Mode Selection for Measuring Modal Dispersion in Stokes Space

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Abstract— **The appropriate choice of mode combinations is crucial to the accuracy of modal dispersion characterization techniques. We compute quasi-orthogonal launch modes that minimize the noise error in modal dispersion vector measurements using the mode-dependent signal delay method.**

I. Introduction

Space division multiplexing (SDM) and multimode or multicore fibers (MMFs) can provide a significant link capacity upgrade that can satisfy near-to-mid-term data traffic demands [1]. SDM MMFs are designed to exhibit modal dispersion (MD) comparable to polarization-mode dispersion (PMD) [2]. Several experimental methods have been proposed for MD characterization which are similar to PMD characterization techniques for single-mode fibers (SMFs) [3]. For instance, MD can be estimated by measuring the Jones transfer matrix of MMFs using swept-wavelength interferometry [3], [4]. However, this requires a complex experimental setup based on a coherent optical receiver [3], [4]. In contrast, the mode-dependent signal delay method can be used to measure the MD of SDM MMFs using a simpler experimental setup based on direct detection [5], [6].

The MD of an N-mode MMF can be geometrically represented by a vector (the so called MD vector) [7], [8] in a generalized N^2-1 dimensional Stokes space. The MD vector can be defined in terms of the principal modes (PMs) and the corresponding differential mode group delays (DMGDs) of a MMF [7], [8].

For the measurement of the N^2-1 elements of the MD vector using the mode-dependent signal delay method, it is necessary to launch pulses using mode combinations corresponding to N^2-1 linearly-independent Stokes vectors [4], [5]. Recently, we proposed optimal launch mode combinations that minimize the noise error in the estimation of the MD vector using the mode-dependent signal delay method [8], [9]. In this paper, we revisit this topic proposing an alternative, simplified optimization procedure for computing the optimal launch modes. More specifically, we will show that, by maximizing the volume of the parallelotope formed in Stokes space by the unit vectors representing the launch states of the mode-dependent signal delay method, we simultaneously achieve the signal-to-noise ratio (SNR) optimization of the MD vector.

II. Mathematical model

In [9] and [10] we showed that, in order to minimize the noise error in the estimation of the MD vector, ideally one should select an orthonormal set of Stokes vectors as launch states. Then, the errors of the MD vector components would be uncorrelated.

Due to the incomplete coverage of the Poincaré sphere for $\mathrm{N}>2$, however, it is not possible to find a set of N^2-1 orthonormal Stokes vectors [7], [8]. Therefore, we have to select a set of N^2-1 oblique unit vectors that form a parallelotope in the generalized Stokes space. We want to maximize the volume of the parallelotope while restraining the vectors in the valid portion of the Poincaré sphere. The volume of the parallelotope is given by

$$V=\sqrt{\det[\mathbf{S}\mathbf{S}^T]}, \tag{1}$$

where we defined the auxiliary matrix $\mathbf{S}$ whose rows are the unit vectors in Stokes space $\hat{s}_1,\dots,\hat{s}_{N^2-1}$, corresponding to the launch modes and $\det(.)$ denotes the determinant of a matrix.

Due to the monotonicity of the logarithmic function, we adopt the objective function $\Xi \triangleq \ln V$ instead of (1). We use the gradient ascent method[11] for maximization of the objective function Ξ. Assume that $\mathbf{p}$ denotes a column vector formed by the coordinates of $\hat{s}_1,\dots,\hat{s}_{N^2-1}$. We start from an initial guess of the parameter vector $\mathbf{p}_0$ and take successive steps $\mathbf{p}_k$ in the direction of the gradient of the objective function $\nabla\Xi$ until we reach a local maximum

$$\mathbf{p}_{k+1}=\mathbf{p}_k+\mu_k\nabla\Xi_k\ , \tag{2}$$

where μ_k is the adaptive step size and $\nabla\Xi_k$ is the gradient of the objective function.

For the analytical calculation of the components of $\nabla\Xi_k$, we use Jacobi's formula for the derivative of the determinant of a matrix to obtain

$$\frac{\partial\Xi}{\partial p_i}=\mathrm{Tr}\left[\mathbf{A}\frac{\partial\mathbf{S}}{\partial p_i}\right]\ , \tag{3}$$

where we defined the matrix $\mathbf{A}\triangleq\mathbf{S}^{-1}$ and $\mathrm{Tr}[.]$ represents the trace operator.

Suppose that only the $m-$th Stokes vector depends on p_i. Then, only one element of the main diagonal of the matrix $\mathbf{A}\,\partial\mathbf{S}/\partial p_\mathrm{i}$ is non-zero

$$\frac{\partial\Xi}{\partial p_i}=\frac{\partial\hat{s}_m^T}{\partial p_i}\cdot\mathbf{A}_m,\qquad i=1,\dots,M \tag{4}$$

In the following, we use two different gradient ascent algorithms to compute optimal sets of N^2-1 quasi-orthogonal Stokes vectors $\hat{s}_i$ that correspond to feasible combinations of propagating modes.

In the first algorithm, we parametrize the j-th unit Jones vector $|s_j\rangle$ using $2N-2$ hyperspherical coordinates, i.e.,

$$|s_j\rangle=[\cos(\phi_{j1}),\sin(\phi_{j1})\cos(\phi_{j2})\,\mathrm{e}^{\mathrm{i}\theta_{j1}},\dots,$$
$$\sin(\phi_{j1})\cdots\sin(\phi_{jN-2})\sin(\phi_{jN-1})\,\mathrm{e}^{\mathrm{i}\theta_{jN-1}}]^\mathrm{T}$$

From this expression, we calculate the corresponding Stokes vector $\hat{s}_j$ and its derivatives with respect to ϕ_{jv} and θ_{jv}. Furthermore, we define the vector $\mathbf{p}$ that contains the coordinates ϕ_{jv} and θ_{jv} of all N^2-1 Stokes vectors . Then, we perform unconstrained optimization in a $(N^2-1)\times(2N-2)$ real space using the method of gradient ascent.

In the second algorithm, we parametrize the j-th Jones vector $|s_j\rangle=(s_{jv})_{v=1}^{N}$ using $2N$ real parameters $x_{jv}=\Re(s_{jv})$ and

$y_{jv} = \Im(s_{jv})$. Now, the parameter vector $\mathbf{p}$ contains the coordinates x_{jv} and y_{jv} of all $N^2 - 1$ Stokes vectors. The optimization takes place in a $(N^2 - 1) \times 2N$ real space, where we impose $N^2 - 1$ unit length constraints $\langle s_j | s_j \rangle = 1, j = 1, \ldots, N^2 - 1$. We use the modified update rule $\mathbf{p}_{k+1} = \text{proj}[\mathbf{p}_k + \mu_k \nabla \Xi_k]$, where proj(.) denotes the gradient portion tangential to the constraints (projected gradient ascent).

III. Results and Discussion

Fig. 1 shows that the volume of the parallelotope decreases compared to the one of the unit hypercube as the number of MMF modes increases. Different initial conditions are used for the gradient ascent algorithm: the blue, green, and red curves correspond to starting vectors proposed by Yang and Nolan[6], vectors selected from mutually unbiased bases (MUBs) [12], and symmetric, informationally-complete, positive operator valued measure (SIC-POVM) [13] vectors, respectively. SIC-POVM vectors lead to the best optimization results. This can be attributed to the fact that SIC-POVM vectors have equal pairwise angular separations in Stokes space. This symmetry helps the optimization algorithm to converge rapidly to a better set of quasi-orthogonal vectors.

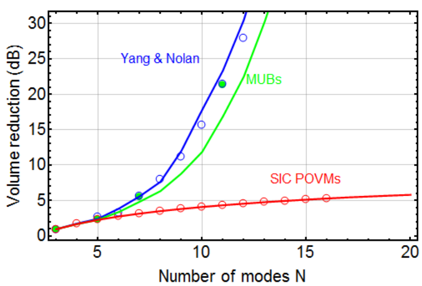
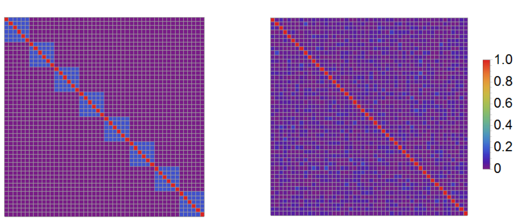


Fig. 1 Parallelotope volume decrease in dB vs. the number of modes. Initial conditions: Red: SIC-POVM vectors; Green: Vectors from MUBs; Blue: Yang and Nolan's vectors. Lines: unconstrained optimization; Points: projected gradient ascent algorithm).

One way to quantify the obliqueness of a vector set is by calculating the dot product of each pair of vectors in the set (Gram matrix). Fig.2 (a),(b) show examples of density plots of the absolute value of the Gram matrix before and after optimization. In this example, we use vectors selected from MUBs as initial conditions. Optimization yields nearly mutually perpendicular vectors, i.e., their Gram matrix is approximately a unit matrix.



(a) (b)

Fig. 2 Density plots of the Gram matrix for $N = 7$ using vectors from MUBs as initial conditions. (a) Before and (b) after optimization.

Fig. 3 shows the number of iterations required for convergence by the unconstrained gradient ascent algorithm vs the number of fiber modes.

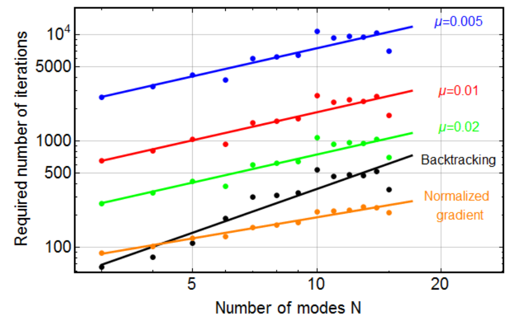


Fig. 3 Comparison of required iterations to achieve convergence vs the number of modes using SIC-POVM vectors as initial conditions ($Condition$: $\epsilon_\xi = 10^{-2}$).

Finally, Fig. 4 shows that using gradient ascent in order to maximize the volume of the parallelotope leads to a set of Stokes vectors that nearly although not quite minimize the noise-based cost function described in [10] , [11]. Therefore, volume maximization constitutes a practical alternative optimization process instead of noise minimization. This approach offers a slight gain in computational efficiency for large N's.

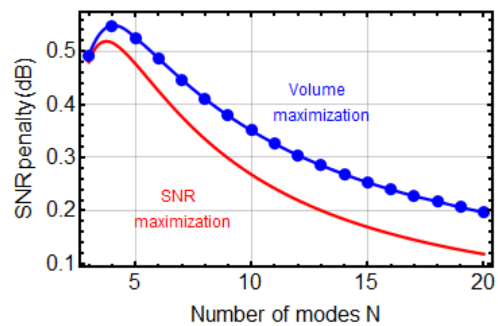


Fig. 4 SNR penalty (dB) vs number of fiber modes for SIC POVM vectors as an initial condition (Symbols: Blue: set of vectors given by volume maximization; Red: given by noise minimization).

IV. Summary

We computed optimal launch mode combinations to be used in the mode-dependent signal delay method for MD measurements. As an orthonormality metric, we used the volume of the parallelotope formed by the Stokes vectors corresponding to the launch states. Volume optimization translates into a nearly optimal SNR in the MD vector estimation.

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