Intermediate Microeconomics
ECNS 301
Fall 2013

Exam #: 2
Version A

Thursday November 7, 2013

Name: ____________________________________________________________

Instructions:
You must answer all of the following questions. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. **No notes, no assistance, no talking, no cell phones, but you can use a calculator.**

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in the exam and your blue book(s). There are two versions of the exam. **Indicate your exam version on your blue book.** It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation.

(a) A firm operating with diminishing total returns cannot be profit maximizing.

**Solution:** True. Diminishing total returns implies the firm could produce more with fewer inputs which would increase their profits.

(b) Cobb-Douglas production functions can never possess varying returns to scale.

**Solution:** True. The Cobb-Douglas production function takes the form \( q = L^\alpha K^\beta \) where the exponents are constant parameters. If you scale the inputs by \( x \), you get \( (xL)^\alpha (xK)^\beta = x^{\alpha+\beta} L^\alpha K^\beta \) which is constant over \( q \).

(c) Technical progress will shift an isoquant outward.

**Solution:** False, technical progress shifts an isoquant inward. With technical progress, you can produce the same quantity with fewer inputs.

(d) After employing her last laborer, Rachel notices that her Average Product has decreased. Her marginal cost is greater than her Average Variable Cost.

**Solution:** True, the decrease in average product (of labor) indicates an increase in average variable cost. The average product of labor is \( AP_L = \frac{q}{L} \), and the average variable cost could be thought of as \( AVC = \frac{wL}{q} \). So, if the average product decreases, then the average variable cost increases. The average variable cost rises when the marginal cost is greater than the AVC.

Short Answer/Numerical

2. A firm’s production function is as follows.

\[ q = KL + 10L \]

(a) What is the marginal product of labor and the average product of labor?

**Solution:**

\[
MP_L = \frac{\partial q}{\partial L} = K + 10
\]

\[
AP_L = \frac{q}{L} = K + 10
\]
(b) What is an equation for any isoquant if \(K\) is on the vertical axis and \(L\) is on the horizontal axis?

**Solution:**

\[
\begin{align*}
q &= KL + 10L \\
KL &= q - 10L \\
K &= \frac{q}{L} - 10
\end{align*}
\]

(c) Find the marginal rate of technical substitution as a function of just \(K\) and \(L\).

**Solution:**

\[
\begin{align*}
K &= \frac{q}{L} - 10 \\
\frac{\partial K}{\partial L} &= -\frac{q}{L^2} \\
\frac{\partial K}{\partial L} &= -\frac{KL + 10L}{L^2} \\
\frac{\partial K}{\partial L} &= -\frac{K + 10}{L}
\end{align*}
\]

(d) Does this production function exhibit increasing returns to scale, constant returns to scale or decreasing returns to scale and why?

**Solution:** Scale the inputs by \(a\).

\[
\begin{align*}
q &= KL + 10L \\
q' &= (aK)(aL) + 10(aL) \\
q' &= a^2 KL + 10aL \\
q' &= a(aKL + 10L)
\end{align*}
\]

For \(a > 1\), \(q' > aq\) so the production function exhibits increasing returns to scale.
3. Consider the following utility function: $U = x_1^2 x_2$. The consumer has income of $M$, the price of $x_1$ is $P_1$, and the price of $x_2$ is $P_2$. (25)

(a) What is the marginal rate of substitution?

**Solution:** The marginal rate of substitution is the slope of the indifference curve. The utility function is

$$U = x_1^2 x_2$$

and if the utility level is $\bar{u}$ then we have

$$\bar{u} = x_1^2 x_2.$$ 

The equation for any indifference curve is

$$x_2 = \frac{\bar{u}}{x_1^2}$$

because we put $x_2$ on the vertical axis. You would get higher or lower indifference curves by increasing or decreasing $\bar{u}$. The slope of the indifference curve is found by taking the derivative of the indifference curve equation.

$$\frac{dx_2}{dx_1} = -\frac{2\bar{u}}{x_1^3}$$

We could simplify this by substituting in $\bar{u} = x_1^2 x_2$.

$$\frac{dx_2}{dx_1} = -\frac{2x_1^2 x_2}{x_1^3}$$

You also get the same thing when you take the ratio of the marginal utilities.

(b) What is the equation for the budget constraint?

**Solution:** The budget constraint is

$$M = P_1 x_1 + P_2 x_2.$$ 

Solving for $x_2$ we get

$$x_2 = \frac{M}{P_2} - \frac{P_1}{P_2} x_1.$$
(c) What is the optimal consumption bundle if \( M = 60 \), \( P_1 = 1 \) and \( P_2 = 2 \)? What level of utility is achieved?

**Solution:** This is the solution to the consumer’s problem. The optimization problem is

\[
\max_{x_1, x_2} U = x_1^2 x_2
\]

subject to \( M = P_1 x_1 + P_2 x_2 \).

Write the Lagrangian as

\[
\max_{x_1, x_2, \lambda} L(x_1, x_2, \lambda) = x_1^2 x_2 + \lambda (M - P_1 x_1 - P_2 x_2)
\]

and the first order conditions are as follows.

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= 2x_1 x_2 - P_1 \lambda = 0 \\
\frac{\partial L}{\partial x_2} &= x_1^2 - P_2 \lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= 100 - P_1 x_1 - P_2 x_2 = 0
\end{align*}
\]

Combine the first two first order conditions to get

\[
\frac{2x_1 x_2}{P_1} = \frac{x_1^2}{P_2} \\
2P_2 x_2 = P_1 x_1.
\]

Now plug this into the last first order condition (the budget constraint) to get

\[
\begin{align*}
M &= P_1 x_1 + P_2 x_2 \\
M &= (2P_2 x_2) + P_2 x_2 \\
M &= 3P_2 x_2 \\
x_2^* &= \frac{M}{3P_2} \\
x_1^* &= \frac{2P_2 x_2}{P_1} = \frac{2M}{3P_1}
\end{align*}
\]

If \( M = 60 \), \( P_1 = 1 \) and \( P_2 = 2 \), then

\[
\begin{align*}
x_2^* &= \frac{M}{3P_2} = \frac{60}{3(2)} = 10 \\
x_1^* &= \frac{2M}{3P_1} = \frac{2(60)}{3(1)} = 40 \\
U &= x_1^2 x_2 = (40)^2 10 = 16000
\end{align*}
\]
(d) Derive the demand function for good \( x_1 \) with income of \( M \), the price of \( x_1 \) is \( P_1 \), and the price of \( x_2 \) is \( P_2 \) (general, not specific).

**Solution:** We did this in the last part.

\[ x_1^* = \frac{2M}{3P_1} \]

4. In many countries it is not uncommon for children to work instead of attending school. Working provides income for the family and sending children to school costs $10 per week. Governments and families realize that it’s much better for children to go to school, but families are income constrained and the average annual income per family is $100. The annual demand for schooling is

\[ q = 10mp^{-2} \]

where \( q \) is the number of weeks per year of schooling desired.

(a) What’s the price elasticity of demand for schooling?

**Solution:** The price elasticity of demand is the percentage change in quantity demanded divided by the percentage change in the price.

\[ PED = \frac{\% \Delta \text{ in } Q_D}{\% \Delta \text{ in } p} = \frac{\partial q}{\partial p} \frac{p}{q} = \frac{\partial q}{\partial p} \frac{q}{p} \]

From the demand function given,

\[ \frac{\partial q}{\partial p} = -20mp^{-3} \]

and putting it all together we get

\[ PED = -20mp^{-3} \frac{p}{10mp^{-2}} = -2. \]
(b) If the government subsidizes the cost per week of schooling so that children are sent to school for 40 weeks out of the year, what’s the weekly subsidy?

**Solution:** From the problem, \( m = 100 \), so \( q = 1000p^{-2} \). Now we want to find \( p \) such that \( q = 40 \).

\[
\begin{align*}
40 &= 1000p^{-2} \\
p^2 &= \frac{100}{4} \\
p &= \frac{10}{2} = 5
\end{align*}
\]

At a price of $5, 40 weeks of school per year would be purchased. The current price is $10, so you need a subsidy of $10 - $5 = $5.

(c) How much does this subsidy cost the government per year per family?

**Solution:** The $5 per unit subsidy would be provided for all the schooling purchased (40 weeks), so the per year cost per family to the government is \( 40 \times 5 = 200 \).

(d) Would it be cheaper to setup government run schools and provide 40 weeks of schooling per year for free? Assume that the government is not more efficient than the market.

**Solution:** No, it’s always cheaper to subsidize a good than to pay for it outright. Think partial subsidy versus full subsidy. The partial subsidy costs $200 per family per year and the full subsidy would cost as least $400 per family per year.