Accurate Model of the Semiconductor Laser Nonuniform FM Response for the Study of Coherent Optical Systems

I. Roudas, Y. Jaouën, J. Prado, R. Vallet, *Member, IEEE*, and P. Gallion

Abstract—**This paper proposes an accurate simulation model of the semiconductor laser nonuniform FM response, to be used in the computer-aided design of coherent optical communication systems. The FM response of the laser is modeled as a recursive digital filter derived directly from measurements. Least squares fitting and the impulse invariant transformation method are used to calculate digital filter coefficients. The procedure is applied in the case of a conventional DFB laser. The calculated model of the DFB is used to study the influence of the nonuniform FM response on the performance of a coherent heterodyne CPFSK system with differential receiver operating at 1 Gb/s. Theoretical and experimental results are in excellent agreement.**

I. Introduction

Continuous Phase Frequency Shift Keying (CPFSK) is an attractive modulation format for coherent optical multichannel communication systems [1]. One of its major advantages is that it can be generated directly by injecting a small non-return to zero (NRZ) modulation current in a distributed feedback (DFB) laser. However, the FM response of conventional single-electrode DFB lasers is non-uniform, causing intersymbol interference (ISI) which degrades the performance of coherent optical systems [2]–[5].

In [2]–[5], different models were used to simulate the nonuniform FM response. The diversity of these previous approaches shows that there exists no widely acceptable simulation model.

This paper proposes an accurate, fast, and communications engineer oriented model. The principle idea of our approach is that the FM response of the laser can be approximated by a recursive digital filter derived directly from measurements. The procedure is divided into two steps: 1) measurements of the FM response are fitted by a rational function using a least squares error criterion; 2) the rational function is used to calculate the digital filter coefficients by means of the impulse invariant transformation method.

This procedure is applied in the case of a DFB laser. With this DFB model, we study the impact of the nonuniform FM response on the spectrum, the waveform at the output of the differential receiver, and the error probability of a coherent heterodyne CPFSK system operating at 1 Gb/s. The experiment verifies the theoretical results.

Manuscript received July 6, 1994.

The authors are with the Communications Department, Ecole Nationale Supérieure des Télécommunications, 75634 Paris Cedex 13, France.

IEEE Log Number 9406051.

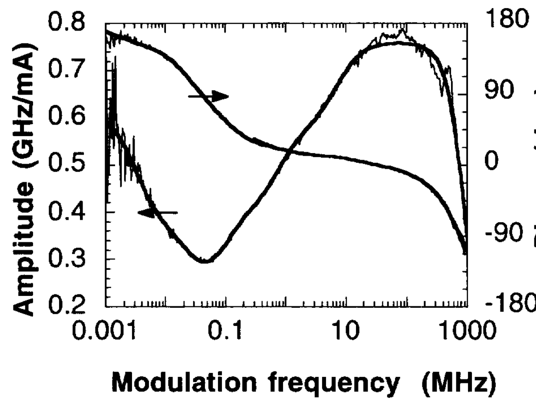

Fig. 1. Measurements (thin lines) and approximation (bold lines) of the FM response of a DCPBH-DFB laser as a function of the modulation frequency.

II. Modeling Procedure

A. Approximation

A typical FM response of a double-channel planar buried heterostructure (DCPBH) DFB laser [6] is shown in Fig. 1 (thin lines). It presents a magnitude dip in the 1 kHz to 10 MHz region and a phase transition from about 170° to less than −90° as the modulation frequency increases.

The approximation problem can be stated as follows: given a set of measurements of the amplitude and phase of the laser FM response $H(\omega)$, find a rational function $r(s)|_{s=i\omega}$ which best fits these values (the restriction to a rational function is implied by the digitization method used). From a physical point of view, $r(s)$ can be considered as the transfer function of an analog filter whose input is the laser injection current and whose output is the instantaneous optical frequency.

The choice of the approximating function is motivated by the physics of the device. The FM response is due to two different mechanisms, the temperature modulation effect and the carrier density modulation effect [6]. The former dominates at low modulation frequencies and the latter at high modulation frequencies. Under small signal approximation the two effects simply add and we can write $r(s) = r_C(s) + r_T(s)$ where $r_C(s)$, $r_T(s)$ denote the carrier and thermal FM contributions respectively.

The transfer function of the carrier induced FM $r_C(s)$ can be derived from rate equations [6]. It is well known that $r_C(s)$ is

1041-1135/94$04.00

uniform until several gigahertz and presents a magnitude peak at the relaxation resonance frequency. In practice, however, the parasitics of the laser drive circuit and package, limit the laser modulation bandwidth and cause a roll-off at high frequencies as shown in Fig. 1. Assuming that the parasitics form an equivalent low-pass filter with a series inductance and resistance and a shunt capacitance $r_C(s)$ can be written in the form

$$r_C(s; a, b, c) = \frac{a}{s^2 + bs + c} \tag{1}$$

where a, b, c are positive real adjustable parameters.

Several analytical approximations have been proposed for the transfer function of the thermal induced FM $r_T(s)$ [5]–[8]. To our experience, the model proposed by [5] gives the most satisfactory approximation in all cases. The inconvenience of this model is that $r_T(s)$ is an irrational function, rendering the digitization very difficult.

For this reason a new model for $r_T(s)$ is proposed here. The function $r_T(s)$ must be lowpass, stable and Hermitian symmetric. A possible form which satisfies the above requirements and is also convenient for digitization is

$$r_T(s; M, \mathbf{r}, \mathbf{p}) = \sum_{i=1}^{M} \frac{r_i}{s - p_i} \qquad r_i, p_i \in R, p_i < 0 \tag{2}$$

where the number of fractions M, the residues $\mathbf{r}$ and the poles $\mathbf{p}$ are adjustable parameters.

This form provides always a very efficient approximation of the thermal FM response. To see this, we can rewrite $r(s)$, at the low frequency region (far below the high-frequency roll-off), in the form

$$r(s) \simeq \sum_{i=1}^{M} \frac{r_i}{s - p_i} + \frac{a}{c} = \frac{a}{c} \prod_{i=1}^{M} \frac{s - z_i}{s - p_i} \tag{3}$$

where $\mathbf{z}$ are the zeros of $r(s)$ at low frequencies.

The last equality in (3) can be used to make an asymptotic Bode plot. From this, it is relatively easy to find approximately M, $\mathbf{z}$, $\mathbf{p}$ in order to create a staircase approximation of the magnitude dip. By alternating real poles and zeros, it is possible to create each side of the dip. By placing two successive zeros it is possible to create the bottom of the dip. The major advantage of this method in comparison with [6]–[8] is that any dip with arbitrary depth and side slopes can be approximated.

The least squares algorithm by Levenberg–Marquardt [9] is used for the calculation of the adjustable parameters of $r_C(s)$, $r_T(s)$. As the two modulation effects act in different frequency regions, $r_C(s)$ and $r_T(s)$ are fitted successively and independently. In the high frequency region, it is sufficient to approximate only the magnitude of the FM response, since $r_C(s)$ is a minimum phase all-pole causal filter and its magnitude and phase are related by the Bayard–Bode relationships [10]. In the low frequency region, we approximate the measurements with $r(s)$ as given by (3), in order to calculate the parameters M, $\mathbf{r}$, $\mathbf{p}$. It is necessary to approximate both the magnitude and the phase of $r(s)$ since the zeros are not a priori in the left half of the s-plane [10]. The initial conditions can be derived analytically for the carrier FM response and by Bode plots for the thermal FM response.

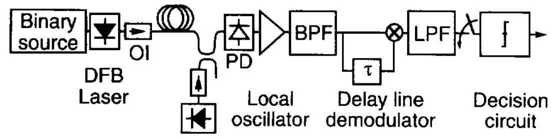

Fig. 2. Block diagram of a coherent heterodyne CPFSK system with differential receiver operating at 1 Gb/s. (Abbreviations: OI = optical isolator, PD = photodiode, BPF = bandpass filter, LPF = lowpass filter, τ = delay).

B. Digitization

The digitization problem can be stated as follows: given the rational transfer function of an analog filter $r(s)$, find a digital filter with the same characteristics.

Several methods for digitizing an analog filter exist in the literature. We have used a technique called *impulse invariant transformation* [11]. The most attractive property of the impulse invariant transformation is that it preserves both magnitude and phase characteristics of the analog filter by adequate choice of the sampling period T_s.

The straightforward application of the procedure described in [11] leads to the following recursive relations for the instantaneous optical frequency $f(nT_s)$

$$f(nT_s) = T_s \sum_{i=1}^{M+2} f_i(nT_s) \tag{4}$$

$$f_i(nT_s) = r_i i(nT_s) + e^{p_i T_s} f_i[(n-1)T_s] \tag{5}$$

where $i(nT_s)$ is the instantaneous injection current, $f_i(nT_s)$ are auxiliary variables for the recursion with $f_i(0) = 0$, and r_{M+1}, r_{M+2}, p_{M+1}, p_{M+2}, are the residues and poles respectively resulting from the partial fraction expansion of $r_C(s)$.

III. Results

In Fig. 1, the amplitude and the phase of the transfer function of the digital filter $H_d(z)|_{z=e^{i\omega T_s}}$ (bold lines) are plotted together with the measurements (thin lines). The number of fractions of $r_T(s)$ is $M = 5$. The sampling rate was chosen equal to 100 GHz in order to avoid aliasing.

The above digital filter is used to simulate the nonuniform FM response of the transmitter laser in a coherent heterodyne CPFSK system with differential receiver (Fig. 2). Experimental results are obtained with the set-up described in [12].

In Fig. 3 we compare theoretical (bold line) and experimental (thin line) spectra of the CPFSK at a bit rate $R_b = 1$ Gb/s and for a modulation index $m = 1$. In this case, an ideal CPFSK spectrum consists of a central lobe $3R_b$ large and two discrete spectral lines at $\pm R_b/2$ around the central frequency [13]. The effect of the nonuniform FM response is that the discrete lines have been smeared and the separation between the central and the secondary lobes has disappeared. Note that the experimental spectrum is slightly asymmetric. This is due to the residual AM modulation of the optical signal which enhances the spectrum at lower frequencies. It is possible to

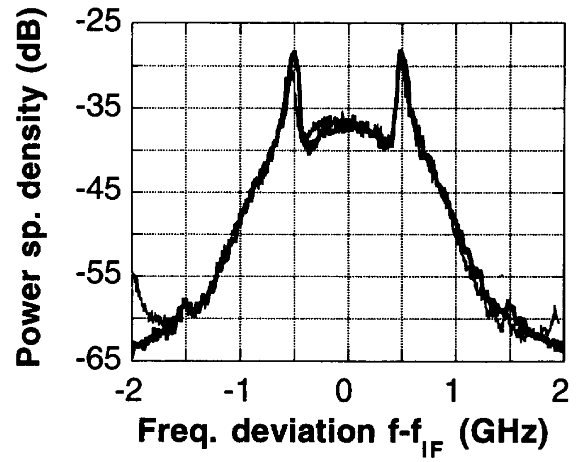

Fig. 3. Theoretical (bold line) and experimental (thin line) spectra of the CPFSK.

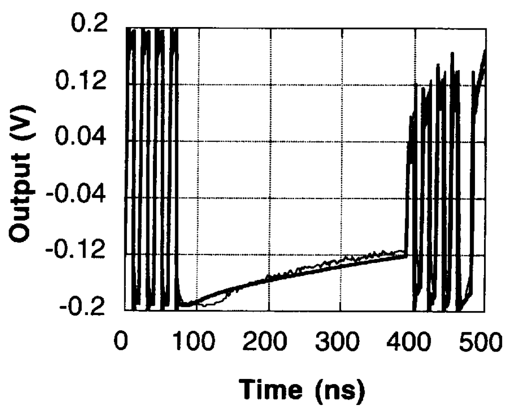

Fig. 4. Theoretical (bold line) and experimental (thin line) output waveform of the differential receiver.

incorporate this spurious effect in the simulation, modeling the AM response by an additional digital filter designed with the procedure described in Section II.

Comparison between theoretical (bold line) and experimental (thin line) waveforms at the output of the differential receiver is shown in Fig. 4. The transmitted sequence contains 32 consecutive zeros. The bit rate for this measurement is equal to $R_b = 100$ Mb/s. Obviously, the nonuniform FM response causes a reduction of the amplitude of the output signal whenever a long sequence of consecutive 1 or 0 is transmitted.

The above results show that the proposed simulation model of the nonuniform FM response can be a very efficient tool in the computer-aided design of coherent optical systems. The simulation can include a semi-analytical technique [14] in order to evaluate low error probabilities ($\sim 10^{-9}$). With this technique it is found that the sensitivity penalties at 10^{-9} in comparison with an ideal CPFSK system are 2.8 dB for the sequence 1010, and 4 dB for the sequence 2^7-1. A probability of error equal to 10^{-9} can not be achieved with a sequence of $2^{15}-1$ because of the appearance of an error floor.

IV. Conclusion

This letter presents an accurate simulation model of the nonuniform FM response of semiconductor lasers for coherent optical systems design. The modeling procedure is applied in the case of a conventional DFB laser. The influence of the nonuniform FM response on the spectrum, the waveform at the output of the receiver, and the error probability of a coherent optical CPFSK system is studied both theoretically and experimentally. The excellent agreement between theoretical and experimental results confirms the validity of the approach.

References

[1] R. A. Linke and A. H. Gnauck, "High-capacity coherent lightwave systems," *IEEE J. Lightwave Technol.*, vol. LT-5, pp. 1750–1769, Nov. 1988.
[2] S. B. Alexander, D. Welford, and D. vL. Marquis, "Passive equalization of semiconductor diode laser frequency modulation," *IEEE J. Lightwave Technol.*, vol. 7, pp. 11–23, Jan. 1989.
[3] R. S. Vodhanel, A. F. Elferaie, M. Z. Iqbal, R. E. Wagner, J. L. Gimlett, and S. Tsuji, "Performance of directly modulated DFB lasers in 10 Gb/s ASK, FSK, and DPSK lightwave systems," *IEEE J. Lightwave Technol.*, vol. 8, pp. 1379–1386, Sept. 1990.
[4] G. Jacobsen, K. Emura, T. Ono, and S. Yamazaki, "Requirements for LD FM characteristics in an optical CPFSK system," *IEEE J. Lightwave Technol.*, vol. 9, pp. 1113–1123, Sept. 1991.
[5] N. Caponio, P. Gambini, M. Puleo, and E. Vezzoni, "Impact of DFB-LD FM response distortion, in amplitude and phase, on FSK coherent systems," *CSELT Tech. Rep.*, vol. 19, pp. 313–316, Nov. 1991.
[6] K. Petermann, *Laser Diode Modulation and Noise.* Boston, MA: Kluwer Academic, 1989.
[7] S. Saito, O. Nilsson, and Y. Yamamoto, "Coherent FSK transmitter using a negative feedback stabilised semiconductor laser," *Elect. Lett.*, vol. 20, no. 17, pp. 703–704, Aug. 1984.
[8] G. S. Pandian and S. Dilwali, "On the thermal FM response of a semiconductor laser diode," *IEEE Photon. Technol. Lett.*, vol. 4, pp. 130–133, Feb. 1992.
[9] W. Press, B. Flannery, S. Teukolsky, and W. Vetterling, *Numerical Recipes*. Cambridge, U.K.: Cambridge Univ. Press, 1986.
[10] H. W. Bode, *Network Analysis and Feedback Amplifier Design.* Princeton, NJ: Van Nostrand, 1945.
[11] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975.
[12] Y. Jaouën, I. Roudas, and P. Gallion, "Experimental reduction of phase-noise influence for an optical CPFSK system with I. F. filtering," *Microwave and Optic. Tech. Lett.* , vol. 6, no. 16, pp. 903–905, Dec. 1993.
[13] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1989.
[14] I. Roudas, Y. Jaouën, and P. Gallion, "Optimum IF filter bandwidth for coherent optical heterodyne CPFSK differential receivers," presented at CLEO'94, Anaheim, CA, May 1994, paper CThI33.