Intermediate Microeconomics
ECNS 301
Fall 2015

Homework #: 2

Due by the beginning of class on: Thursday September 10, 2015

Name: ____________________________________________________________

Instructions:
There are 5 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
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Optimization

1. Find the value(s) of \( x \) which maximize (or minimize) \( f(x) \).
   (a) \( f(x) = -2x^2 + 4x + 9 \)
   (b) \( f(x) = 5x^2 + x \)
   (c) \( f(x) = x^2 + 3 \)
   (d) \( f(x) = 3x^2 - 6x + 2 \)

2. Find all the values of \((x, y)\) that result in a critical point of \( f(x, y) \).
   (a) \( f(x, y) = x^2 + 2y^2 - 6x + 8y - 1 \)
   (b) \( f(x, y) = x^2 - 2y^2 - 6x + 8y + 3 \)
   (c) \( f(x, y) = x^2 + 6xy + 2y^2 - 6x + 10y - 2 \)
   (d) \( f(x, y) = x^2 - xy - 2y^2 + 7x - 8y + 3 \)
   (e) \( f(x, y) = -x^2 - 2xy - 2y^2 + 6x - 10y + 5 \)

Constrained Optimization

3. Find the values of \( x \) and \( y \) that maximize (or minimize) \( f(x, y) \) subject to the following constraints.
   (a) \( f(x, y) = xy, \) subject to \( x + 2y = 2 \)
   (b) \( f(x, y) = x(y + 4), \) subject to \( x + y = 8 \)
   (c) \( f(x, y) = x - 3y - xy, \) subject to \( x + y = 6 \)
   (d) \( f(x, y) = 7 - y + x^2, \) subject to \( x + y = 0 \)
   (e) \( f(x, y) = x + y^2, \) subject to \( x^2 + y^2 = 4 \)

Applications

4. Jane is running a lemonade stand at the farmer’s market. The farmer’s market only allows one stand, so she faces no competition. She currently sells lemonade for $1. Tarzan tells her that this price is too high and suggests that if she lowers the price she can sell more lemonade. Jane does not know her costs, but she does know that her marginal cost is positive. The inverse demand for lemonade is \( P = 3 - 0.01Q^2 \).
   (a) What is Jane’s revenue as a function of quantity?
   (b) What price will maximize revenue?
   (c) Is Tarzan right? How will lowering the price affect profits?
5. A firm’s output, $Q$, depends on the number of workers it hires, $L$. Its production function is $Q = 4L^{0.5}$. It can sell its output for $2, and workers are paid $1. It has no costs other than what it pays its workers.

(a) What are profits expressed in terms of the number of workers hired?

(b) What is the optimal number of workers it should hire? What are profits when the optimal number of workers are hired?

(c) The firm unionizes, and the union receives 25% of ownership of the firm. The union does not care about profits, but only cares about the number of workers hired ($L$). The other 75% of firm owners continue to care only about profits. The new objective function of the firm is as follows.

$$0.75\pi(L) + 0.25L$$

How many workers are hired with the union? What are the firm’s profits?