Intermediate Microeconomics
ECNS 301
Fall 2014

Exam #: 1
Version A

Thursday September 25, 2014

Name: ____________________________________________

Instructions:
You must answer all of the following questions. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in your blue book(s). There are two versions of the exam. Indicate your exam version on your blue book. It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation.

   (a) If a good is not produced, then there is no demand for it.

   Solution: False, demand is independent of supply.

   (b) Orange juice sells for $2 per gallon and gasoline sells for $1 per gallon. Although we don’t know the consumer’s utility function, we do know that if a consumer buys both goods, she receives twice as much utility from orange juice as from gasoline.

   Solution: False. The price difference describes the marginal rate of transformation. In equilibrium we know that the marginal rate of transformation equals the marginal rate of substitution and that the marginal rate of substitution is the ratio of marginal utilities. The statement is false because it refers to the overall level of utility rather than the marginal levels.

   (c) Indifference curves on the same indifference map can have different shapes.

   Solution: True, there’s nothing that says they have to have the same shapes. Indifference curves can’t cross, be thick, and they must be convex.

   (d) An increase in a consumer’s income will increase the marginal rate of transformation.

   Solution: False, the budget constraint shifts out, but the ratio of prices stays the same.

Short Answer/Numerical

2. The market supply and demand functions for a particular market are as follows.

   \[\begin{align*}
   Q &= 550 - 20p \\
   Q &= 5p - 50
   \end{align*}\]

   Quantitatively (with numbers) answer the following questions.

   (a) Describe the market equilibrium price, quantity, and welfare measures.

   Solution: Set demand equal to supply and solve for price.

   \[\begin{align*}
   550 - 20p &= 5p - 50 \\
   600 &= 25p \\
   p &= 24
   \end{align*}\]
When $p = 24$,

$$Q = 550 - 20(24) = 70$$
$$Q = 5(24) - 50 = 70$$

Now we need to find the welfare measures: consumer surplus, producer surplus, and total surplus. Note that for demand $550$ is the intercept on the $Q$/horizontal axis. We want to find the intercept on the $p$/vertical axis. When $Q = 0$, $20p = 550$ so $p = 27.5$. Consumer surplus is

$$CS = \frac{1}{2}(27.5 - 24)70 = 122.5.$$

Using supply when $Q = 0$, $5p = 50$, so $p = 10$. Producer surplus is

$$PS = \frac{1}{2}(24 - 10)70 = 490.$$

Total surplus is $122.5 + 490 = 612.5$. Alternatively, you can find total surplus as

$$TS = \frac{1}{2}(27.5 - 10)70 = 612.5.$$

(b) What happens to the market equilibrium when the government imposes an ad-valorem tax on sellers of 20%?

**Solution:** For an ad-valorem tax on sellers, the price for sellers is $p = p_t - \tau p_t$ where $\tau = 0.2$. We use a minus sign because the tax on sellers results in them getting a lower price. Setting demand equal to supply we get

$$550 - 20p_t = 5(p_t - 0.2p_t) - 50$$
$$600 = 24p_t$$
$$p_t = 25.$$  

When $p_t = 25$, buyer pay 25 and sellers get $25 - (0.2)25 = 20$. Putting 25 into the demand equation or 20 into the supply equation yields the equilibrium quantity with the tax:

$$Q = 550 - 20(25) = 50$$
$$Q = 5(20) - 50 = 50.$$
(c) A policy maker watches a rerun of Robin Hood on TV (the one with Kevin Costner and Morgan Freeman). In their dream that night, they get a crazy idea to steal from the rich and give to the poor. For some reason they think that firms are rich and consumers are poor. What happens to the market equilibrium when they keep the ad-valorem tax on sellers of 20% and give an ad-valorem subsidy of 20% to buyers?

**Solution:** The ad-valorem tax on sellers is modeled in the same fashion as above. The ad-valorem subsidy for buyers lowers the price they have to pay so that \( p = p_t - \tau p_t \) where \( \tau = 0.2 \). Setting demand equal to supply we get

\[
550 - 20(p_t - 0.2p_t) = 5(p_t - 0.2p_t) - 50
\]

\[
600 = 20p_t
\]

\[
p_t = 30.
\]

When \( p_t = 30 \), buyers pay \( 30 - (0.2)30 = 24 \) and sellers get \( 30 - (0.2)30 = 24 \). The equilibrium quantity is 70.

(d) As an adviser to the policy maker would you advise them to keep the Robin Hood style tax/subsidy or to just keep the ad-valorem tax on sellers? Why?

**Solution:** With the ad-valorem tax on sellers, you generate \( 50(5) = 250 \) of tax revenue, but the tax also creates a deadweight loss. With the Robin Hood style tax/subsidy, no tax revenue is generated as all tax revenue is given back to the consumers in the form of a subsidy. The Robin Hood style tax/subsidy is efficient as the free market equilibrium is achieved.

If the policy maker is more concerned with efficiency, I would advise them to use the Robin Hood style tax/subsidy. Although in this case it would be better to have no tax at all.

If the policy maker needs to generate some tax revenue, I would advise them to just keep the ad-valorem tax on sellers.
3. Your derelict cousin only buys food and lottery tickets. The price of food is $2 and the price of a lottery ticket is $1. Somehow your cousin has $100 per month to spend on food and lottery tickets.

(a) Graph and provide an equation for your cousin’s budget constraint.

\[ 2f + l = 100. \]

Let’s put \( l \) on the vertical axis and \( f \) on the horizontal. Solving for \( l \), we get \( l = 100 - 2f \). Now we need to graph this equation.

If your cousin spent all of his money on food, he could buy \( f = 50 \) food. If he spent all of his money on lottery tickets, he could buy \( l = 100 \) lottery tickets. These are the two intercepts for the budget constraint.

(b) Now your cousin gets $150 per month in food stamps, which cannot be sold. Graph or provide an equation for the new budget constraint.
**Solution:** If your cousin spent all of his money on food, he could buy \( f = 125 \) food. If he spent all of his money on lottery tickets, he could buy \( l = 100 \) lottery tickets. These are the two intercepts for the budget constraint. Using just the food stamps on food, your cousin could buy \( l = 100 \) lottery tickets and \( f = 75 \) of food. This is the point where the slope changes.

The equation for the budget constraint is as follows.

\[
l = \begin{cases} 
100 & \text{for } 0 \leq f \leq 75 \\
250 - 2f & \text{for } 75 \leq f \leq 125 
\end{cases}
\]

(c) After getting on food stamps, you think your idiot cousin is now buying more lottery tickets. Is this possible, assuming that no laws are being broken? If so, why, if not, why not?

**Solution:** Yes, but it depends on where the indifference curve is tangent to the budget constraint. It’s likely that after receiving food stamps, your cousin will consume more food and more lottery tickets. Your cousin can take some of his $100 per month and spend it on lottery tickets instead of food as the food
stamps will allow him to purchase food. If before receiving food stamps, your cousin was consuming $f = 0$ and $l = 100$, then he wouldn’t buy more lottery tickets when receiving food stamps.

(d) Your cousin now develops a scheme to sell food stamps at a 25% discount. Graph or provide an equation for the new budget constraint.

**Solution:** If you sell $150 in food stamps at a 25% discount, you could convert the food stamps into $150 \times 0.75 = 112.5$ dollars. If your cousin spent all of his money on food, he could buy $f = 125$ food. Now if he spent all of his money on lottery tickets, he could buy $l = 212.5$ lottery tickets. These are the two intercepts for the budget constraint. Using just the food stamps on food, your cousin could buy $l = 100$ lottery tickets and $f = 75$ of food. This is the point where the slope changes.

The equation for the budget constraint is as follows.

$$l = \begin{cases} 
212.5 - \frac{3}{2}f & \text{for } 0 \leq f \leq 75 \\
250 - 2f & \text{for } 75 \leq f \leq 125 
\end{cases}$$
4. A consumer has the following utility function.

\[ U(x, y) = \ln(x) + y \] (25)

Given prices \( p_x = 1 \), \( p_y = 10 \) and income \( m = 30 \):

(a) find the marginal rate of substitution,

**Solution:** You can find the marginal rate of substitution (MRS) two different ways.

First way: The marginal rate of substitution is the ratio of the marginal utilities times minus one.

\[ MRS = -\frac{\partial U}{\partial y} \frac{\partial U}{\partial x} \]

where \( \frac{\partial U}{\partial x} = \frac{1}{x} \) and \( \frac{\partial U}{\partial y} = 1 \) The MRS is

\[ MRS = -\frac{1}{x} = -\frac{1}{x} \]

or \( MRS = -x \) depending on which marginal utility was in the numerator.

Second way: Solve an equation for an indifference curve and then find the slope. For an indifference curve with a utility level of \( \bar{u} \), the equation for this indifference curve is

\[ y = \bar{u} - \ln(x) \]

and the slope of this indifference curve is

\[ \frac{\partial y}{\partial x} = -\frac{1}{x} \]

which is the MRS.

(b) find the interior solution to the consumer’s utility maximization problem,

**Solution:** The interior solution has positive amounts of \( x \) and \( y \) consumed and is found using the Lagrangian. Since the price of good \( y \) changes later on, I’m going to solve for the Lagrangian using \( p_y \) for the price of good \( y \).

The consumer’s problem is

\[ \max_{x,y} U(x, y) = \ln(x) + y \text{ subject to } x + p_y y = 30 \]

and the Lagrangian is

\[ \max_{x,y,\lambda} \mathcal{L}(x, y, \lambda) = \ln(x) + y + \lambda(30 - x - p_y y). \]
The first order conditions from the Lagrangian are as follows.

\[
\frac{\partial L}{\partial x} = \frac{1}{x} - \lambda \\
\frac{\partial L}{\partial y} = 1 - p_y \lambda \\
\frac{\partial L}{\partial \lambda} = 30 - x - p_y y
\]

Set each first order condition equal to zero. The first equation yields \( \lambda = \frac{1}{x} \) and the second equation results in \( \lambda = \frac{1}{p_y} \). Set the \( \lambda \)'s equal to each other to get \( \frac{1}{x} = \frac{1}{p_y} \) or \( x = p_y \).

The third equation is the same as the budget constraint where \( x + p_y y = 30 \).

When \( x = p_y \),
\[
x + p_y y = p_y + p_y y = p_y (1 + y) = 30.
\]
Divide by \( p_y \) so that \( 1 + y = \frac{30}{p_y} \) and then solve for \( y \) to get
\[
y = \frac{30}{p_y} - 1 = \frac{30 - p_y}{p_y}.
\]

When \( p_y = 10 \), the interior solution is \( x = 10 \) and \( y = 2 \).

(c) find the corner solutions to the consumer’s utility maximization problem,

**Solution:** There are two corner solutions and each corner solution is characterized by consuming zero of one good. When \( x = 0 \), you spend all of your money on \( y \) and you can buy \( y = 30/10 = 3 \). When \( y = 0 \), you spend all of your money on \( x \) and you can buy \( x = 30/1 = 30 \).

(d) find the optimal consumption bundle of \( x \) and \( y \).

**Solution:** We have three different bundles: one interior and two corner solutions. The optimal consumption bundle is the bundle of \( x \) and \( y \) that results in the highest utility level. The utility function is \( U(x, y) = \ln(x) + y \).

The utility of the interior solution is \( U(10, 2) = 4.3 \), the utility of the corner solution where you buy all \( x \) is \( U(30, 0) = 3.4 \) and the utility of the corner solution where you buy all \( y \) is \( U(0, 3) \) which is undefined (The log of zero approaches negative infinity). The optimal consumption bundle is \( x = 10 \) and \( y = 2 \).
(e) If the price of good $y$ is $p_y = 2$, how does your answer to part d change?

**Solution:** The interior solution is $x = 2$ and $y = (30 - 2)/2 = 14$. This has a utility level of $U(2, 14) = 14.69$. The corner solution where you buy all $x$ still results in a utility level of $U(30, 0) = 3.4$, and the corner solution where you buy all $y$ still has an undefined utility level. Our answer to part d does not change.