfare under the *commitment equilibrium*. Label it (4).

The ***discretionary equilibrium*** occurs if the government does not commit to any particular inflation rate initially. People have to form some inflation expectation. Assume that individuals' expectations of inflation are rational.

Exercise 3: Find the optimal rate of inflation under discretion by differentiating (3) with respect to π. Label it (5).

Rationality assumes that individuals will expect the government to commit to (5) when it comes time to making this choice. Thus, if a 5% rate of inflation is thought to be best, individuals expect this rate. Therefore, expected inflation is

$$E[\pi] = d / 2c \tag{6}$$

Exercise 4: Using (6), find the unemployment rate. Label it (7).

Exercise 5: Find social welfare using (6). Label it (8).

The difference is that under discretion, inflation is different from zero so social welfare is lower.

5.3. Comparison between discretion and commitment

The *commitment equilibrium* is better than the *discretionary equilibrium* from a welfare perspective. If the government has two options: commit and have everyone observe that commitment and leave no possibility of not honoring that commitment, or do not commit and have everyone believe that next period's short term optimum will be chosen, then the planner should commit.

If discretion prevails and commitment does not, it is best that the agency that chooses the inflation rate to have a bias against inflation (by making c high). The effect is to make the choice of inflation, given by (5), low.

The nature of the equilibrium should be made clear. If the planner has two choices, commitment (rules) or no commitment (discretion), the choice in this example is for commitment.

5.4. Time inconsistency that maximizes social welfare

Supposed individuals have expectations that the government is committed to zero inflation. If the government can fool them, is this a pareto improvement?

Exercise 6: Use (3) to demonstrate that social welfare can be larger if the government can fool the individuals' expectations.

This can be achieved if you can fool all of the people some of the time. It is difficult to see how a government can systematically promise and break promises for low inflation and still have these promises affect beliefs. Time inconsistency is present because the optimal choice (about whether the government should have discretion in period two) is one thing (no) in period one and something different (yes) in period two.

6. Government input in the production function

The government's roles in the economy are diverse. One role is to maintain infrastructure and other essential government inputs into the production process. This can be described as a regular input in the production function.

This section describes the modeling of the government sector and solves for the per capita steady state physical capital stock.

6.1. The model

Consider the basic two-period overlapping generations model with two periods (young and old) as specified in the lecture notes with log preferences in adulthood and old age, no population growth, no growth in technological progress, i.e., $A_t = A \ \forall t$, and depreciation of physical capital of δ.

Suppose that the government provides government input into the production process:

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha} G_t^{\theta} \tag{1}$$

where G_t is the government input and $\theta \in (0,1)$ is the return on this input in the production of output and $0 < \alpha < 1$, $0 < \beta < 1$, and $\alpha + \beta < 1$. The law of motion for the government input is

$$G_{t+1} = (1-\delta_G)G_t + I_t^G \tag{2}$$

In this set up $\delta_G = 1$.

This government input is financed through taxation of all income sources at rate τ.

<u>Exercise 1</u>: Provide the government's budget constraint. Label it (3).

6.1.1. The household's problem

Exercise 2: State the household utility maximization problem assuming they work and consume in the first period of life, and consume off of savings in retirement.

Exercise 3: Solve the household's maximization problem. Label it (4).

Exercise 4: State the firm's profit maximization problem and solve the first order conditions. Label them (5) – (6).

- The marginal product of capital and labor are affected by the level of government input.

Exercise 5: State market clearing conditions.

The *competitive equilibrium* is defined as a sequence of allocations and prices such that households maximize utility taking prices and government policies as given, firms maximize profits taking prices as given, markets clear, and the government budget constraint is satisfied.

We can solve for the equilibrium.

Exercise 6: Solve for the law of motion of physical capital as a function of exogenous parameters, state variables, and G_t. Label it (7).

One issue is that G_t is endogenous and must be removed.

Exercise 7: Divide the law of motion for government infrastructure by (7) to find a constant fraction of government input. Label it (8).

Exercise 8: Normalize the size of the population to 1 and use (7) and (8) to solve for the law of motion of physical capital. Label it (9).

Exercise 9: Interpret the results.

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Midterm Exam Information

Bring: Pencils, eraser, drink.
Do not use: cellphone, headphones, books, notes

Procedure

1. Write name on front page.
2. Use a new sheet for each question.
3. One each sheet of paper:
   a. Write question and subquestion number.
   b. Write page number.
4. Show all work.
5. If you get stuck, explain what you would do to complete the problem.
6. When you make a mistake:
   a. If you have time, correct it.
   b. If you don't have time, acknowledge it and then proceed to finish the problem writing any assumptions necessary.

Reminders

- Eyes on your own exam.
- The time allotted for this exam is two hours.

Distribution of questions

There are two questions with multiple subparts.

Question 1: A workout problem that requires the application of the Solow Growth Model.
Question 2: A workout problem that requires the application of the Neoclassical Growth Model.

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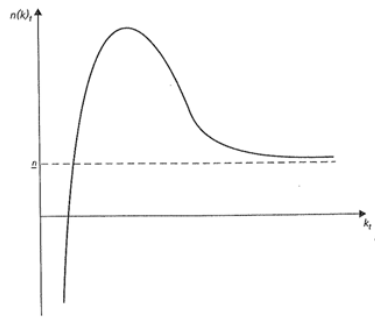
Sample Midterm Exam

1. Consider the Solow growth model from the lecture notes except that n is no longer exogenous, but rather an endogenous variable that depends on prosperity. Hence, n is replaced with n_t, and an additional equation, $n_t = h(y_t)$ where $y_t = Bk_t^{\alpha}$.

    A) Solve for the fundamental difference equation of the Solow growth model.

    B) Solve for the growth rate, γ_t.

    C) Graph the physical capital trajectory towards the steady state given the following path of n below.



    D) How does model relate to conditional convergence?

2. Consider a household with the following preferences over consumption levels at two dates:

$$U(c_1, c_2) = \left(\frac{c_1^{1-\sigma} - 1}{1-\sigma} \right) + \beta \left(\frac{c_2^{1-\sigma} - 1}{1-\sigma} \right)$$

The per period budget constraints of the household are

$$c_1 + s = W_1,$$
$$c_2 = W_2 + s(1+r)$$

where r is the interest rate and W_1 and W_2 are wage income in both periods.

a) Construct the lifetime budget constraint.

b) Characterize the utility maximizing choice of s. How does savings decision depend upon the interest rate r?

Final Exam Information

Bring: Pencils, eraser, drink.
Do not use: cellphone, headphones, books, notes

Procedure

7. Write name on front page.
8. Use a new sheet for each question.
9. One each sheet of paper:
    a. Write question and subquestion number.
    b. Write page number.
10. Show all work.
11. If you get stuck, explain what you would do to complete the problem.
12. When you make a mistake:
    a. If you have time, correct it.
    b. If you don't have time, acknowledge it and then proceed to finish the problem writing any assumptions necessary.

Reminders

- Eyes on your own exam.
- The time allotted for this exam is the entire class time.

Distribution of questions

There are two questions with multiple subparts.

Question 1: A workout problem that requires the application of either the Solow Growth Model or Neoclassical Growth Model
Question 2: A workout problem that requires the application of the Overlapping Generations Model.

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Sample Final Exam

1. Productivity and Employment

Consider the Neoclassical economy with one modification. Suppose that there is a tax man that requires a tax on inelastically supplied labor income at rate τ_t. Assume that the per capita production function is given by

$$f(k) = Ak_t^{\alpha} g_t^{\beta}$$

where $0 < \alpha < 1$, $0 < \beta < 1$, and $\alpha + \beta < 1$. Government spending, g_t, is included in the production function and is the only government spending (it can be interpreted as infrastructure or other productive services). With this modification, preferences and the law of motion for physical capita are given by

$$\sum_{t=0}^{\infty} \beta^t \ln c_t,$$
$$k_{t+1} = (1-\delta)k_t + i_t.$$

a. Does the 1st and 2nd welfare theorems hold?

b. State the individual's per period budget constraint.

c. State the individuals' maximization problem.

d. State the Euler equation(s).

e. State the firm's problem.

f. State the firm's first-order condition(s). How is the wage rate determined?

g. State the competitive equilibrium.

h. Solve for steady-state c, y, g, and k.

i. *Extra credit*. Given steady-state c, y, g, and k, how is the optimal tax rate found?

2. Consider a basic two-period (young and old) OLG model with standard preferences provided by

$$\ln c_t^y + \beta \ln c_{t+1}^o + \delta \ln T_t$$

where c_t^y is consumption when young, c_{t+1}^o is consumption when old, T_t is a bequest from a young individuals to his parent (to help out with retirement). This is a form of altruism with the parameter δ dictating how much an individual cares about their parent's income when the parents are old.

a. State the individual's per period budget constraints.

b. State the individual's lifetime budget constraint.

c. Set-up the individual's problem assuming that individuals receive an income, w, in the young period of life only, choose the amount of savings, s_t and transfers, T_t. All savings earn interest at rate R.

d. Solve the individual's problem and state the first order conditions.

e. Solve for the equilibrium amount of savings and private transfer.

Yes, $r > g$. So what?
By N. Gregory Mankiw

This essay was prepared the Annual Meeting of the American Economic Association, January 2014.

Thomas Piketty's book *Capital in the Twenty-First Century* captured the public's attention in a way that few books by economists have. Though its best-seller status was a surprise, probably even to its author, it has the ingredients that foster wide appeal. The book addresses a pressing issue of the day in a manner that is learned, literary, speculative, provocative, and fascinating from beginning to end. While largely a work of economic history, it does not stop there. Piketty ultimately leads the reader to a vision of what the future may hold and advice about what policymakers should do about it. That vision is a dystopia of continually increasing economic inequality due to the dynastic accumulation of capital, leading to a policy recommendation of a steeply progressive global tax on wealth.

Although I admire Piketty and his book, I am not persuaded by his main conclusions. A chain is only as strong as its weakest links, and several links in Piketty's chain of argument are especially fragile. Other aspects of Piketty's book may well pass the test of time, but the bottom line—his vision of the future and the consequent policy advice—most likely will not.

The book documents that the rate of return on private capital r exceeds the economy's growth rate g, and it argues that this will likely continue to be the case, perhaps by a larger amount in the future. He boldly calls this fact "the central contradiction of capitalism." He reasons that if $r > g$, the wealth of the capitalist class will grow faster than the incomes of workers, leading to an "endless inegalitarian spiral." To someone who views relatively unfettered capitalism as one of the great achievements of human history and the best way to organize a society, as I do, these conclusions present a significant challenge.

The first thing to say about Piketty's logic is that it will seem strange to any economist trained in the neoclassical theory of economic growth. The condition $r > g$ should be familiar. In the textbook Solow growth model, it arrives naturally as a steady-state condition as long as the economy does not save so much as to push the capital stock beyond the Golden Rule level. (Phelps 1961) In this model, $r > g$ is not a problem, but $r < g$ could be. If the rate of return is less than the growth rate, the economy has accumulated an excessive amount of capital. In this dynamically inefficient situation, all generations can be made better off by reducing the economy's saving rate. From this perspective, we should be reassured that we live in a world in which $r > g$ because it means we have not left any dynamic Pareto improvements unexploited.

There is, moreover, good reason to doubt that $r > g$ leads to the "endless inegalitarian spiral" that concerns Piketty. Imagine a wealthy person living in an $r > g$ economy who wants to ensure that he has an endless stream of wealthy descendants. He can pass his wealth on to his children, but to ensure that his descendants remain wealthy, he faces three obstacles.

First, his heirs will consume some of the wealth they inherit. For this purpose, the relevant measure of consumption includes not only food, shelter, and riotous living but also political and philanthropic contributions, which can be sizeable for wealthy families. A plausible estimate of the marginal propensity to consume out of wealth, based on both theory and empirical evidence, is about 3 percent. Thus, if wealth earns a rate of return of r, wealth accumulates at a rate of about $r - 3$.

Second, as wealth is passed down from generation to generation, it is divided among a growing number of descendants. (This would not be a problem for the wealthy patron if his heirs' mating were perfectly assortative—that is, if they all married someone of equal wealth. But matters of the heart are rarely so neat.) To get a rough calibration of this effect, suppose everyone has a typical family of two children, so the number of descendants doubles every generation. Because generations are about 35 years apart, the number of descendants grows at a rate of 2 percent per year. Thus, if family wealth accumulates at a rate of $r - 3$, wealth per descendant grows at a rate of $r - 5$.

Third, many governments impose taxes on both bequests and capital income. In the United States today, the estate tax rate is 40 percent (above a threshold). In Massachusetts, where I live, the state imposes an additional estate tax with a top rate of 16 percent. As a result, at the margin, about half of a family's wealth is taxed away by the government once every generation. If we again assume a generation is 35 years, then estate taxation reduces the accumulation of dynastic wealth by about 2 percent per year. In addition, capital income taxation during a person's life reduces capital accumulation even further. This effect is roughly an additional 1 percent per year, making the total drag of taxes about 3 percent per year. Let's assume, however, that our dynasty has especially good tax planning and put the total tax effect at only 2 percent. Thus, taking taxation into account, wealth per descendant grows at a rate of about $r - 7$.

We can now recalibrate Piketty's logic taking these three effects into account. Piketty reasons that resources of the wealthy would grow relative to the labor income if $r > g$. We can now see, however, that this condition is not sufficient once consumption, procreation, and taxation are accounted for. Instead, to obtain the worrisome "endless inegalitarian spiral," we would need the return on capital r to exceed the economy's growth g by at least 7 percentage points per year.

This scenario is far from what we have experienced. Piketty estimates the real rate of return to be about 4 or 5 percent, which seems plausible for a typical balanced portfolio. Meanwhile, the average growth rate of the U.S. economy has been about 3 percent. So Piketty is right that r has exceeded g, but it has done so by only about 2 percentage points, not the more than 7 percentage points necessary for the creation of Piketty's imagined dystopia.

Moreover, while economists are notoriously bad at predicting the future, especially over long horizons, it seems unlikely that, looking forward, r will start exceeding g by more than 7 percentage points. If the real return remains stable at 5 percentage points, the economy's growth rate would need to become a negative 2 percent. Secular stagnation would not be enough; we would need secular decline. Alternatively, if future growth is 2 percent per year, the real rate of return to capital would need to rise from its historical 5 percent to more than 9 percent. That figure is nowhere near the return that pension and endowment managers are now projecting from a balanced portfolio of stocks and bonds.

Hence, the forces of consumption, procreation, and taxation are, and will probably continue to be, sufficient to dilute family wealth over time. As a result, I don't see it as likely that the future will be dominated by a few families with large quantities of dynastic wealth, passed from generation to generation, forever enjoying the life of the rentier.

But suppose I am wrong. Suppose the dynastic accumulation of capital describes the future, as Pikkety suggests. I would nonetheless remain skeptical of Piketty's proposal to place an additional tax on wealth. A simple, standard neoclassical growth model illustrates the problem with this policy.

Consider an economy composed of two kinds of people—workers and capitalists. Many workers supply labor inelastically and immediately consume their earnings. A few capitalists own the capital stock and, because they represent an infinitely-living dynasty, set their consumption according to the standard model of an optimizing infinitely-lived consumer (as in the Ramsey model). Workers and capitalists come together to produce output, using a production function that experiences labor-augmenting technological progress, and they earn the value of their marginal product. In addition, following the advice of Piketty, the government imposes a tax on capital equal to τ per period, the revenue from which is transferred to workers.

To oversimplify a bit, let's just focus on this economy's steady state. Using mostly conventional notation, it is described by the following equations:

(1) $c_w = w + \tau k$

(2) $c_k = (r - \tau - g)nk$

(3) $r = f'(k)$

(4) $w = f(k) - rk$

(5) $g = \sigma(r - \tau - \rho)$

where c_w is consumption of each worker, c_k is the consumption of each capitalist, w is the wage, r is the (before-tax) rate of return on capital, k is the capital stock per worker, n is the number of workers per capitalist (so nk is the capital stock per capitalist), $f(k)$ is the production function for output (net of depreciation), g is the rate of labor-augmenting technological change and thus the steady-state growth rate, σ is the capitalists' intertemporal elasticity of substitution, and ρ is the capitalists' rate of time preference. Equation (1) says that workers consume their wages plus what is transferred by the government. Equation (2) says that capitalists consume the return on their capital after paying taxes and saving enough to maintain the steady-state ratio of capital to effective workers. Equation (3) says that capital earns its marginal product. Equation (4) says that workers are paid what is left after capital is compensated. Equation (5) is derived from the capitalists' Euler equation; it relates the growth rate of capitalist's consumption (which is g in steady state) to the after-tax rate of return.

Because the steady-state return on capital in this economy is $r = g/\sigma + \tau + \rho$, the condition $r > g$ arises naturally. A plausible calibration might be $g = 2$, $\tau = 2$, $\rho = 1$, and $\sigma = 1$, which leads to $r = 5$. In this economy, even though $r > g$, there is no "endless inegalitarian spiral." Instead, there is a steady-state level of inequality. (Optimizing capitalists consume enough to prevent their wealth from growing faster than labor income.) If we assume the number of workers per capitalist n is large, then capitalists will enjoy a higher standard of living. In this natural case, c_w/c_k, the ratio of workers' consumption to capitalists' consumption, can be used as a proxy for inequality. A more egalitarian outcome is then associated with a higher ratio c_w/c_k.

Now consider the policy question: What level of capital taxation τ should the government set? Not surprisingly, the answer depends on the objective function.

If policymakers want to maximize the consumption of workers c_w subject to equations (1) through (5) as constraints, they would choose $\tau = 0$. This result of zero capital taxation is familiar from the optimal tax literature. (Chamley 1985, Judd 1985, and Atkeson, Chari, and Kehoe 1999, recently reconsidered by Straub and Werning 2014.) In this economy, because capital taxation reduces capital accumulation, labor productivity, and wages, it is not desirable even from the standpoint of workers who hold no capital and who get the subsidies that capital taxation would finance.

By contrast, suppose the government in this economy were a plutocracy, concerned only about the welfare of the capitalists. In this case, it would choose τ to maximize c_k subject to the above five equations as constraints. The best plutocratic policy is a capital subsidy financed by taxes on workers. That is, plutocrats would make τ as negative as it can be. If there is some minimum subsistence level for workers, the labor tax and capital subsidy would be driven so high as to push workers' consumption down to subsistence.

Now consider a government concerned about inequality between workers and capitalists. In particular, suppose that policymakers want to increase the ratio c_w/c_k. In this case, a positive value for the capital tax τ is optimal. Indeed, if maximizing c_w/c_k is the only goal, then the capital tax should be as large as it can be. Taxing capital and transferring the proceeds to workers reduces the steady-state consumption of both workers and capitalists, but it impoverishes the capitalists at a faster rate. For a standard production function ($f' > 0$ and $f'' < 0$), a higher capital tax always raises c_w/c_k.

Thus, in this simple neoclassical growth model, a positive tax on capital has little to recommend it if we care only about levels of consumption, but it may look attractive if we are concerned about disparities. To misquote Winston Churchill: the inherent vice of the free-market equilibrium is the unequal sharing of blessings; the inherent virtue of capital taxation is the more equal sharing of miseries.

So far, I have included only one policy instrument—the one recommended by Piketty—but we can consider others. A better way to pursue equality in this model economy, and I believe the real economy as well, is a progressive tax on consumption. Such a tax could equalize living standards between workers and capitalists without distorting the intertemporal margin and thereby discouraging capital accumulation. Under a progressive consumption tax, the capitalists would be just as wealthy as they are without it, but they would not fully enjoy the fruits of their wealth.

With this model as background, let's move to the big question: Why should we be concerned about inequality in wealth? Why should anyone care if some families have accumulated capital and enjoy the life of the rentier? Piketty writes about such inequality as if we all innately share his personal distaste for it. But before we embark on policies aimed at reducing wealth inequality, such as a global tax on capital, it would be useful to explore why this inequality matters.

One place to look for answers is Occupy Wall Street, the protest movement that drew attention to growing inequality. This movement was motivated, I believe, by the sense that the affluence of the financial sector was a threat to other people's living standards. In the aftermath of a financial crisis followed by a deep recession, this sentiment was understandable. Yet the protesters seemed not to object to affluence itself. If they had, Occupy Wall Street would have been accompanied by Occupy Silicon Valley, Occupy Hollywood, and Occupy Major League Baseball. From this perspective, the rentier lifestyle of capitalists should not be a concern. As we have seen, in a standard neoclassical growth model, the owners of capital earn the value of their marginal contribution to the production process, and their accumulation of capital enhances the productivity and incomes of workers.

Another possibility is that we object to wealth inequality because it is not fair. Why should someone be lucky enough to be born into a family of capitalists whereas someone else is born into a family of workers? The disparity between workers and capitalists is inconsistent with the ideal of equal opportunity. Yet that ideal conflicts with another—the freedom of parents to use their resources to help their children. (Fishkin 1984) Moreover, in considering Piketty's proposal of a global capital tax, we have to ask: Would you rather be born into a world in which we are unequal but prosperous or a world in which we are more equal but all less prosperous? Even if equal opportunity is a goal, one might still prefer unequal opportunities to be rich over equal opportunities to be poor.

A final possibility is that wealth inequality is somehow a threat to democracy. Piketty alludes to this worry throughout his book. I am less concerned. The wealthy includes supporters of both the right (the Koch brothers, Sheldon Adelson) and the left (George Soros, Tom Steyer), and despite high levels of inequality, in 2008 and 2012 the United States managed to elect a left-leaning president committed to increasing taxes on the rich. The fathers of American democracy, including George Washington, Thomas Jefferson, John Adams, and James Madison, were very rich men. With estimated net worth (in today's dollars) ranging from $20 million to $500 million, they were likely all in the top 0.1 percent of the wealth distribution, demonstrating that the accumulation of capital is perfectly compatible with democratic values. Yet, to the extent that wealth inequality undermines political ideals, reform of the electoral system is a better solution than a growth-depressing tax on capital.

My own view—and I recognize that this is a statement of personal political philosophy more than economics—is that wealth inequality is not a problem in itself. And I do not see anything objectionable if the economically successful use their good fortune to benefit their children rather than spending it on themselves. As a society, we should help those at the bottom of the economic ladder through such policies as a well-functioning educational system and a robust social safety net (funded with a progressive consumption tax). And we should help people overcome impediments to saving, thereby allowing more workers to become capitalists. But if closing the gap between rich and poor lowers everyone's standard of living, as I believe Piketty's global tax on capital would do, I see little appeal to the proposal.

A Century of US Central Banking: Goals, Frameworks, Accountability
By Ben Bernanke

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The Great Experiment

The original goal of the Great Experiment that was the founding of the Fed was the preservation of financial stability. In the words of one of the authors of the Federal Reserve Act, Robert Latham Owen (1919, p. 24), the Federal Reserve was established to "provide a means by which periodic panics which shake the American Republic and do it enormous injury shall be stopped."

At the time, the standard view of financial panics was that they were triggered when the needs of business and agriculture for liquid funds outstripped the available supply — as when seasonal plantings or shipments of crops had to be financed, for example— and that panics were further exacerbated by the incentives of banks and private individuals to hoard liquidity during such times (Warburg 1914). The new institution was intended to relieve such strains by providing an "elastic" currency: that is, by providing liquidity as needed to individual member banks through the discount window. Commercial banks, in turn, would then be able to accommodate their customers. Interestingly, although congressional advocates hoped the creation of the Fed would help prevent future panics, they did not fully embrace the idea that the Fed should help end ongoing panics by serving as lender of last resort, as had been famously recommended by the British economist and writer Walter Bagehot (1873 [1897]), the source of the classic dictum that central banks should address panics by lending freely at a penalty rate (see also Willis 1923, p. 1407; Carlson and Wheelock 2012; Bordo and Wheelock 2013). Instead, legislators imposed limits on the Federal Reserve's ability to lend in response to panics, for example, by denying nonmember banks access to the discount window and by restricting the types of collateral that the Fed could accept.

Soon after the Federal Reserve was founded in 1913, its mission shifted to supporting the war effort and then to managing the unwinding of that support. The year 1923 was thus one of the first in which the Federal Reserve confronted normal peacetime financial conditions, and it took the opportunity to articulate its views on the appropriate conduct of policy in such conditions in the Tenth Annual Report of the Federal Reserve Board (Board of Governors 1924).

The framework that the Federal Reserve employed in these early years to promote financial stability reflected in large measure the fact that the United States was on the gold standard as well as the influence of the so-called "real bills" doctrine.

In the real bills doctrine, the Federal Reserve saw its function as meeting the needs of business for liquidity— consistent with the idea of providing an elastic currency— with the ultimate goal of supporting financial and economic stability. When business activity was increasing, the Federal Reserve would seek to accommodate the need for credit by supplying liquidity to banks; when business was contracting and less credit was needed, the Fed would then reduce the liquidity in the system. The policy framework of the Fed's early years has been much criticized in retrospect. Economic historians have pointed out that, under the real bills doctrine, the Fed increased the money supply precisely at those times at which business activity and upward pressures on prices were strongest; that is, monetary policy was procyclical. Thus, the Fed's actions tended to increase rather than decrease the volatility in economic activity and prices (Friedman and Schwartz 1963; Humphrey 1982; Meltzer 2003).

As noted, the Federal Reserve pursued its real bills approach in the context of the gold standard. In the 1920s, Federal Reserve notes were redeemable in gold on demand, and the Fed was required to maintain a gold reserve equal to 40 percent of outstanding notes. In principle, the gold standard should limit discretion by monetary policymakers, but in practice US monetary policy did not appear to be greatly constrained in the years after the Fed's founding. Indeed, the large size of the US economy, together with the use of market interventions that prevented inflows and outflows of gold from being fully translated into changes in the domestic money supply, gave the Federal Reserve considerable scope during the 1920s to conduct monetary policy according to the real bills doctrine without much hindrance from the gold standard.

I've discussed the original mandate and early policy framework of the Federal Reserve. What about its accountability to the public? When the Federal Reserve was established, the question of whether it should be a private or a public institution was highly contentious. The compromise solution created a hybrid Federal Reserve System. The system was headed by a federally appointed Board of Governors, which initially included the Secretary of the Treasury and the Comptroller of the Currency. However, the 12 regional Reserve Banks were placed under a mixture of public and private oversight, including board members drawn from the private sector, and they were given considerable scope to make policy decisions that applied to their own districts. For example, Reserve Banks were permitted during this time to set their own discount rates, subject to a minimum set by the Board of Governors.

While the founders of the Federal Reserve hoped that this new institution would provide financial and hence economic stability, the policy framework and the institutional structure would prove inadequate to the challenges the Fed would soon face.

The Great Depression

The Great Depression was the Federal Reserve's most difficult test. Tragically, the Fed failed to meet its mandate to maintain financial stability. In particular, although the Fed provided substantial liquidity to the financial system following the 1929 stock market crash, its response to the subsequent banking panics of the 1930s was limited at best; the widespread bank failures and the collapse in money and credit that ensued were major sources of the economic downturn. Bagehot's dictum to lend freely at a penalty rate in the face of panic appeared to have few adherents at the Federal Reserve of that era (Friedman and Schwartz 1963).

Economists have also identified a number of instances from the late 1920s to the early 1930s when Federal Reserve officials, in the face of the sharp economic contraction and financial upheaval, either tightened monetary policy or chose inaction. Some historians trace these policy mistakes to the early death in 1928 of Benjamin Strong, Governor of the Federal Reserve Bank of New York, which left the decentralized system without an effective leader (for example, Friedman and Schwartz 1963, chapter 7). This hypothesis, whether valid or not, raises the interesting question of what intellectual framework an effective leader would have drawn on at the time to develop and justify a more activist monetary policy. The degree to which the gold standard actually constrained US monetary policy during the early 1930s is debated; but, in any case, the gold standard philosophy clearly did not encourage the sort of highly expansionary policies that were needed. The same can be said for the real bills doctrine, which apparently led policymakers to conclude, on the basis of low nominal interest rates and low borrowings from the Fed, that monetary policy was appropriately supportive and that further actions would be fruitless (Meltzer 2003; Romer and Romer 2013). Historians have also noted the prevalence at the time of yet another counterproductive doctrine: the so-called "liquidationist view" that depressions perform a necessary cleansing function (as discussed, for example, in DeLong 1990). It may be that the Federal Reserve suffered less from lack of leadership in the 1930s than from the lack of an intellectual framework for understanding what was happening and what needed to be done.

The Fed's inadequate policy framework ultimately collapsed under the weight of economic failures, new ideas, and political developments. The international gold standard was abandoned during the 1930s. The real bills doctrine lost prestige after the disaster of the 1930s; for example, the Banking Act of 1935 amended section 12A(c) of the Federal Reserve Act so as to instruct the Federal Reserve to use open market operations with consideration of "the general credit situation of the country," not just to focus narrowly on short-term liquidity needs. The Congress also expanded the Fed's ability to provide credit through the discount window, allowing loans to a broader array of counterparties, secured by a broader variety of collateral.

The experience of the Great Depression had major ramifications for all three aspects of the Federal Reserve I am discussing: its goals, its policy framework, and its accountability to the public. With respect to goals, the high unemployment of the Depression—and the fear that high unemployment would return after World War II— elevated the maintenance of full employment as a goal of macroeconomic policy. The Employment Act of 1946 made the promotion of employment a general objective for the federal government. Although the Fed did not have a formal employment goal until the Federal Reserve Reform Act of 1977 codified "maximum employment," along with "stable prices," as part of the Fed's so-called dual mandate, earlier legislation nudged the central bank in that direction.7 For example, legislators described the intent of the Banking Act of 1935 as follows: "To increase the ability of the banking system to promote stability of employment and business, insofar as this is possible within the scope of monetary action and credit administration" (US Congress 1935). At the same time, the Federal Reserve became less focused on its original mandate of preserving financial stability, perhaps in part because it felt superseded by the creation during the 1930s of the Federal Deposit Insurance Corporation and the Securities and Exchange Commission, along with other reforms intended to make the financial system more stable.

In the area of governance and accountability to the public, policymakers also recognized the need for reforms to improve the Federal Reserve's structure and decision-making. The Banking Act of 1935 simultaneously bolstered the legal independence of the Federal Reserve and provided for stronger central control by the Federal Reserve Board. In particular, the act created the modern configuration of the Federal Open Market Committee (FOMC), giving the Board the majority of votes on the Committee, while removing the Secretary of the Treasury and the Comptroller of the Currency from the Board. In practice, however, the US Treasury continued to have considerable sway over monetary policy after 1933, with Meltzer (2003) describing the Fed as "in the back seat." During World War II, the Federal Reserve used its tools to support the war financing efforts by holding interest rates and government borrowing costs low. Even after the war, Federal Reserve policy remained subject to considerable Treasury influence. It was not until the 1951 Accord with the Treasury that the Federal Reserve began to recover genuine independence in setting monetary policy.

The Great Inflation and Disinflation

Once the Federal Reserve regained its policy independence, its goals centered on the price stability and employment objectives laid out in the Employment Act of 1946. In the early post–World War II decades, the Fed used open market operations and the discount rate to influence short-term market interest rates; the federal funds interest rate (that is, the interest rate that depository institutions pay each other for loans, usually overnight, to make sure that they hold sufficient reserves at the Fed) gradually emerged as the preferred target for conducting monetary policy. Low and stable inflation was achieved for most of the 1950s and the early 1960s. However, beginning in the mid-1960s, inflation began a long climb upward, partly because policymakers proved to be too optimistic about the economy's ability to sustain rapid growth without inflation (for discussion, see Orphanides 2003; Meltzer 2009a).

Two mechanisms might have mitigated the damage from that mistaken optimism. First, a stronger policy response to rising inflation—more like that observed in the 1950s — certainly would have helped (Romer and Romer 2002b). Indeed, empirical estimates of the response of the federal funds rate to inflation for the 1970s generally show only a weak reaction (Judd and Rudebusch 1998; Taylor 1999a; Clarida, Galí, and Gertler 2000). Second, Fed policymakers could have reacted to continued high readings on inflation by adopting a more realistic and less optimistic assessment of the economy's productive potential (Lars Svensson in the discussion following Stokey 2003, p. 63). Instead, policymakers chose to emphasize so-called cost-push and structural factors as sources of inflation and saw wage- and price- setting as having become insensitive to economic slack (for example, Poole 1979; Romer and Romer 2002a, 2013; Bernanke 2004; Nelson 2005). This perspective, which contrasted sharply with Milton Friedman's (1963, p. 17) famous dictum that "inflation is always and everywhere a monetary phenomenon," led to Fed support for measures such as wage and price controls rather than monetary solutions to address inflation. A further obstacle was the view among many economists during the 1970s, as discussed in DeLong (1997) and Taylor (1997), that the gains from low inflation did not justify the costs of achieving it.

The consequence of the monetary framework of the 1970s was two bouts of double-digit inflation during that decade. Moreover, by the end of the decade, lack of commitment to controlling inflation had clearly resulted in inflation expectations becoming "unanchored," or unstable, with high estimates of trend inflation embedded in longer-term interest rates.

Under the leadership of Chairman Paul Volcker, the Federal Reserve in 1979 fundamentally changed its approach to the issue of ensuring price stability. This change involved an important rethinking on the part of policymakers. By the end of the 1970s, Federal Reserve officials increasingly accepted the view that inflation is a monetary phenomenon, at least in the medium and longer term; they became more alert to the risks of excessive optimism about the economy's potential output; and they placed renewed emphasis on the distinction between real—that is, inflation- adjusted— and nominal interest rates (for discussion, see Meltzer 2009b). The change in policy framework was initially tied to a change in operating procedures that put greater focus on growth in bank reserves, but the critical change—the willingness to respond more vigorously to inflation—endured even after the Federal Reserve resumed its traditional use of the federal funds rate as the policy instrument (Axilrod 1982). The new regime also reflected an improved understanding of the importance of providing a firm anchor for the inflation expectations of the private sector, secured by the credibility of the central bank. Finally, it entailed a changed view about the dual mandate, in which policymakers regarded achievement of price stability as helping to provide the conditions necessary for sustained maximum employment (Lindsey, Orphanides, and Rasche 2005).

The Great Moderation

Volcker's successful battle against inflation set the stage for the so-called Great Moderation of 1984 to 2007, during which the Fed enjoyed considerable success in achieving both objectives of its dual mandate. Financial stability remained a goal, of course. The Federal Reserve monitored threats to financial stability and responded when the financial system was upset by events such as the 1987 stock market crash and the terrorist attacks of 2001. More routinely, the Fed shared supervisory duties with other banking agencies. Nevertheless, for the most part, financial stability did not figure prominently in monetary policy discussions during these years. In retrospect, it is clear that, during that period, macroeconomists— both inside and outside central banks —relied too heavily in their modeling and analysis on variants of the so-called Modigliani and Miller (1958) theorem, which shows that —under a number of restrictive assumptions — the value of a firm is not related to how that firm is financed. Influenced by the logic of Modigliani–Miller, many monetary economists and central bankers concluded that the details of the structure of the financial system could be largely ignored when analyzing the behavior of the broader economy.

An important development of the Great Moderation was the increasing emphasis that central banks around the world put on communication and transparency, as economists and policymakers reached consensus on the value of communication in attaining monetary policy objectives (Woodford 2005). Federal Reserve officials, like those at other central banks, had traditionally been highly guarded in their public pronouncements. They believed, for example, that the ability to take markets by surprise was important for influencing financial conditions (for example, Goodfriend 1986; Cukierman and Meltzer 1986). Although Fed policymakers of the 1980s and early 1990s had become somewhat more explicit about policy objectives and strategy (Orphanides 2006), the same degree of transparency was not forth- coming on monetary policy decisions and operations. The release of a post-meeting statement by the Federal Open Market Committee, a practice that began in 1994, was therefore an important watershed. Over time, these statements were expanded to include more detailed information about the reason for the policy decision and an indication of the balance of risks (Lindsey 2003).

In addition to improving the effectiveness of monetary policy, these developments in communications also enhanced the public accountability of the Federal Reserve. Accountability is, of course, essential for continued policy independence in a democracy. Moreover, central banks that are afforded policy independence in the pursuit of their mandated objectives tend to deliver better economic outcomes (Alesina and Summers 1993; Debelle and Fischer 1994).

One cannot look back at the Great Moderation today without asking whether the sustained economic stability of the period somehow promoted the excessive risk-taking that followed. The idea that this long period of relative calm lulled investors, financial firms, and financial regulators into paying insufficient attention to risks that were accumulating must have some truth in it. I don't think we should conclude, though, that we therefore should not strive to achieve economic stability. Rather, the right conclusion is that, even in (or perhaps, especially in) stable and prosperous times, monetary policymakers and financial regulators should regard safeguarding financial stability to be of equal importance as—indeed, a necessary prerequisite for—maintaining macroeconomic stability.

Macroeconomists and historians will continue to debate the sources of the remarkable economic performance during the Great Moderation: for a sampling of the debate, one might start with Stock and Watson (2003); Ahmed, Levin, and Wilson (2004); Dynan, Elmendorf, and Sichel (2006); and Davis and Kahn (2008). My own view is that the improvements in the monetary policy framework and in monetary policy communication, including, of course, the better management of inflation and the anchoring of inflation expectations, were important reasons for that strong performance. However, we have learned in recent years that while well- managed monetary policy may be necessary for economic stability, it is not sufficient.

The Financial Crisis, the Great Recession, and Today

It has now been about six years since the first signs of the financial crisis appeared in the United States in 2007, and the economy still has not fully recovered from its effects. What lessons should we take for the future from this experience, particularly in the context of a century of Federal Reserve history?

The financial crisis and the ensuing Great Recession reminded us of a lesson that we learned both in the nineteenth century and during the Depression, but had forgotten to some extent, which is that severe financial instability can do grave damage to the broader economy. The implication is that a central bank must take into account risks to financial stability if it is to help achieve good macroeconomic performance. Today, the Federal Reserve sees its responsibilities for the maintenance of financial stability as coequal with its responsibilities for the management of monetary policy, and we have made substantial institutional changes in recognition of this change in goals. In a sense, we have come full circle, back to the original goal of the Federal Reserve of preventing financial panics (Bernanke 2011).

How should a central bank seek to enhance financial stability? One means is by assuming the lender-of-last-resort function that Bagehot (1873 [1897]) understood and described 140 years ago, under which the central bank uses its power to provide liquidity to ease market conditions during periods of panic or incipient panic. The Fed’s many liquidity programs played a central role in containing the crisis of 2008 to 2009. However, putting out the fire is not enough; it is also important to foster a financial system that is sufficiently resilient to withstand large financial shocks. Toward that end, the Federal Reserve, together with other regulatory agencies and the Financial Stability Oversight Council, is actively engaged in monitoring financial developments and working to strengthen financial institutions and markets. The reliance on stronger regulation is informed by the success of New Deal regulatory reforms, but current reform efforts go even further by working to identify and defuse risks not only to individual firms but to the financial system as a whole, an approach known as “macroprudential regulation.”

Financial stability is also linked to monetary policy, though these links are not yet fully understood. Here the Fed’s evolving strategy is to make monitoring, supervision, and regulation the first line of defense against systemic risks; to the extent that risks remain, however, the Federal Open Market Committee strives to incorporate these risks in the cost–benefit analysis applied to all monetary policy actions (Bernanke 2002).

What about the monetary policy framework? In general, the Federal Reserve’s policy framework inherits many of the elements put in place during the Great Moderation. These features include the emphasis on preserving the Fed’s inflation credibility, which is critical for anchoring inflation expectations, and a balanced approach in pursuing both parts of the Fed’s dual mandate in the medium term. We have also continued to increase the transparency of monetary policy. For example, the Federal Open Market Committee's communications framework now includes a statement of its longer-run goals and monetary policy strategy. In the statement issued January 25, 2012, the Committee indicated that it judged that inflation at a rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures) is most consistent over the longer run with the FOMC's dual mandate. FOMC participants also regularly provide estimates of the longer-run normal rate of unemployment; those estimates currently have a central tendency of 5.2 to 6.0 percent. By helping to anchor longer-term expectations, this transparency gives the Federal Reserve greater flexibility to respond to short-run developments. This framework, which combines short-run policy flexibility with the discipline provided by the announced targets, has been described as constrained discretion (for example, as discussed in Bernanke and Mishkin 1997, in this journal). Other communication innovations include early publication of the minutes of FOMC meetings and quarterly post-meeting press conferences by the Chairman.

The framework for implementing monetary policy has evolved further in recent years, reflecting both advances in economic thinking and a changing policy environment. Notably, following the ideas of Svensson (2003) and others, the Federal Open Market Committee has moved toward a framework that ties policy settings more directly to the economic outlook, a so-called forecast-based approach. In a forecast-based approach, monetary policymakers inform the public of their medium- term targets — say, a specific value for the inflation rate —and attempt to vary the instruments of policy as needed to meet that target over time. In contrast, an instrument-based approach involves providing the public information about how the monetary policy committee plans to vary its policy instrument—typically, a short-term interest rate, like the federal funds interest rate—in response to economic conditions. In particular, the FOMC has released more detailed statements following its meetings that have related the outlook for policy to prospective economic developments and has introduced regular summaries of the individual economic projections of FOMC participants (including for the target value of the federal funds interest rate). The provision of additional information about policy plans has helped Fed policymakers deal with the constraint posed by the effective lower bound on short-term interest rates; in particular, by offering guidance about how policy will respond to economic developments, the Committee has been able to increase policy accommodation, even when the short-term interest rate is near zero and cannot be meaningfully reduced further (as elaborated in Yellen 2012). The Committee has also sought to influence interest rates of securities that mature farther into the future (that is, farther out on the "yield curve"), notably through its securities purchases. Other central banks in advanced economies that also confronted the situation that short-term interest rates had been lowered to their effective lower bound of near-zero percent have taken similar measures.

In short, the recent crisis has underscored the need both to strengthen monetary policy and financial stability frameworks and to better integrate the two. We have made progress on both counts, but more needs to be done. In particular, the complementarities among regulatory and supervisory policies (including macro- prudential policy), lender-of-last-resort policy, and standard monetary policy are increasingly evident. Both research and experience are needed to help the Fed and other central banks develop comprehensive frameworks that incorporate all of these elements. The broader conclusion is what might be described as the over- riding lesson of the Federal Reserve's history: that central banking doctrine and practice are never static. We and other central banks around the world will have to continue to work hard to adapt to events, new ideas, and changes in the economic and financial environment.

Literature Review Semester Project

The literature review project will be conducted on a potential thesis topic. A literature review consists of an organized summary/discussion of published scientific contributions in a particular subject area and includes a synthesis of the works presented. A summary is a recap of the important information of the source and is only a part of a literature review. A literature review is a synthesis, a re-organization, or a reshuffling to tie the literature together. The review consists of at least 50 academic sources that are summarized in tables by categories (determined by you) and a one-page summary. The summary evaluates the sources and advises the reader on the most pertinent or relevant contributions as well as papers that may have questionable results. See dropbox folder for templates and examples.

Writing a Literature Review

Step 1: Decide on a topic

On one sheet of paper, type:

1. Description of topic: 1 quarter-page length paragraph (minimum).
2. Structure of analysis: Include bullet points on main points.
3. Cite any sources.

Step 2: Gather sources on this topic. To collect your sources, I would suggest using Zotero or Bibtex file.

There are a few common ways to search for economic sources:

1. Scholar.google.com – tell citation counts and click through citations for other papers.
2. Web of science – crawls through available sources by keyword search
3. Econ lit. search – tells topic summary of papers.
4. JSTOR –for any paper, to the right of it will be other related papers.
5. Other - SSRN, NBER Working papers, ideas.repec.org, AEA database.

For inter-disciplinary assistance, set appointments with disciplines' professors.

Step 3: Categorize the papers into similar themes and summarize them in tables for each category.

- Use excel or another program that can be used to construct tables.
- Include theories, methodologies, relationships to other works (debates), and key findings.
- Categorize the papers by key determinants.
  - How do the papers relate?
  - Are there major debates or 'sides'?
  - Are there relationships between the contributions?
  - Are there branches/extensions?

Category	Subcategory	Author(s)	Data	Statistical Method	Variable(s) of interest	Outcome Variable(s)	Assumption(s)	Result(s)	Implication(s)

Step 4: Construct paper outline

- Based on organization of sources from *Step 3*
- Review categories and determine if adjustments need to be made.

Step 5: Write the 3-4 page literature review summary

- Introduction
  - Introduce topic
  - Include a thesis statement
  - Methods and standards for literature review
  - Current situation, history of field, and any caveats that the reader should know
  - Explain organization (*Step 4*)
- Provide key findings in body paragraphs
  - Use evidence and quotes
  - Be selective
  - Keep your own voice and don't cut and paste large volumes of other's work
- Conclusion
  - Highlight what is important for the reader
  - Include questions for future research (typically motivation for your own thesis).

After writing the rough draft, evaluate your paper outline and decide if you have overemphasized or underemphasized certain categories.

Specifications on review summary

The literature review is a technical paper that is written for Economists. This implies that you will need to combine technical writing skills with the language of economics (including economic terms instead of plain English). For example, Economists do not say "willing to sacrifice" but rather "forgo" or "opportunity cost". The more you write like an Economist, the more succinct your writing will be for your target audience.

Specific Formatting

Times New Roman Font
12 point font
1.5 spacing
Justified paragraphs
1" margins
No spaces before/after paragraphs
Page numbers on bottom right (excluding title page) of same font and font size
Footnotes in the same font style as text.
Proper citations in APA and alphabetize reference citations.
Separate title page which includes your name, title of literature review, and date.
Title all figures and tables and reference them in your paper.

Suggestions

Below are some suggestions from reading students work over the years that I think may be quite beneficial as you construct your literature review.

- Your target readers are Economists with Ph.Ds. Write to your audience.
- Do not speak in past or future tense, only present tense. The paper is now and all studies are as if they happened now.
- Never involve the reader. Never use <u>We</u>, <u>You</u>, <u>Us</u>, <u>Our</u>.
- <u>Contrary to popular belief</u>, <u>As you know</u> and other phrases are unacceptable. Most in academia are not persuaded by any popular beliefs. Remember your audience.
- Generalizations are plagiarism and typically start with <u>it goes without saying…</u> or <u>it is commonly understood</u>. Nothing goes without saying by someone and you have to properly give them credit.
- Quotation marks are used to quote someone, not emphasize a word or phrase. Emphasizing should be done using italics or underline only.
- Don’t quote a phrase that can be twisted however the writer choses (typically quoting just a few words), e.g., speaking on the U.S. economy’s ability to rebound, Ben Bernanke recently said that “it will be rough.”
- Rhetoric is not permitted.
- Don’t make judgment calls on ideas, e.g., <u>a good idea…</u>, <u>a thought-provoking point…</u>
- Don’t use <u>So</u> to begin a sentence.
- Tables do not typically span two pages and they certainly don’t get cut off half way through one. Use landscape page layout if you find that they are too long.
- All sections and paragraphs must have an introduction. Never start a paragraph with an author’s name or a table unless it somehow is the introduction to the paragraph.
- Any side comments are footnotes and are not included in the main text.
- Do not use the word <u>thing</u> and <u>things</u>. Everything has a name, use it.
- One idea per paragraph.
- <u>Very</u>, <u>a lot</u>, <u>works well</u>, among many others are never used to explain a model or idea.
- There are no people in economics. There are households, firms, policymakers, individuals, etc.

Literature Review Rubric

Title page		Title, name, and date	5%
3-4 pg. summary	**Introduction**	The introduction is engaging and states how literature review was conducted (e.g., what was the search criteria and main topics explored).	15%
	Thesis Statement	Clearly and concisely states the paper's purpose in a single sentence (engaging, and thought provoking).	
	Organization of tables	Discusses logical sequencing of tables and elaborates on how papers are categorized, i.e., previews the structure of the tables.	
	Body paragraphs	Thoughtful discussion and supporting details on the literature and how the papers are collectively related.	
	Conclusion	The conclusion is engaging and indicates what was learned.	
Tables	**Organization**	Demonstrates logical and subtle sequencing of ideas.	40%
	Content	Depth and complexity of ideas supported by rich, engaging and pertinent details; evidence analysis and insights.	
Grammar and Sentence Structure	**Mechanics**	No errors in punctuation, capitalization and spelling.	25%
	Usage	No errors in sentence structure and word usage.	
	Citation	All cited works, both text and visual, are done in the correct format with no errors.	
References		APA formatting with no errors.	15%

Presentation Requirements

1. Presentation expectations

1. Each student is assigned 12-15 minutes.
2. Construct a presentation using Powerpoint, Prezi, or other presentation software.
3. Include visuals and graphics to explain points.

2. Layout of presentation is as follows

Slide 1: Title page
Slide 2: Introduce topic

- This should include both an oral and visual (if possible) descriptions of the research topic (about 2-3 minutes).
- Research topic is mandatory sentence in the introduction

For example: This literature review will analyze anti-price gouging laws, crime, and natural disasters and how they relate to each other

Slide 3: Organization of literature review

For example

1) Overview of price controls
2) Overview of natural disasters
    - Price controls during natural disasters
3) Overview of crime
    - Natural disasters and crime
    - Price controls and crime

Slide 4: Broad overview on findings

- This may include historical contexts, economic framework for the topic, various benefits and costs, other findings which are relevant to the topic

Second last slide: Conclusion

- What you learned! (about 2 minutes)

Last slide: Questions

- Permit 2 minutes for questions.

3. Tips

1. May find it helpful to write out what you are going to say and practice it a few times.
2. Can bring 3x5 cards as a helper, but you cannot read verbatim off of the cards.
3. Visual aids greatly assist in explaining what you are saying.
4. Example of previous year’s presentation will be provided in dropbox.
5. Follow presentation rubric.
6. Time your presentation as there is a penalty for going over.

Literature Review Presentation Rubric

Organization	Well-structured and presented in a logical sequence, used time wisely.	12.5%
Subject Knowledge	Show a full understanding of the topic.	12.5%
Comprehension	Able to accurately answer almost all questions about the topic.	12.5%
Clarity	Clear and easily understood.	12.5%
Value	Presentation had an exceptional amount of valuable material and was extremely beneficial to the class.	12.5%
Depth	Shows depth of thought.	12.5%
Quality of Material	Presentation materials were high quality and useful.	12.5%
Mechanics	Great volume, posture, and eye contact. Individuals are confident and engaging.	12.5%