Intermediate Microeconomics
ECNS 301
Spring 2012

Exam #: 2
Version A

Friday March 30, 2012

Name: ________________________________

Instructions:
Answer all of the following questions. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. **No notes, no assistance, no talking, no cell phones, but you can use a calculator.**

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in the exam and your blue book(s). There are two versions of the exam. **Indicate your exam version on your blue book.** It is your responsibility to make sure your version of the exam is different from the students next to you. If you have the same version as any of the students next to you, you will be asked to move.
## Intermediate Microeconomics
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True/False/Uncertain Plus Explanation

1. For each of the following, state whether it is true, false or uncertain and explain your answer. No points are given without explanation. (25)

(a) You should stop studying for your economics exam once you reach the point of diminishing returns.

**Solution:** True, diminishing returns implies that you will do worse by studying more. This is different than diminishing marginal returns.

(b) If inputs into production cannot be substituted for each other but have to be employed in fixed-proportions isoquants are straight, downward-sloping lines.

**Solution:** False, the isoquants will be horizontal and vertical lines that form right angles.

(c) The minimum point of a short-run average cost curve will be on the long-run average cost curve.

**Solution:** True, if the minimum point of a short-run average cost curve was not on the long-run average cost curve, costs would not be minimized in the long run.

(d) If the government wishes to increase the utility of consumers by a specific amount, it is less expensive to do that through a cash gift than through a price subsidy on a commonly purchased good (such as food).

**Solution:** True, with a cash gift there is no substitution effect, but with a price subsidy there is a substitution effect.
Short Answer/Numerical

2. A consumer has the following utility function.

\[ U(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}} \]  

(a) Find the general demand functions for \( x \) and \( y \) which are described as \( x^* = x(p_x, p_y, m) \) and \( y^* = y(p_x, p_y, m) \).

**Solution:** To solve the utility maximization problem, setup the Lagrangian.

\[ \max_{x, y, \lambda} L(x, y, \lambda) = x^{\frac{1}{2}} y^{\frac{1}{2}} + \lambda (m - P_x x - P_y y) \]

The three first order conditions are as follows.

\[ \frac{\partial L}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}} - \lambda P_x \]

\[ \frac{\partial L}{\partial y} = \frac{1}{2} x^{\frac{1}{2}} y^{-\frac{1}{2}} - \lambda P_y \]

\[ \frac{\partial L}{\partial \lambda} = m - P_x x - P_y y \]

Setting each first order condition equal to zero and rearranging we get the following.

\[ \frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2P_x} = \lambda \]

\[ \frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2P_y} = \lambda \]

\[ m = P_x x + P_y y \]

Combining the first two first order conditions...

\[ \frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2P_x} = \frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2P_y} \]

\[ P_y x^{-\frac{1}{2}} y^{\frac{1}{2}} = P_x x^{\frac{1}{2}} y^{-\frac{1}{2}} \]

\[ P_y y = P_x x \]

Now substitute this result into the third first order condition...

\[ m = P_x x + P_y y \]

\[ m = P_x x + P_x x = 2P_x x \]

\[ x^* = \frac{m}{2P_x} \]
which is the general demand for good $x$. Since $P_y y = P_x x$, $y = \frac{P_x x}{P_y}$ and the general demand for good $y$ is 

$$y^* = \frac{m}{2P_y}$$

(b) The consumers current income is $m = 100$ and current prices are $p_x = 1$, $p_y = 1$. What happens to the optimal bundles when $p_x$ decreases to $p_x' = \frac{1}{4}$?

**Solution:** Substituting in the original prices and income given by the question

$$y^* = \frac{m}{2P_y} = \frac{100}{2(1)} = \frac{100}{2} = 50$$

$$x^* = \frac{m}{2P_x} = \frac{100}{2(1)} = \frac{100}{2} = 50$$

and substituting in the new prices after the price change we get the following.

$$y^* = \frac{m}{2P_y} = \frac{100}{2(\frac{1}{4})} = \frac{100}{\frac{1}{2}} = 200$$

$$x^* = \frac{m}{2P_x} = \frac{100}{2 \left( \frac{1}{4} \right)} = \frac{100}{\frac{1}{2}} = 200$$

The optimal bundles change from $(50, 50)$ to $(200, 50)$.

(c) What is the total effect of the change in $p_x$ on $x^*$?

**Solution:** The decrease in $p_x$ changes $x^*$ from 50 to 200, so the magnitude of the total effect is 150. If we want to think about the sign of the total effect, we know that the decrease in price increases the quantity demanded so the sign of the total effect is negative.

(d) How much income do you have to take away from the consumer to compensate them for the price change? (Hint: Use the new prices with the old utility level to find the change in income.)

**Solution:** Utility is $U(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$. With the original consumption bundle of $(50, 50)$, utility is $U(50, 50) = 50^{\frac{1}{2}}50^{\frac{1}{2}} = 50$. If we are going to compensate the consumer for the price change, we want to keep their level of utility at 50.

Demands for the two goods are $x^* = \frac{m}{2P_x}$ and $y^* = \frac{m}{2P_y}$. With the new prices of $P_x = \frac{1}{4}$ and $P_y = 1$, the demands are $x^* = 2m$ and $y^* = \frac{m}{2}$.

We can rewrite the utility function $U(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$ as $U(x, y) = (xy)^{\frac{1}{2}}$. Substituting in the demand functions with the new prices ($x^* = 2m$ and $y^* = \frac{m}{2}$), we
get \( U(x^*, y^*) = ((2m)^{\frac{1}{2}})^{\frac{1}{2}} = (m^2)^{\frac{1}{2}} = m \). Keeping the utility level at 50, we find that a price decrease of \( P_x \) from 1 to \( \frac{1}{4} \) is the same as decreasing income from 100 to 50. In order to compensate the consumer for the price change, we have to take away $50. This is the compensating variation of the price change.

(e) Decompose the total effect reported in part c into a substitution effect and an income effect.

**Solution:** If we keep the consumer at the initial utility level, we can see how they would substitute \( y \) for \( x \) due to the price change. To keep the consumer at the initial utility level, we have to use the new prices of \( P_x = \frac{1}{4} \) and \( P_y = 1 \) and take away $50 of income so \( m = 100 - 50 = 50 \).

The general demand for good \( x \) is \( x^* = \frac{m}{2p_x} \). When \( m = 50 \) and \( P_x = \frac{1}{4} \), \( x^* = 100 \). Note that before the price change, \( x^* = 50 \) (see part b). The magnitude of the substitution effect is 100 − 50 = 50. If we want to think about the sign of the substitution effect, the price of good \( x \) decreases which causes the consumer to substitute good \( y \) for good \( x \), so the substitution effect is negative. Since the magnitude of the total effect is 150 (see part c) and since the magnitude of the substitution effect is 50, the magnitude of the income effect is 100. The sign of the income effect is positive (good \( x \) is a normal good) since according to the handout I gave you the total effect equals the substitution effect minus the income effect.

3. In many countries it is not uncommon for children to work instead of attending school. Working provides income for the family and sending children to school costs $10 per week. Governments and families realize that it’s much better for children to go to school, but families are income constrained and the average annual income per family is $100. The annual demand for schooling is

\[
q = 10mp^{-2}
\]

where \( q \) is the number of weeks per year of schooling desired.

(a) What’s the price elasticity of demand for schooling?

**Solution:** The price elasticity of demand is the percentage change in quantity demanded divided by the percentage change in the price.

\[
PED = \frac{\% \Delta \text{ in } Q}{\% \Delta \text{ in } p} = \frac{\partial q}{\partial p} \frac{p}{q} = \frac{\partial q}{\partial p} \frac{p}{q}
\]

From the demand function given,

\[
\frac{\partial q}{\partial p} = -20mp^{-3}
\]
and putting it all together we get

\[ PED = -20mp^{-3} \frac{p}{10mp^{-2}} = -2. \]

(b) If the government subsidizes the cost per week of schooling so that children are sent to school for 40 weeks out of the year, what’s the weekly subsidy?

**Solution:** From the problem, \( m = 100 \), so \( q = 1000p^{-2} \). Now we want to find \( p \) such that \( q = 40 \).

\[
40 = 1000p^{-2} \\
p^2 = \frac{100}{4} \\
p = \frac{10}{2} = 5
\]

At a price of $5, 40 weeks of school per year would be purchased. The current price is $10, so you need a subsidy of $10 - $5 = $5.

(c) How much does this subsidy cost the government per year per family?

**Solution:** The $5 per unit subsidy would be provided for all the schooling purchased (40 weeks), so the per year cost per family to the government is \( 40 \times 5 = 200 \).

(d) Would it be cheaper to setup government run schools and provide 40 weeks of schooling per year for free? Assume that the government is not more efficient than the market.

**Solution:** No, it’s always cheaper to subsidize a good than to pay for it outright. Think partial subsidy versus full subsidy. The partial subsidy costs $200 per family per year and the full subsidy would cost as least $400 per family per year.

4. A firm has the following production function

\[ q = K^{\frac{1}{2}}L^{\frac{1}{2}}. \]

The wage rate and the rental rate of capital are both equal to $1. The firm would like to know the minimum cost of producing 1,000 units of output.

(a) Find the combination of inputs that minimizes the cost of producing 1,000 units.
Solution: Consider the firm’s cost minimization problem.

\[
\min_{K,L} wL + rK \\
\text{subject to } q = K^{\frac{1}{2}}L^{\frac{1}{2}}
\]

Now setup the Lagrangian.

\[
\min_{K,L,\lambda} \mathcal{L}(K, L, \lambda) = wL + rK + \lambda(q - K^{\frac{1}{2}}L^{\frac{1}{2}})
\]

Differentiate the Lagrangian with respect to the choice variables.

\[
\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \frac{1}{2} K^{-\frac{1}{2}}L^{\frac{1}{2}} \\
\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \frac{1}{2} K^{\frac{1}{2}}L^{-\frac{1}{2}} \\
\frac{\partial \mathcal{L}}{\partial \lambda} = q - K^{\frac{1}{2}}L^{\frac{1}{2}}
\]

Set each first order condition equal to zero.

\[
0 = r - \lambda \frac{1}{2} K^{-\frac{1}{2}}L^{\frac{1}{2}} \\
0 = w - \lambda \frac{1}{2} K^{\frac{1}{2}}L^{-\frac{1}{2}} \\
0 = q - K^{\frac{1}{2}}L^{\frac{1}{2}}
\]

Rearrange each equation to get the following.

\[
\lambda = 2r \frac{K^{\frac{1}{2}}}{L^{\frac{1}{2}}} \\
\lambda = 2w \frac{L^{\frac{1}{2}}}{L^{\frac{1}{2}}} \\
q = K^{\frac{1}{2}}L^{\frac{1}{2}}
\]

Use the first two equations to get the following.

\[
2r \frac{K^{\frac{1}{2}}}{L^{\frac{1}{2}}} = 2w \frac{L^{\frac{1}{2}}}{L^{\frac{1}{2}}} \\
wL = rK
\]

In the problem above, \( w = 1 \) and \( r = 1 \), so \( L = K \) in equilibrium. Substituting this equilibrium condition into the third first order condition yields the following.

\[
q = K = L
\]

If you want to produce \( q = 1000 \), you should use \( K = 1000 \) and \( L = 1000 \).
(b) What is the minimum cost of producing 1,000 units?

Solution: The cost is \( wL + rK \) and since \( w = 1 \) and \( r = 1 \) the cost is \( 1000 + 1000 = 2000 \).

(c) Now the firm wants to know how much output it can produce for a cost of $5,000. Find the output-maximizing input combination and the maximum output that can be produced.

Solution: With \( w = 1 \) and \( r = 1 \) the cost is \( L + K \). Since \( q = K = L \) in equilibrium, the cost function is \( C(q) = 2q \). If the cost is $5,000, then \( 5000 = 2q \) so \( q = 2500 \).

(d) The firm decides to purchase 1,000 units of capital. What are the firm’s short run and long run cost curves?

Solution: The firm’s long run cost curve was found above to be \( C(q) = 2q \). For the short run cost curve, we solve for \( L \) as a function of \( q \) given that \( K = 1000 \).

\[
q = K^{\frac{1}{2}}L^{\frac{1}{2}} \\
q = 1000^{\frac{1}{2}}L^{\frac{1}{2}} \\
q^2 = 1000L \\
L = \frac{q^2}{1000}
\]

With \( w = 1 \) and \( r = 1 \) the cost is \( L + K \). In the short run, \( K = 1000 \) so the costs are \( 1000 + L \). Since \( L = \frac{q^2}{1000} \) in the short run, the short run cost curve is as follows.

\[
C_{SR}(q) = 1000 + \frac{q^2}{1000}
\]