Intermediate Microeconomics
ECNS 301
Fall 2014

Exam #: 2
Version A

Thursday November 6, 2014

Name: ________________________________

Instructions:
You must answer all of the following questions. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in your blue book(s). There are two versions of the exam. Indicate your exam version on your blue book. It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
**True/False/Uncertain Plus Explanation**

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation.

   (a) The change in total welfare from a 10% increase in price will depend only on the elasticity of demand.

   **Solution:** False, this also depends on the elasticity of supply as we were looking at the total welfare. Note that a price increase could occur from an increase in demand or a decrease in supply.

   (b) The length of the short run is the same for all firms.

   **Solution:** False, the length of the short run depends on the length of time it takes to change all of their inputs.

   (c) You should stop studying for your economics exam once you reach the point of diminishing returns.

   **Solution:** True, diminishing returns implies that you will do worse by studying more. This is different than diminishing marginal returns.

   (d) A firm may express increasing, constant and decreasing returns to scale for various levels of output.

   **Solution:** True. As output increases a firm can experience all types of returns to scale.

**Short Answer/Numerical**

2. A firm’s production function is as follows.

   \[ q = KL^2 + 13L + 4 \]

   (a) What is the marginal product of labor and the average product of labor?

   **Solution:**

   \[ MP_L = \frac{\partial q}{\partial L} = 2KL + 13 \]
   \[ AP_L = \frac{q}{L} = KL + 13 + \frac{4}{L} \]
(b) Find the marginal rate of technical substitution as a function of just $K$ and $L$.

**Solution:** The MRTS is the ratio of the marginal products. You found $MP_L$ above. Now find $MP_K = L^2$. Taking the ratio of the marginal products, the MRTS is

$$MRTS = \frac{MP_L}{MP_K} = \frac{2KL + 13}{L^2}.$$  

If you found MRTS as $MRTS = \frac{MP_K}{MP_L}$, that works too.

(c) Find the value of $L$ that minimizes the average product of labor.

**Solution:** The $AP_L = KL + 13 + \frac{4}{L}$ (from above) and if you want to find the value of $L$ that minimizes this, then take the derivative and set it equal to zero.

$$\min L \left( KL + 13 + \frac{4}{L} \right)$$

$$0 = K - \frac{4}{L^2}$$

$$L^2 = \frac{4}{K}$$

$$L = \left( \frac{4}{K} \right)^{\frac{1}{2}}$$

(d) At what value of $K$ does the average product of labor equal the marginal product of labor?

**Solution:** At $K = \frac{4}{L^2}$ as the average product of labor equals the marginal product of labor where the average product of labor is minimized. If you want to set $AP_L = MP_L$ and solve for $K$, we have:

$$AP_L = MP_L$$

$$KL + 13 + \frac{4}{L} = 2KL + 13$$

$$\frac{4}{L} = KL$$

$$K = \frac{4}{L^2}$$
3. Consumer’s consume food and other goods. The amount of food consumed is denoted $f$ with price $p_f$ and the amount of other goods is denoted $y$ with price $p_y$. In order to support farmers (and low income consumers), the state of Montana is considering subsidizing the price of food so that the quantity of food consumed by every consumer is 30. With the price subsidy the price of food becomes $p_f' = p_f - \tau$ where $\tau$ is the amount of the per unit subsidy. There are 1 million people in Montana and each person has the following preferences.

$$U(f, y) = \min\{f, 2y\}$$

$p_y$ is normalized to 1, $p_f = 7$, income is $m = 90$, and the price subsidy considered is $\tau = 5$.

(a) How does the price subsidy change the optimal consumption bundle of each consumer? What was it before the subsidy and after?

**Solution:** The preferences described by the utility function exhibit perfect complements. In equilibrium we know that $f = 2y$, and the budget constraint is $m = p_f f + p_y y$. Combining the two equations, we get $m = p_f 2y + p_y y$ and solving for $y$ yields the demand function $y^* = \frac{m}{2p_f + p_y}$. The demand function for good $f$ is $f^* = \frac{2m}{2p_f + p_y}$.

Before the price subsidy, $p_y = 1, p_f = 7$ and $m = 90$. The consumption bundles of each consumer before the price subsidy are as follows.

$$y^* = \frac{m}{2p_f + p_y} = \frac{90}{2(7) + 1} = 6$$
$$f^* = \frac{2m}{2p_f + p_y} = \frac{2(90)}{2(7) + 1} = 12$$

After the price subsidy, $p_y = 1, p_f = 2$ and $m = 90$. The consumption bundles of each consumer after the price subsidy are as follows.

$$y^* = \frac{m}{2p_f + p_y} = \frac{90}{2(2) + 1} = 18$$
$$f^* = \frac{2m}{2p_f + p_y} = \frac{2(90)}{2(2) + 1} = 36$$

(b) Will the food subsidy achieve it’s objective?

**Solution:** The objective was to have $f^* = 30$ and with the policy $f^* = 40$ so the policy leads to the consumption of too much food.
(c) What are the substitution effects and income effects associated with the price subsidy for both \( y \) and \( f \)?

**Solution:** The total effect for \( y \) is \( 18 - 6 = 12 \). The total effect for \( f \) is \( 36 - 12 = 24 \). Remember that the Slutsky equation tells us that the total effect is the substitution effect plus the income effect. Since the two goods are perfectly complements, the substitution effect in each case is zero and the income effect for each good equals the total effect.

(d) What is the change in consumer welfare due to the price subsidy?

**Solution:** For this part, you could either find the change in consumer surplus, the compensating variation, or the equivalent variation. What you can’t do is find the change in utility as the magnitude change in utility is meaningless.

To find the compensating variation, use the old utility level and the new prices to find the decrease in income required to get you back to your original utility level. The initial utility level was 12 and the new prices are 1 and 2. In this case \( U(f, y) = \min\{f, 2y\} \) which is \( U = f \), and

\[
12 = \frac{2(m + CV)}{2(2) + (1)}.
\]

Note that \( m = 90 \) so solving for compensating variation yields \( CV = -60 \).

To find the equivalent variation, use the new utility level and the old prices to find the increase in income equivalent to the new utility level. The new utility level is 36 and the old prices are 1 and 7. In this case \( U(f, y) = \min\{f, 2y\} \) which is \( U = f \), and

\[
36 = \frac{2(m - EV)}{2(7) + (1)}.
\]

Note that \( m = 90 \) so solving for equivalent variation yields \( EV = 180 \).

To find the change in consumer surplus, we use the demand for good \( f \), \( f = \frac{2m}{2p_f + p_y} \). The price of good \( y \) is 1 and \( m = 90 \). Evaluate the change in consumer surplus due to the price decrease in \( p_f \) from 7 to 2. Demand for good \( f \) is now \( f = \frac{180}{2p_f + 1} \). The change in consumer surplus is

\[
\Delta CS = \int_2^7 \frac{180}{2p_f + 1} dp_f
\]

\[
\Delta CS = 180\left[\log\left(\frac{2p_f + 1}{2}\right)\right]^7_2
\]

\[
\Delta CS = 180\left(\frac{\log(15)}{2} - \frac{\log(2)}{2}\right)
\]

\[
\Delta CS = 90(\log(15) - \log(5))
\]

\[
\Delta CS = 98.88.
\]
4. A firm’s production function is $q = K^2L$ where $q$ is the quantity produced, $K$ is the amount of capital used, and $L$ is the amount of labor used. $w$ is the wage rate of labor and $r$ is the rental rate of capital.

(a) In the short run when capital is fixed, how much labor should the firm hire?

**Solution:** If capital is fixed $q = K^2L$, and $L = \frac{q}{K^2}$. $L$ is how much labor the firm should hire.

(b) In the short run when capital is fixed, what is the short run cost function?

**Solution:** The cost function is $wL + rK$. In the short run,

$$C_{SR}(q) = \frac{wq}{K^2} + rK$$

when the short run choice of labor is substituted into the cost function.

(c) In the long run, how much labor and capital should the firm use?

**Solution:** Setup the cost minimization problem and take the first order conditions.

$$\min_{L,K,\lambda} L(L, K, \lambda) = wL + rK + \lambda(q - K^2L)$$

$$\frac{\partial L}{\partial L} = w - \lambda K^2 = 0$$

$$\frac{\partial L}{\partial K} = r - 2\lambda KL = 0$$

$$\frac{\partial L}{\partial \lambda} = q - K^2L = 0$$

From the first first order condition, we know that $\lambda = \frac{w}{K^2}$, and from the second first order condition $\lambda = \frac{r}{2KL}$. Combining these two equations together we get

$$\frac{w}{K^2} = \frac{r}{2KL}$$

$$2wL = rK$$

$$L = \frac{rK}{2w}.$$
Substitute our value for $L$ into the production function.

$$q = K^2 \frac{rK}{2w}$$
$$q = K^3 \frac{r}{2w}$$
$$K^3 = \frac{2wq}{r}$$
$$K = \left(\frac{2wq}{r}\right)^{\frac{1}{3}}$$

This is how much capital the firm should use in the long run.

Now we need to figure out how much labor the firm should use in the long run. From above, we found that

$$L = \frac{rK}{2w}$$

and

$$K = \left(\frac{2wq}{r}\right)^{\frac{1}{3}},$$

so

$$L = \frac{r}{2w} \left(\frac{2wq}{r}\right)^{\frac{1}{3}}.$$ 

You don’t have to simplify this, but

$$L = q^{\frac{1}{3}} \left(\frac{r}{2w}\right)^{\frac{2}{3}}.$$ 

(d) What are the long run costs?

**Solution:** Costs are $wL + rK$. Now substitute in our long run choices of capital and labor to get

$$C_{LR}(q) = wq^{\frac{1}{3}} \left(\frac{r}{2w}\right)^{\frac{2}{3}} + r \left(\frac{2wq}{r}\right)^{\frac{1}{3}}.$$ 

You don’t have to simplify this, but

$$C_{LR}(q) = (r^2 wq)^{\frac{1}{3}} \left(2^{-\frac{2}{3}} + 2^{\frac{2}{3}}\right).$$