Intermediate Microeconomics  
ECNS 301  
Fall 2015  

Homework #: 1  

Due by the beginning of class on: Thursday September 3, 2015  

Name:  

Instructions:  
There are 12 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
Math for Math’s Sake

The following is meant to be a review of the math you’re supposed to know but might need some refreshing.

1. Find $f'(x)$ or $\frac{df(x)}{dx}$ for the following functions:

   (a) $f(x) = Ax^n$ where $n$ and $A$ are constants
   
   Solution: $f'(x) = nAx^{n-1}$

   (b) $f(x) = x^{0.5}$

   Solution: $f'(x) = 0.5x^{-0.5} = \frac{0.5}{\sqrt{x}}$

   (c) $f(x) = 10x^{-3}$

   Solution: $f'(x) = -30x^{-4} = \frac{-30}{x^4}$

   (d) $f(x) = 5x^4 + 6x^2 + 7x + 3 + \frac{22}{x}$

   Solution: $f'(x) = 20x^3 + 12x + 7 - \frac{22}{x^2}$

   (e) $f(x) = g(x)h(x)$

   Solution: $f'(x) = g'(x)h(x) + g(x)h'(x)$ (The product rule)

   (f) $f(x) = (x^2)(x^3 - 4)$

   Solution: $f'(x) = (2x)(x^3 - 4) + (x^2)(3x^2) = 5x^4 - 8x$

   (g) $f(x) = (3x^3)(\ln(x))$

   Solution: $f'(x) = 9x^2 \ln(x) + 3x^3 \left(\frac{1}{x}\right)$

   (h) $f(x) = \frac{g(x)}{h(x)}$

   Solution: $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$ (The quotient rule)

   (i) $f(x) = \frac{\ln(x)}{5x^2}$

   Solution: $f'(x) = \frac{\frac{5x^2}{x} - \ln(x)10x}{25x^4} = \frac{5x-\ln(x)10x}{25x^4} = \frac{1-2\ln(x)}{5x^3}$
(j) \( f(x) = g(h(x)) \)

**Solution:** \( f'(x) = g'(h(x))h'(x) \) (The chain rule)

(k) \( f(x) = (x^5 - 10x)^a \) where \( a \) is a constant

**Solution:** \( f'(x) = a(x^5 - 10x)^{a-1}(5x^4 - 10) \)

(l) \( f(x) = \ln(2x) \)

**Solution:** \( f'(x) = \frac{1}{2x}(2) = \frac{1}{x} \)

2. In question 1.c above, what is the slope of \( f(x) \) when \( x = 2? \)

**Solution:** From question 1.c, \( f'(x) = -30x^{-4} \) and \( f'(2) = -30(2)^{-4} = -\frac{15}{8} = -1.875 \)

3. Find all the local extrema for the following functions and indicate whether the point is a local maximum or a local minimum.

(a) \( f(x) = 5x^2 + 30x + 3 \)

**Solution:** \( f'(x) = 10x + 30 \) and \( f''(x) = 10 > 0 \) ⇒ Convex function (min)
\( f'(x) = 0 \) ⇒ \( 10x + 30 = 0 \) ⇒ \( x = -3 \) is a local minimizer

(b) \( f(x) = -8x^2 + 4x - 50 \)

**Solution:** \( f'(x) = -16x + 4 \) and \( f''(x) = -16 < 0 \) ⇒ Concave function (max)
\( f'(x) = 0 \) ⇒ \( -16x + 4 = 0 \) ⇒ \( x = \frac{1}{4} \) is a local maximizer

4. Find the inverse of the following functions:

(a) \( y = f(x) \rightarrow y = x^2 + 4 \) with \( x \geq 0 \)

**Solution:** \( x = \sqrt{y - 4} \)

(b) \( p = f(Q) \rightarrow p = 3 - 6Q \)

**Solution:** \( Q = \frac{3-p}{6} \)
5. Let \( f(x) \) be defined as follows where \( x \in [0, 5] \) means \( 0 \leq x \leq 5 \):
\[
f(x) = \begin{cases}
-2x + 15 & \text{for } x \in [0, 5], \\
-4x + 25 & \text{for } x \in [5, 6].
\end{cases}
\]

(a) What is the slope of \( f(x) \) when \( x = 4 \)?

**Solution:** \(-2\)

(b) What is the slope of \( f(x) \) when \( x = 5.5 \)?

**Solution:** \(-4\)

(c) What is the slope of \( f(x) \) when \( x = 5 \)?

**Solution:** The slope is undefined when \( x = 5 \). For a derivative to exist, the function must be continuous and smooth. The above function is continuous, but has a kink so it is not smooth (The limit from the right does not equal the limit from the left.).

6. Solve the following integrals:

(a) \( \int_0^5 2x \, dx \)

**Solution:** \[
\int_0^5 2x \, dx = [x^2]_0^5 = 25
\]

(b) \( \int_1^4 \frac{1}{x} \, dx \)

**Solution:** \[
\int_1^4 \frac{1}{x} \, dx = [\ln(x)]_1^4 = \ln(4)
\]

7. Simplify the following:
\[
x \frac{y^{-2}}{x^{1/3} y^{2/3}}
\]

**Solution:**
\[
\frac{x^{1/3} y^{-2}}{x^{1/3} y^{2/3}} = \frac{x^{1/3} x^{2/3}}{y^{2/3} y^{1/3}} = \frac{x^{1+2/3}}{y^{2+1/3}} = \frac{x}{y}
\]
8. Solve the following for $x$: $\frac{x}{2} = \frac{8}{x^3}$ 

**Solution:** cross multiply to get $x^4 = 16$ so $x = \pm 2$

9. Solve the system of two equations and two unknowns and graph the two equations:

$$Q = 2p - 40$$

$$Q = 200 - p.$$ 

**Solution:**

\[
\begin{align*}
2p - 40 &= 200 - p \\
3p &= 240 \\
p &= 80 \\
Q &= 200 - (80) \\
Q &= 2(80) - 40 \\
Q &= 120
\end{align*}
\]

Set both $Q$’s equal to each other. Rearrange. Solve for $p$. Substitute $p = 80$ to solve for $Q$.

![Graph of supply and demand curves](image)
10. Solve the system of two equations and two unknowns:

\[ q_1 + 3q_2 = 10 \]
\[ 4q_1 + 8q_2 = 4. \]

**Solution:**

\begin{align*}
q_1 &= 10 - 3q_2 \\
4(10 - 3q_2) + 8q_2 &= 4 \\
40 - 12q_2 + 8q_2 &= 4 \\
36 &= 4q_2 \\
q_2 &= 9 \\
q_1 &= 10 - 3(9) = -17 \\
\end{align*}

Substitute \( q_2 = 9 \) into the equation for \( q_1 \).

11. Take the derivative of the following with respect to \( x_1 \) and \( x_2 \) where \( a, b \) and \( c \) are constants:

(a) \( U = 2x_1 + 3x_2 \)

**Solution:**

\[
\frac{\partial U}{\partial x_1} = 2 \quad \frac{\partial U}{\partial x_2} = 3
\]

(b) \( U = x_1^{\frac{1}{2}} + x_2 \)

**Solution:**

\[
\frac{\partial U}{\partial x_1} = \frac{1}{2}x_1^{-\frac{1}{2}} \quad \frac{\partial U}{\partial x_2} = 1
\]

(c) \( U = x_1 + \frac{1}{x_2} \)

**Solution:**

\[
\frac{\partial U}{\partial x_1} = 1 \quad \frac{\partial U}{\partial x_2} = -\frac{1}{x_2^2}
\]

(d) \( U = x_1^a + x_2^b \)

**Solution:**

\[
\frac{\partial U}{\partial x_1} = ax_1^{a-1} \quad \frac{\partial U}{\partial x_2} = bx_2^{b-1}
\]

(e) \( U = x_1^{0.3} + x_2^{0.7} \)

**Solution:**

\[
\frac{\partial U}{\partial x_1} = 0.3x_1^{-0.7} \quad \frac{\partial U}{\partial x_2} = 0.7x_2^{-0.3}
\]
(f) \( U = x_1^2 x_2^3 \)

Solution: \( \frac{\partial U}{\partial x_1} = 2x_1 x_2^3 \quad \frac{\partial U}{\partial x_2} = 3x_1^2 x_2^2 \)

(g) \( U = c x_1^a x_2^b \)

Solution: \( \frac{\partial U}{\partial x_1} = ac x_1^{a-1} x_2^b \quad \frac{\partial U}{\partial x_2} = bc x_1^a x_2^{b-1} \)

Applications

12. Find the slope of each of the following production functions, \( y = f(L) \) where \( L \) is the amount of labor. Draw a graph of the production function and a graph of the change in output with respect to a change in labor (this is the marginal product of labor). Give an economic interpretation of the slope of the derivative function.

(a) \( y = aL \) with \( a > 0 \)

Solution:

\[
MPL = \frac{dy}{dL} = a \\
\frac{d^2 y}{dL^2} = 0
\]

The MPL is constant so as you increase the amount of labor, you get the same increase in output.
(b) \( y = 10L^{\frac{3}{4}} \)

\begin{align*}
MPL &= \frac{dy}{dL} = \frac{20}{3} L^{-\frac{1}{3}} \\
\frac{d^2 y}{dL^2} &= -\frac{20}{9} L^{-\frac{4}{3}}
\end{align*}

The MPL always positive and is decreasing, so as you increase the amount of labor, output increases but at a decreasing rate.
(c) $y = 12L^2 - L^3$

Solution:

$$MPL = \frac{dy}{dL} = 24L - 3L^2$$

$$\frac{d^2y}{dL^2} = 24 - 6L$$

The MPL is positive for $L \leq 8$ and negative for $L > 8$. As you increase the amount of labor, output increases until $L = 8$ and then output decreases as $L$ increases. For $L > 12$, output is negative! The slope of the MPL is positive for $L \leq 4$ so additional labor increases output at an increasing amount. For $L \geq 4$, the slope of the MPL is negative so additional labor increases output (until $L > 8$) but at a decreasing rate.
\[ y = 12L^2 - L^3 \]

\[ \frac{dy}{dL} = 24L - 3L^2 \]