Intermediate Microeconomics
ECNS 301
Spring 2012

Exam #: 1
Version A

Friday February 10, 2012

Name: ____________________________

Instructions:
Answer all of the following questions. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in the exam and your blue book(s). There are two versions of the exam. Indicate your exam version on your blue book. It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
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True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation. (25)

(a) If the government increases the tax on alcohol, use of marijuana will increase.

Solution: Uncertain, it depends on whether marijuana and alcohol are substitutes or complements.

(b) A consumer with convex, ‘well-behaved’ indifference curves is indifferent between two bundles of X and Y: (6,10) and (8,6). She therefore prefers the bundle (7,7) to either of the first two.

Solution: Uncertain. Draw a line through the two bundles and (7,7) is to the southwest of the line. If the two goods are perfect substitutes, then the statement is false. However, if there’s enough curvature in the indifference curve, the statement could be true.

(c) Both Tums and Rolaids will cure David’s heartburn, and he regards them as perfect substitutes. Therefore, his indifference curves will be linear with a slope of −1.

Solution: False. With perfect substitutes, the indifference curves are linear with a negative slope, but the slope is not necessarily −1. For instance if David likes the flavor of the tropical fruit flavored tums better, the utility function could be \( u(T, R) = 10T + R \), where Tums and Rolaids are perfect substitutes but the slope is not −1.

(d) The equilibrium price of elbow grease is $5 per kilogram, but the government has in place a price ceiling at $3 per kilogram. Shoulder grease is a substitute for elbow grease. If the price of shoulder grease falls, then the shortage of elbow grease will grow larger.

Solution: False. If the price of shoulder grease falls, demand for elbow grease, a substitute, will decrease. The decrease in the demand for elbow grease causes the shortage of elbow grease to decrease.
Short Answer/Numerical

2. The market demand function for a particular good is

\[ Q = 50 - 2p + 8p_r \]

where \( Q \) is the market quantity, \( p \) is the market price, and \( p_r \) is the price of a related good. The market supply curve is described by the following.

\[ Q = 6 + 2p \]

(a) Are the two goods substitutes or complements?

**Solution:** The two goods are substitutes because an increase in \( p_r \) leads to an increase in the quantity demanded of that good: \( \frac{\partial Q}{\partial p_r} = 8 > 0 \).

(b) Find the market equilibrium price and quantity when \( p_r = 7 \), and when \( p_r = 13 \).

**Solution:** Set the quantity demanded equal to the quantity supplied and solve for the equilibrium price as a function of \( p_r \).

\[
Q_D = Q_S \\
50 - 2p + 8p_r = 6 + 2p \\
50 - 6 + 8p_r = 2p + 2p \\
44 + 8p_r = 4p \\
p = 11 + 2p_r
\]

From the supply function, we know that \( Q = 6 + 2p \) so that

\[ Q = 6 + 2(11 + 2p_r) = 6 + 22 + 4p_r = 28 + 4p_r. \]

Considering the different values of \( p_r \), the solution to the problem is

\[
\begin{align*}
p_r = 7 & \quad p_r = 13 \\
p = 11 + 2(7) = 25 & \quad p = 11 + 2(13) = 37 \\
Q = 28 + 4(7) = 56 & \quad Q = 28 + 4(13) = 80
\end{align*}
\]

(c) What is the comparative static \( \frac{dp}{dp_r} \)?

**Solution:** Since we found that \( p = 11 + 2p_r \), \( \frac{dp}{dp_r} = 2 \).
(d) When thinking about the demand for this good, are consumers more sensitive to a change in the price, \( p \) or more sensitive to a change in the price of the related good, \( p_r \).

**Solution:** We want to compare the price elasticity of demand, \( \frac{\partial Q}{\partial p} \), to the cross price elasticity of demand, \( \frac{\partial Q}{\partial p_r} \).

The relevant slopes are

\[
\frac{\partial Q}{\partial p} = -2 \quad \quad \quad \frac{\partial Q}{\partial p_r} = 8
\]

and we also know that \( p = 11 + 2p_r \).

Ignoring the sign of the elasticities (we know that the price elasticity of demand is negative by the law of demand, and that the cross price elasticity of demand is positive since the goods are substitutes), compare the magnitudes of the two elasticities to see which one has the larger magnitude and thus is more sensitive. We want to compare the magnitude of \( -2\frac{\partial p}{\partial Q} \) to \( 8\frac{\partial p_r}{\partial Q} \). Since we just want compare the magnitudes, take the absolute values to get \( 2\frac{\partial p}{\partial Q} \) and \( 8\frac{\partial p_r}{\partial Q} \). Since the \( Q \)'s are the same in each equation, we can just compare \( 2p \) to \( 8p_r \). If \( p > 4p_r \), consumers will be more sensitive to a change in the price, \( p \). Additionally, we know that \( p = 11 + 2p_r \) so that when \( 11 + 2p_r > 4p_r \) or when \( 11 > 2p_r \) consumers are more sensitive to a change in the price, \( p \), and when \( 11 < 2p_r \) consumers are more sensitive to a change in the price of the related good, \( p_r \).

3. Consider the following constrained multivariate optimization problem.

\[
\max_{x,y} f(x, y) = 5x^2y
\]

subject to \( 20x + 15y = 90 \)

For the parts below, always consider \( y \) to be on the vertical axis and \( x \) to be on the horizontal axis.

(a) What is the Lagrangian for this problem?

**Solution:**

\[
\max_{x,y,\lambda} \mathcal{L}(x, y, \lambda) = 5x^2y + \lambda(90 - 20x - 15y)
\]
(b) Find the optimal values of $x$ and $y$.

**Solution:** Differentiate the Lagrangian with respect to all of the choice variables, $x$, $y$, and $\lambda$. Then set all of the first order conditions equal to zero.

\[
\begin{align*}
\frac{\partial L}{\partial x} &= 10xy - 20\lambda = 0 \\
\frac{\partial L}{\partial y} &= 5x^2 - 15\lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= 90 - 20x - 15y = 0
\end{align*}
\]

Solve for $\lambda$ in each of the first two equations.

\[
\lambda = \frac{10xy}{20} = \frac{5x^2}{15}
\]

Set $\lambda = \lambda$ to get

\[
\begin{align*}
\frac{10xy}{20} &= \frac{5x^2}{15} \\
\frac{xy}{2} &= \frac{x^2}{3} \\
3xy &= 2x^2 \\
3y &= 2x
\end{align*}
\]

Now use $3y = 2x$ in the budget constraint. Since $3y = 2x$, $30y = 20x$. The budget constraint has $90 = 20x + 15y$, so after substitution $90 = 30y + 15y = 45y$. Solving for $y$, we get $y = 2$ and if $y = 2$, then $3(2) = 2x$ so $x = 3$.

(c) What is the value of $f(x, y)$ evaluated at the optimal values of $x$ and $y$? Call this value $A$ where $A = f(x^*, y^*)$.

**Solution:**

\[
f(x, y) = 5x^2y
\]

If $x = 3$ and $y = 2$

\[
f(3, 2) = 5(3)^2(2) = 90,
\]

so $A = 90$. 

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(d) Find an equation for a curve \((y \text{ as a function of } x)\) such that \(f(x, y) = A\) where \(A\) is the value you found in the part above.

**Solution:** If \(90 = 5x^2y\) and we solve for \(y\), we get

\[
y = \frac{90}{5x^2} = \frac{18}{x^2}
\]

(e) Show that the slope of the curve you found above by setting \(f(x, y) = A\) is equal to the slope of the constraint at the optimal values of \(x\) and \(y\).

**Solution:** The equation for the constraint is \(90 = 20x + 15y\). Solve the constraint for \(y\), and then find \(\frac{dy}{dx}\).

\[
\begin{align*}
90 &= 20x + 15y \\
15y &= 90 - 20x \\
y &= 6 - \frac{4}{3}x \\
\frac{dy}{dx} &= \frac{4}{3}
\end{align*}
\]

The indifference curve found in the last part was \(y = \frac{18}{x^2}\). The slope of this indifference curve is

\[
\frac{dy}{dx} = -2\frac{18}{x^3} = -\frac{36}{x^3}
\]

and when we use our optimal value of \(x = 3\), we know that \(x^3 = 27\) and

\[
\frac{dy}{dx} = -\frac{36}{27} = -\frac{4}{3}
\]

so the slope of the indifference curve is equal to the slope of the budget constraint at the optimal values of \(x\) and \(y\).

4. You decide to quit school, travel back in time, and work at a mine. Your mining job pays $50 per month. The company store only sells cans of beans for $1 and bottles of whiskey for $5. Every month you buy 15 cans of beans and 7 bottles of whiskey.

(a) What’s an equation describing your budget line and what’s the slope of your budget line?

**Solution:** Let \(Q_b\) be the number of cans of beans consumed and let \(Q_w\) be the number of the bottles of whiskey consumed. An equation for the budget line is as follows.

\[
50 = Q_b + 5Q_w
\]
(b) Assuming prices don’t change, the government imposes a 10% tax on whiskey, but decides to give poor miners (that’s you) a subsidy of $3.50 per month. What’s an equation describing your new budget line?

**Solution:**

\[50 + 3.5 = Q_b + 5Q_w(1 + 0.1)\]

or

\[53.5 = Q_b + 5.5Q_w\]

(c) Do you prefer the status quo or the new policy with the whiskey tax and subsidy, and why?

**Solution:** With the status quo, you bought 15 cans of beans and 7 bottles of whiskey. With the new prices,

\[15 + 5.5(7) = 53.5\]

so you can still afford your old bundle. Since you can still afford the same bundle as before, the new policy doesn’t make you worse off. If with the new policy, you can purchase a more preferred bundle, then you like the new policy better.

(d) The next election year comes around and the government now decides to give everybody a subsidy worth 10% of their income. An unintended consequence of this policy is that the inflation rate increases and prices are 8% higher. Are consumers better off or worse off and why?

**Solution:** Consumers are better off because their income has grown by more than prices so their budget constraint has shifted out.

Consider any budget constraint for income \(m\) and goods \(x\) and \(y\)

\[m = p_x x + p_y y.\]

Now if income increases by 10% and prices increase by 8%, we have

\[1.1m = 1.08p_x x + 1.08p_y y\]

\[1.1m = 1.08(p_x x + p_y y)\]

\[\frac{1.1}{1.08}m = p_x x + p_y y\]

Since \(\frac{1.1}{1.08} > 1\) income has increased by more than the price increase.