Intermediate Microeconomics
ECNS 301
Spring 2014

Exam #: 1
Version A

Friday February 7, 2014

Name: ____________________________________________

Instructions:
You must answer all of the following questions. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in your blue book(s). There are two versions of the exam. Indicate your exam version on your blue book. It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation.

   (a) The government places a tax of $5 per unit on beeswax and we find that the price buyers pay increases by $2.50. If the tax is increased to $15, then the price buyers pay will increase by $7.50.

   **Solution:** Uncertain. We could certainly create an example with linear demand and supply curves where this is true, but it will not be true in all cases.

   (b) If the government subsidizes one good, this may cause consumers to purchase more of all goods.

   **Solution:** True. By subsidizing one good, the government has increased overall purchasing power which may result in consumers purchasing more of all goods.

   (c) When Hurricane Katrina ripped through Mississippi in 2005, the wages of carpenters doubled. This price indicates a shortage of carpenters.

   **Solution:** False/Uncertain. The doubling of wages was likely due to an increase in demand. (Shortages are typically created by some policy that prevents price adjustment so that the quantity supplied does not equal the quantity demanded. There’s no good reason as to why the price would not adjust.)

   (d) If you know the slope of the budget constraint for two goods, then you know the prices of the two goods.

   **Solution:** False. You don’t know the prices, only the relative prices.

   (e) Both Tums and Rolaids will cure David’s heartburn, and he regards them as perfect substitutes. Therefore, his indifference curves will be linear with a slope of $-1$.

   **Solution:** False. With perfect substitutes, the indifference curves are linear with a negative slope, but the slope is not necessarily $-1$. For instance if David likes the flavor of the tropical fruit flavored tums better, the utility function could be $u(T, R) = 10T + R$, where Tums and Rolaids are perfect substitutes but the slope is not $-1$. 


2. Consider the following constrained multivariate optimization problem.

\[
\max_{x,y} f(x, y) = 2x^{\frac{1}{2}}y^2
\]
subject to \( px + 8y = 50 \)

\( p > 0 \) is a variable. For the parts below, always consider \( y \) to be on the vertical axis and \( x \) to be on the horizontal axis.

(a) What is the Lagrangian for this problem?

\[
\text{Solution:} \quad \max_{x,y,\lambda} L(x, y, \lambda) = 2x^{\frac{1}{2}}y^2 + \lambda(50 - px - 8y)
\]

(b) Find the optimal values of \( x \) and \( y \).

\[
\text{Solution:} \quad \text{Differentiate the Lagrangian with respect to all of the choice variables, } x, y, \text{ and } \lambda. \text{ Then set all of the first order conditions equal to zero.}
\]

\[
\begin{align*}
\frac{\partial L}{\partial x} &= x^{-\frac{1}{2}}y^2 - p\lambda = 0 \\
\frac{\partial L}{\partial y} &= 4x^{\frac{1}{2}}y - 8\lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= 50 - px - 8y = 0
\end{align*}
\]

Solve for \( \lambda \) in each of the first two equations.

\[
\begin{align*}
\lambda &= \frac{y^2}{px^{\frac{1}{2}}} \\
\lambda &= \frac{x^{\frac{1}{2}}y}{2}
\end{align*}
\]

Set \( \lambda = \lambda \) to get

\[
\begin{align*}
\frac{y^2}{px^{\frac{1}{2}}} &= \frac{x^{\frac{1}{2}}y}{2} \\
\frac{y^2}{p} &= \frac{xy}{2} \\
2y^2 &= px y \\
2y &= px
\end{align*}
\]
Now use \(2y = px\) in the budget constraint. The budget constraint has \(50 = px + 8y\), so after substitution \(50 = 2y + 8y = 10y\). Solving for \(y\), we get \(y = 5\) and if \(y = 5\), then \(2(5) = px\) so \(x = \frac{10}{p}\).

(c) What are the comparative statics of the problem: \(\frac{dx^*}{dp}\) and \(\frac{dy^*}{dp}\)?

**Solution:** From above, \(y = 5\) so \(y\) does not depend on \(p\). This makes \(\frac{dy^*}{dp} = 0\). Also from above, \(x = \frac{10}{p}\). Differentiating with respect to \(p\), we get \(\frac{dx^*}{dp} = -\frac{10}{p^2}\).

(d) What is the value of \(f(x, y)\) evaluated at the optimal values of \(x\) and \(y\)? Call this value \(A\) where \(A = f(x^*, y^*)\).

**Solution:**
\[
f(x, y) = 2x^2 y^2
\]
If \(x = \frac{10}{p}\) and \(y = 5\)
\[
f\left(\frac{10}{p}, 5\right) = 2 \left(\frac{10}{p}\right)^\frac{3}{2} (5)^2 = 50(10)^\frac{3}{2}(p)^{-\frac{3}{2}},
\]
so \(A = 50(10)^{\frac{3}{2}}(p)^{-\frac{3}{2}}\).

(e) What is the comparative static of the value of \(f(x, y)\) evaluated at the optimal values of \(x\) and \(y\)? You can think of this as \(\frac{df(x^*, y^*)}{dp}\) or \(\frac{dA}{dp}\).

**Solution:** If \(A = 50(10)^{\frac{3}{2}}(p)^{-\frac{3}{2}}\), we get
\[
\frac{dA}{dp} = -25(10)^{\frac{3}{2}}(p)^{-\frac{3}{2}}.
\]

3. There are two goods: \(x\) and \(y\). The price of good \(y\) is $1 and the price of good \(x\) is $10. You have $150 to spend on good \(x\) and \(y\). You’ve decided to consume \(x = 5\) and \(y = 100\).

(a) What’s the slope of the budget line and what is the economic term we associate with the slope?

**Solution:** The economic term associated with the slope of the budget line is the marginal rate of transformation. An equation for the budget line is \(y + 10x = 150\). Solving for \(y\), we get \(y = 150 - 10x\) and the slope of this line is \(-10\). Alternatively if you solved for \(x\), you get \(x = 15 - \frac{1}{10}y\) and the slope is \(-\frac{1}{10}\).
(b) The government levies a $2 per-unit tax on good \( y \) (on consumers) along with an income subsidy of $200. What’s an equation describing your new budget line?

**Solution:** With the $2 per-unit tax on good \( y \), the price of \( y \) is effectively $3. With the income subsidy of $200, income is effectively $350. An equation for this budget line is \( 3y + 10x = 350 \) or \( y = \frac{350}{3} - \frac{10}{3}x \).

(c) The government levies an ad-valorem tax of 25\% on good \( y \) (on consumers) and a per-unit subsidy of $5 on good \( x \). What’s an equation describing your new budget line?

**Solution:** Income is $150. The effective price of good \( x \) is $10 - $5 = $5. The effective price of good \( y \) is $1.25 or \( \frac{5}{4} \). An equation for this budget line is \( \frac{5}{4}y + 5x = 150 \) or \( y = 120 - 4x \).

(d) Considering no policy, the policy in part B and the policy in part C, what is the most preferred and least preferred policy?

**Solution:** Note that \( x = 5 \) and \( y = 100 \) are points on all three budget lines. The slope and \( y \)-intercepts for all three equations are different. Part B and part C allow the consumer choices that were not possible with no policy and they also allow for the bundle chosen under no policy. For these two reasons, the least preferred policy is no policy. The most preferred policy is the policy in part B as it provides more choices that we not possible with no policy when compared to the policy in part C.

4. The market supply and demand functions for a particular market are as follows.

\[
Q = 50 - p \\
Q = 2p - 4
\]

The government is considering a per-unit tax of \( \tau \) to be levied on sellers.

(a) What are the equilibrium prices and quantity with the tax expressed as a function of \( \tau \)?

**Solution:** With a per-unit tax of \( \tau \) levied on sellers, the price sellers get is \( p - \tau \) and the price buyers pay is \( p \). To find the equilibrium, set \( D(p) = S(p - \tau) \) and
solve for $p$.

$$D(p) = S(p - \tau)$$

$$50 - p = 2(p - \tau) - 4$$

$$3p = 54 + 2\tau$$

$$p = 18 + \frac{2}{3}\tau$$

The price buyers pay is $p = 18 + \frac{2}{3}\tau$. The price sellers get is $p = 18 - \frac{1}{3}\tau$. The market quantity is:

$$Q = 50 - (18 + \frac{2}{3}\tau) = 32 - \frac{2}{3}\tau$$

$$Q = 2(18 - \frac{1}{3}\tau) - 4 = 32 - \frac{2}{3}\tau.$$ 

(b) Show that as the tax rate increases the price buyers pay increases, the price sellers get decreases and the equilibrium quantity decreases.

**Solution:** Let $p_B$ be the price buyers pay and $p_S$ be the price sellers get.

$$\frac{dp_B}{d\tau} = \frac{2}{3} > 0$$

$$\frac{dp_S}{d\tau} = -\frac{1}{3} < 0$$

$$\frac{dQ}{d\tau} = -\frac{2}{3} < 0$$

(c) What value of $\tau$ maximizes tax revenue and how much tax revenue is generated?

**Solution:** For tax revenue, the government gets $\tau$ for each good sold, so tax revenue is $\tau Q$. This reduces to $32\tau - \frac{2\tau^2}{3}$ and the optimization problem is as follows.

$$\max_{\tau} 32\tau - \frac{2\tau^2}{3}$$

The first order condition is

$$32 - \frac{4}{3}\tau = 0$$

so the optimal tax rate is $\tau = 24$. When $\tau = 24$, $Q = 16$ and tax revenue is 384.
(d) Based on your answers above, what do we know about the relative price elasticity of supply and demand?

**Solution:** By examining the prices, we know that buyers pay \( \frac{2}{3} \) of the tax and sellers pay \( \frac{1}{3} \) of the tax. Since buyers have a larger tax incidence, we know that demand is more inelastic than supply or that supply is more elastic than demand.