Intermediate Microeconomics  
ECNS 301  
Fall 2012

Exam #: 1  
Version A

Thursday September 27, 2012

Name: ________________________________

Instructions:
You must either answer all of the following questions or answer exactly 4 of the following 5 questions. Make your choice clear. Regardless of your choice each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in the exam and your blue book(s). There are two versions of the exam. Indicate your exam version on your blue book. It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation. (20)

(a) The equilibrium price of elbow grease is $5 per kilogram, but the government has in place a price ceiling at $3 per kilogram. If technological improvements are made in the elbow grease-making process, the price of elbow grease will fall.

Solution: Uncertain. Technological improvements will increase the supply, but we don’t know by how much. For a small increase in supply, the price ceiling will still be binding and the price will not change. For a large increase in supply, the price ceiling will not be binding and the price will fall.

(b) A consumer with convex, ‘well-behaved’ indifference curves is indifferent between two bundles of X and Y: (6, 10) and (8, 10). She therefore prefers the bundle (7, 11) to either of the first two.

Solution: True. The bundle (7, 11) is preferred to bundle (6, 10) because preferences are monotonic (more is better). Since the consumer is indifferent between (6, 10) and (8, 10), and because preferences are transitive, (7, 11) is preferred to (8, 10). You could also base your answer on the fact that indifference curves are decreasing and convex.

(c) The government places a tax of $5 per unit on beeswax and we find that the price buyers pay increases by $2.50. If the tax is increased to $15, then the price buyers pay will increase by $7.50.

Solution: Uncertain. We could certainly create an example with linear demand and supply curves where this is true, but it will not be true in all cases.

(d) A decrease in income of $2 is equivalent to all prices increasing by $2.

Solution: False. The budget constraint has constant returns to scale (homogeneity of degree 1), but this only applies for multiplicative factors. Starting with a budget constraint of

\[ m = p_x x + p_y y \]

a decrease in income of $2 is

\[ m - 2 = p_x x + p_y y. \]

If all prices increased by $2, then the budget constraint is

\[ m = (2 + p_x) x + (2 + p_y) y. \]
which is equal to

\[ m = 2x + 2y + p_x x + p_y y \]
\[ m - 2x - 2y = p_x x + p_y y. \]

If a decrease in income of $2 is equivalent to all prices increasing by $2, then it must be that

\[ m - 2 = m - 2x - 2y \]
\[ 2 = 2x + 2y \]
\[ 1 = x + y. \]

We got this equation by setting the \( p_x x + p_y y \) terms equal to each other for each of the two scenarios. Note that in this example if \( x + y = 1 \), then it is uncertain instead of false, but in general the statement is false.

**Short Answer/Numerical**

2. Consider the following constrained multivariate optimization problem.

\[ \max_{x,y} f(x, y) = 13x^2 y^5 \]
\[ \text{subject to } 2x + y = 35 \]

For the parts below, always consider \( y \) to be on the vertical axis and \( x \) to be on the horizontal axis.

(a) What is the Lagrangian for this problem?

**Solution:**

\[ \max_{x,y,\lambda} \mathcal{L}(x, y, \lambda) = 13x^2 y^5 + \lambda(35 - 2x - y) \]
(b) Find the optimal values of \( x \) and \( y \).

**Solution:** Differentiate the Lagrangian with respect to \( x \), \( y \), and \( \lambda \), set each first order condition equal to zero and solve for \( x \) and \( y \).

\[
\begin{align*}
\frac{\partial L}{\partial x} &= 26xy^5 - 2\lambda = 0 \\
\frac{\partial L}{\partial y} &= 65x^2y^4 - \lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= 35 - 2x - y = 0
\end{align*}
\]

Rearrange the first first order condition to get

\[\lambda = 13xy^5\]

and rearrange the second first order condition to get

\[\lambda = 65x^2y^4.\]

Combine these two equations to get

\[\lambda = \lambda\]

\[13xy^5 = 65x^2y^4\]

\[xy^5 = 5x^2y^4\]

\[xy = 5x^2\]

\[y = 5x.\]

Substitute this into the third first order condition (the budget constraint) to get

\[35 = 2x + y\]

\[35 = 2x + 5x\]

\[35 = 7x\]

\[x^* = 5\]

\[y^* = 25 = 5x^*\]
(c) Find the optimal values of $x$ and $y$ using $\log(f(x, y))$ as the objective function where $\log$ refers to the natural logarithm.

**Solution:** The objective function we want to use is

$$\log(f(x, y)) = \log(13x^2y^5) = \log(13) + 2\log(x) + 5\log(y).$$

The Lagrangian is

$$\max_{x, y, \lambda} L(x, y, \lambda) = \log(13) + 2\log(x) + 5\log(y) + \lambda(35 - 2x - y).$$

Differentiate the Lagrangian with respect to $x$, $y$, and $\lambda$, set each first order condition equal to zero and solve for $x$ and $y$.

$$\frac{\partial L}{\partial x} = \frac{2}{x} - 2\lambda = 0$$
$$\frac{\partial L}{\partial y} = \frac{5}{y} - \lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = 35 - 2x - y = 0$$

Rearrange the first first order condition to get

$$\lambda = \frac{1}{x}$$

and rearrange the second first order condition to get

$$\lambda = \frac{5}{y}.$$ 

Combine these two equations to get

$$\lambda = \lambda$$
$$\frac{1}{x} = \frac{5}{y}$$
$$y = 5x.$$ 

Substitute this into the third first order condition (the budget constraint) to get

$$35 = 2x + y$$
$$35 = 2x + 5x$$
$$35 = 7x$$
$$x^* = 5$$
$$y^* = 25 = 5x^*$$
(d) Compare the optimal values of $x$ and $y$ you found. Explain any differences or similarities.

**Solution:** For each part the optimal values of $x$ and $y$ are $x^* = 5$ and $y^* = 25$. They are the same. This is because the natural logarithm is an increasing monotonic transformation. If you take an increasing monotonic transformation of the objective function is does not change the optimal values.

3. You go to a conference in New Orleans to present your research. After your conference duties are over you want to eat oyster po-boys and drink sazeracs (the official cocktail of New Orleans). Sazeracs cost $3 each and oyster po-boys are $7. Your lousy MSU per-diem only provides $32 for the whole trip (this is the amount of money you can spend on food and drinks). You’ve decided that while on your trip you’re going to drink 6 sazeracs and eat 2 oyster po-boys.

(a) What’s an equation describing your budget line and what’s the slope of your budget line.

**Solution:** Let the quantity of oyster po-boys you eat be $o$ and let the quantity of sazeracs you drink be $s$. From the problem we know that $m = 32$, $p_o = 7$, and $p_s = 3$. The equation for your budget line is

\[ m = p_o o + p_s s \]
\[ 32 = 7o + 3s. \]

If we solve for $s$, we get

\[ s = \frac{32}{3} - \frac{7}{3}o \]

and the slope is $-\frac{7}{3}$.

If we solve for $o$, we get

\[ o = \frac{32}{7} - \frac{3}{7}s \]

and the slope is $-\frac{3}{7}$.

(b) Now MSU changes their per-diem policy. They give you an extra $6 to spend but require you to submit receipts and charge you $1 for every drink you buy. What’s an equation describing your new budget line?

**Solution:**

\[ m + 6 = p_o o + (p_s + 1)s \]
\[ 38 = 7o + 4s. \]
(c) Now MSU decides they can’t afford the extra $6 and changes their per-diem policy again. Instead of an extra $6 dollars they’ll only give you an extra $3, but now you don’t have to submit receipts and there is no $1 per drink penalty. What’s an equation describing your new budget line?

**Solution:**

\[ m + 3 = p_o o + p_s s \]
\[ 35 = 7o + 3s. \]

(d) What per-diem policy would you prefer and why?

**Solution:** Let Policy A be the original policy, Policy B be the policy with the $1 drink tax and the extra $6, and let Policy C be the policy with the extra $3. With Policy A, you chose \( o = 2 \) and \( s = 6 \). You can still afford this bundle with Policy B, so you prefer Policy B to Policy A. You prefer Policy C to Policy A, since Policy C is an increase in income from Policy A. It’s not clear whether you prefer Policy C to Policy B or Policy B to Policy C and it depends on your preferences. There are some bundles you can afford with Policy C that you can’t afford with Policy B, and some bundles you can afford with Policy B that you can’t afford with Policy C.

4. The market supply and demand functions for a particular market are as follows. 

\[ Q = 74 - p \]
\[ Q = p - 2 \]

The government is considering a per-unit tax of \( \tau \) to be levied on buyers.

(a) What are the equilibrium prices and quantity with the tax expressed as a function of \( \tau \)?

**Solution:** With a per-unit tax of \( \tau \) levied on buyers, the price sellers get is \( p \) and the price buyers pay is \( p + \tau \). To find the equilibrium, set \( D(p + \tau) = S(p) \) and solve for \( p \).

\[ D(p + \tau) = S(p) \]
\[ 74 - p - \tau = p - 2 \]
\[ 2p = 76 - \tau \]
\[ p = 38 - \frac{\tau}{2} \]
The price sellers get is \( p = 38 - \frac{\tau}{2} \). The price buyers pay is \( p = 38 + \frac{\tau}{2} \). The market quantity is:

\[
Q = 74 - (38 + \frac{\tau}{2}) = 36 - \frac{\tau}{2}
\]
\[
Q = 38 - \frac{\tau}{2} - 2 = 36 - \frac{\tau}{2}.
\]

(b) Show that as the tax rate increases the price buyers pay increases, the price sellers get decreases and the equilibrium quantity decreases.

**Solution:** Let \( p_B \) be the price buyers pay and \( p_S \) be the price sellers get.

\[
\frac{dp_B}{d\tau} = \frac{1}{2} > 0
\]
\[
\frac{dp_S}{d\tau} = -\frac{1}{2} < 0
\]
\[
\frac{dQ}{d\tau} = -\frac{1}{2} < 0
\]

(c) What value of \( \tau \) maximizes tax revenue and how much tax revenue is generated?

**Solution:** For tax revenue, the government gets \( \tau \) for each good sold, so tax revenue is \( \tau Q \). This reduces to \( 36\tau - \frac{\tau^2}{2} \) and the optimization problem is as follows.

\[
\max_{\tau} 36\tau - \frac{\tau^2}{2}
\]

The first order condition is

\[
36 - \tau = 0
\]

so the optimal tax rate is \( \tau = 36 \). When \( \tau = 36 \), \( Q = 18 \) and tax revenue is 648.
(d) Instead of using a per-unit tax, the government decides to use an ad-valorem tax of $\tau$ to be levied on buyers. What tax rate should be set to maximize tax revenue?

**Solution:** Before when we set the tax rate to maximize tax revenue, the size of the tax was $36$, the price buyers paid was $p_B = 56$ and the price sellers got was $p_S = 20$.

Now we just need to find an ad-valorem tax levied on buyers that produces the same result. With an ad-valorem tax levied on buyers, sellers get $p$ and buyers pay $p(1 + \tau)$.

\[
\begin{align*}
 p_B &= p_S(1 + \tau) \\
 56 &= 20(1 + \tau) \\
 56 &= 20 + 20\tau \\
 36 &= 20\tau \\
 \tau &= 1.8
\end{align*}
\]

For the al-valorem tax, you should set a tax rate of 1.8 or 180%.

5. The market demand function for a particular good is

\[ Q = 50 - 2p + 8p_r \]

where $Q$ is the market quantity, $p$ is the market price, and $p_r$ is the price of a related good. The market supply curve is described by the following.

\[ Q = 6 + 2p \]

(a) Are the two goods substitutes or complements?

**Solution:** The two goods are substitutes because an increase in $p_r$ leads to an increase in the quantity demanded of that good: $\frac{dQ}{dp_r} = 8 > 0.$
(b) Find the market equilibrium price and quantity when $p_r = 7$, and when $p_r = 13$.

**Solution:** Set the quantity demanded equal to the quantity supplied and solve for the equilibrium price as a function of $p_r$.

\[
Q_D = Q_S \\
50 - 2p + 8p_r = 6 + 2p \\
50 - 6 + 8p_r = 2p + 2p \\
44 + 8p_r = 4p \\
p = 11 + 2p_r
\]

From the supply function, we know that $Q = 6 + 2p$ so that

\[
Q = 6 + 2(11 + 2p_r) = 6 + 22 + 4p_r = 28 + 4p_r.
\]

Considering the different values of $p_r$, the solution to the problem is

<table>
<thead>
<tr>
<th>$p_r$</th>
<th>$p$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>25</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>37</td>
<td>80</td>
</tr>
</tbody>
</table>

(c) What is the comparative static $\frac{dp}{dp_r}$?

**Solution:** Since we found that $p = 11 + 2p_r$, $\frac{dp}{dp_r} = 2$. 
(d) When thinking about the demand for this good, are consumers more sensitive to a change in the price, \( p \) or more sensitive to a change in the price of the related good, \( p_r \).

**Solution:** We want to compare the price elasticity of demand, \( \frac{\partial Q}{\partial p} \), to the cross price elasticity of demand, \( \frac{\partial Q}{\partial p_r} \).

The relevant slopes are

\[
\frac{\partial Q}{\partial p} = -2 \quad \frac{\partial Q}{\partial p_r} = 8
\]

and we also know that \( p = 11 + 2p_r \).

Ignoring the sign of the elasticities (we know that the price elasticity of demand is negative by the law of demand, and that the cross price elasticity of demand is positive since the goods are substitutes), compare the magnitudes of the two elasticities to see which one has the larger magnitude and thus is more sensitive. We want to compare the magnitude of \( \frac{\partial Q}{\partial p} \), to the magnitude of \( \frac{\partial Q}{\partial p_r} \). Substituting in the slopes, we’re comparing \(-2 \frac{p_r}{Q}\) to \(8 \frac{p_r}{Q}\). Since we just want compare the magnitudes, take the absolute values to get \(2 \frac{p_r}{Q}\) and \(8 \frac{p_r}{Q}\). Since the \( Q \)’s are the same in each equation, we can just compare \(2p\) to \(8p_r\). If \( p > 4p_r \), consumers will be more sensitive to a change in the price, \( p \). Additionally, we know that \( p = 11 + 2p_r \) so that when \( 11 + 2p_r > 4p_r \) or when \( 11 > 2p_r \) consumers are more sensitive to a change in the price, \( p \), and when \( 11 < 2p_r \) consumers are more sensitive to a change in the price of the related good, \( p_r \).