Optimization of the mode-dependent signal delay method for the measurement of modal dispersion

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***Abstract*—The mode-dependent signal delay method can be used to estimate the modal dispersion vector of multimode fibers. We compute optimal launch modes minimizing the noise error in this estimate. The electronic SNR is improved asymptotically by almost 6 dB compared to conventional mode combinations.**

I. Introduction

THE exponential growth of internet data traffic in the years to come might eventually lead to a capacity shortage in the global fiber-optic network [1]. To avoid congestion, it is possible to increase link capacity by using spatial division multiplexing (SDM) over multimode and multicore optical fibers (jointly abbreviated below by the composite acronym SDM MMFs) [1].

Assuming that SDM MMFs will eventually become ubiquitous, accurate experimental techniques must be developed for their characterization [2]. In particular, modal dispersion in long SDM MMFs can be described by a set of orthogonal propagation modes called principal modes (PMs) and by their corresponding differential mode group delays (DMGDs) compared to the average mode group delay [3]. These quantities can be geometrically represented by a vector in a generalized Stokes space called modal dispersion (MD) vector [3], [4].

The mode-dependent signal delay method focuses on the measurement of the MD vector using an inexpensive direct-detection receiver [5]. An important question that is left unanswered in the previous articles on the mode-dependent signal delay method [5]-[7] is which launch modes must be used to measure the MD vector.

In the present paper, we answer the above question by analyzing the impact of noise on the MD vector characterization process performed by the mode-dependent signal delay method. Our analysis reveals that using a set of launch modes corresponding to maximally-orthogonal Stokes vectors minimizes the error in the estimation of the MD vector. We develop two optimization algorithms based on the gradient descent method for the selection of maximally-orthogonal launch states. Using these algorithms, we compute optimal sets of launch modes for up to $N = 40$ that maximize the signal-to-noise ratio (SNR) at the direct-detection receiver and enhance the accuracy of the mode-dependent signal delay method. For example, we show that the optimal mode combinations improve asymptotically the noise performance of the mode-dependent signal delay method by almost 6 dB compared to the set of modes proposed by Yang and Nolan [6].

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II. Mathematical model

The mode-dependent signal delay method consists in sending $N^2 - 1$ optical pulses using different combinations of modes though an $N-$mode SDM MMF and measuring the corresponding mode group delays at the fiber output. Fig. 1 shows indicative drawings of the input and output pulses in yellow and various colors, respectively. The group delays τ_{g_i} of the output pulses are related to the input MD vector $\vec{\tau}_s(\omega)$ and the unit Stokes vectors $\hat{s}_i$ representing the launch mode combinations [4].

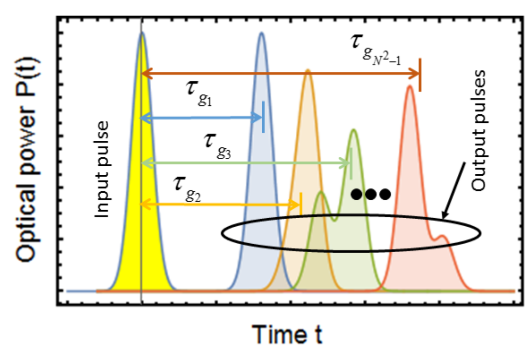

Fig. 1: Input pulse (yellow line) and output pulses (blue, orange, green, and red lines). (Symbols: τ_{g_i} = Group delays of the output pulses calculated as first moments in time).

The input MD vector $\vec{\tau}_s(\omega)$ is estimated by [7]

$$\vec{\tau}_s(\omega) = \mathbf{S^{-1}}\mathbf{T}_{\mathrm{g}}. \tag{1}$$

In (1), we defined the coefficient matrix

$$\mathbf{S}^T := \left[\hat{s}_1, \ldots, \hat{s}_{\ N^2-1}\right], \tag{2}$$

where the columns of $\mathbf{S}^T$ are the launch states represented by the Stokes vectors $\hat{s}_i$, $i = 1, \ldots, N^2 - 1$.

We also defined the column vector of the DMGD's compared to the average group delay τ_0

$$\mathbf{T}_{\mathrm{g}} := 2C_N^2\left[\ \tau_{g,1} - \tau_0, \ldots,\ \tau_{g,N^2-1} - \tau_0\right]^T, \tag{3}$$

where $C_N := \sqrt{N/\left[2\left(N-1\right)\right]}$.

The presence of thermal noise at the individual measurements $\tau_{g,i}$, i=1,..., $N^2 - 1$ can lead to a random offset $\delta\mathbf{T}_{\mathrm{g}}$

in the estimation of the DMGD matrix $\mathbf{T}_{\mathrm{g}}$. The mean square norm of the error in the estimated MD vector is given by [7]

$$E\left\{\|\delta\vec{\tau}_s\|^2\right\} = \sigma^2_{\delta T_g} \mathrm{Tr}\left[\mathbf{A}\mathbf{A}^T\right], \quad (4)$$

where the operator Tr(.) denotes the trace of a matrix, $\sigma^2_{\delta T_g}$ is the variance in the measurement of the individual components of the column vector $\mathbf{T}_{\mathrm{g}}$, and we set $\mathbf{A} = \mathbf{S}^{-1}$ for brevity.

Neglecting $\sigma^2_{\delta T_g}$ in (4), since it is solely dependent on the specific implementation of the direct-detection receiver, we adopt the squared Frobenius norm of $\mathbf{A}$ as a cost function

$$\xi := \|\mathbf{A}\|^2_F = \mathrm{Tr}\left[\mathbf{A}\mathbf{A}^T\right]. \quad (5)$$

We want to minimize ξ subject to the constraint that the Stokes vectors in $\mathbf{S}$ must correspond to valid combinations of modes, taking into account the incomplete coverage of the Poincaré sphere with valid states for $N > 2$ [3], [4].

Assume that ξ is a function of n real parameters $p_1, \ldots, p_n$. We can write the parameters in column vector form as

$$\mathbf{p} := [p_1, \ldots, p_n]^T. \quad (6)$$

The method of gradient descent [10] uses an iterative algorithm to calculate a minimum of the cost function ξ. Starting from a given point $\mathbf{p}^{(0)}$, it makes successive steps to points $\mathbf{p}^{(k)}$ by moving opposite to the direction of the gradient, until it reaches a local minimum:

$$\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} - \mu^{(k)} \nabla\xi\left(\mathbf{p}^{(k)}\right), \quad (7)$$

where $\mu^{(k)}$ is a positive constant (adaptive step size) [10]. The details of the optimization process are given in [11].

III. Results and discussion

Fig. 2 shows plots of the SNR penalty as a function of the number of propagation modes N in the optical fiber for various vector sets compared to the ideal albeit infeasible case of orthonormal Stokes vectors. The results of the numerical optimization are represented by the black curve with circles. Notice that the penalty is initially 0 dB for $N = 2$, reaches a maximum value for $N = 4$, and then falls monotonically to almost 0 dB for $N = 40$. The fact that the penalty is 0 dB for $N = 2$ comes as no surprise: in this case, the whole surface of the Poincaré sphere is covered with valid states. Thus, there exists an infinity of orthonormal vector sets that can be used for the measurement of the MD vector in Stokes space. For larger values of N, it is impossible to find an orthonormal set of $N^2 - 1$ Stokes vectors. For instance, for $N = 4$, we observe that there is 0.517 dB penalty with respect to the ideal case. By further increasing N, we observe a gradual reduction in penalty, reaching 0.046 dB for $N = 40$.

For comparison, we included in the same graph, three additional plots corresponding to vector sets proposed in prior literature in optical communications and quantum mechanics, namely Yang and Nolan's vectors [6], vectors selected from mutually unbiased bases (MUBs) [8], and symmetric, informationally complete, positive operator valued measure (SIC-POVM) vectors [9] in blue, green, and orange, respectively. The main advantage of these three vector sets is that there are relatively simple analytical or numerical algorithms for the evaluation of their coordinates. On the downside, they present much higher penalties than the optimal vector sets.

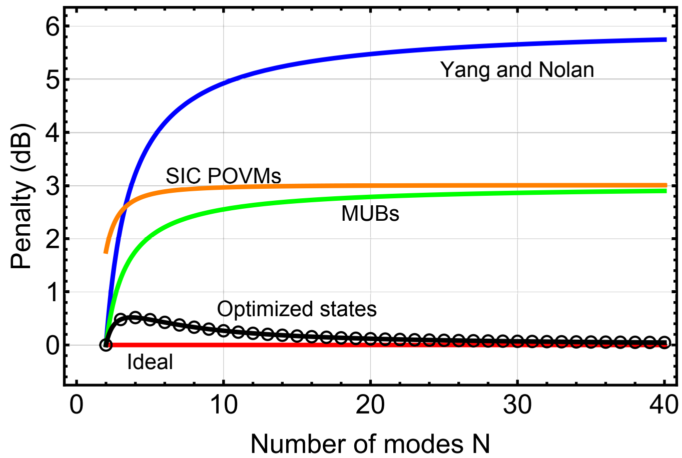

Fig. 2: SNR penalty compared to the ideal case vs the number of modes for four different vector sets (Symbols: Blue line: Yang and Nolan's vectors [6]; Green line: MUBs [8]; Orange line: SIC POVMs [9]; Red line: Orthonormal Stokes vectors; Circles: Numerical optimization).

In summary, the optimal vector sets provided by numerical optimization increase the SNR of the measurements asymptotically by 3 dB for large values of N compared to SIC POVMs and MUBs and by about 6 dB compared to Yang and Nolan's vectors.

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