Intermediate Microeconomics
ECNS 301
Fall 2015

Homework #: 4

Due by the beginning of class on: Thursday September 24, 2015

Name: _______________________________________________________

Instructions:
There are 11 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
Preferences

1. Draw indifference curves for the following two goods to represent each of the following situations:

(a) The two goods are Coke and Pepsi. The individual can’t tell the difference.

Solution: The two goods are perfect substitutes so the indifference curves are linear with a slope of \(-1\). The utility function is \( u = C + P \) where \( C \) is the quantity of Coke and \( P \) is the quantity of Pepsi. If we put the \( C \) on the vertical axis and \( P \) on the horizontal, an equation for the indifference curve is \( C = \bar{u} - P \) for utility level \( \bar{u} \).
(b) The two goods are place-mats and napkins. To set a table, you need exactly the same number of napkins as place-mats, and extras of either cannot be used.

**Solution:** The two goods are perfect complements so the indifference curves form right angles. If you need one place-mat for every napkin, then the utility function is $u = \min\{P, N\}$ where $P$ is the quantity of place-mats and $N$ is the quantity of napkins.
(c) The two goods are money and cocaine. The more cocaine an individual consumes, the more money they are willing to give up to get more.

**Solution:** The indifference curves are concave rather than convex. As the amount of cocaine increases, you’re giving up more and more money.

Note that I just made up the levels of utility for each indifference curve.
(d) Financial assets (like a stock portfolio) get a financial return, but are associated with a certain amount of risk. Less risk is better than more risk. Draw indifference curves for two assets with different levels of risk and return.

**Solution:** The indifference curves are convex and increasing if the individual is risk averse. As you increase the amount of risk, the expected return is getting higher and higher.

2. Max prefers

<table>
<thead>
<tr>
<th>Preference</th>
<th>Max's Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who-Hash</td>
<td>eggs, sausage and Spam</td>
</tr>
<tr>
<td>eggs, sausage and Spam</td>
<td>green eggs and ham</td>
</tr>
<tr>
<td>green eggs and ham</td>
<td>linguine and clams</td>
</tr>
<tr>
<td>linguine and clams</td>
<td>Who-Hash</td>
</tr>
</tbody>
</table>

(a) What basic assumption about consumer preferences does this violate?

**Solution:** Transitivity
(b) Cindy-Lou tries to convince Max that he is irrational, but he insists that in each two-meal comparison above, the value of his more-preferred meal is at least $1 more than the value of his less-preferred meal. Advise Cindy-Lou on how she might make money by taking advantage of Max’s preferences.

Solution: Assume Cindy Lou has each of the meals to start out with, and that Max is starting out with Who-Hash. Max likes linguine with clams better than Who Hash and one dollar. He is willing to give Cindy Lou Who-Hash plus $1 to get linguine. But he is also willing to then trade her linguine plus $1 to get green eggs. And he is willing to trade his green eggs plus $1 to get eggs and sausage. And he will trade that plus $1 to get Who-Hash. But we already know Max will give that plus $1 to Cindy Lou to get linguine. The cycle continues until Cindy Lou has taken all of his money.

Utility

3. Clara’s utility function is \( U = x_A x_B \).

(a) She likes consuming 10 apples and 10 bananas as much as consuming 1 apple and how many bananas?

Solution: Find the level of utility when 10 apples and 10 bananas are consumed.

\[
U(10, 10) = (10)(10) = 100
\]

Now when utility is 100 and \( x_A = 1 \), we get

\[
100 = 1(x_B)
\]

so \( x_B = 100 \).

(b) Does this mean her preferences violate the assumption of convexity (prefer averages to extremes)?

Solution: No, it just means she’s indifferent between these two bundles.
Budget Constraints

4. If you could afford exactly 6 apples and 14 bananas, or 10 apples and 6 bananas, then if you spent all of your income on bananas, how many bananas could you buy?

Solution: The information in the problem gives you two points on the budget constraint. If you draw a line through those two points and then look to see where that line crosses the bananas axis, you’ll know how many bananas you could buy if you spent all your money on bananas.

Even better, we can figure out the equation for the budget constraint. In this problem

\[ m = p_b q_b + p_a q_a \]

is the budget constraint. If we put the quantity of bananas (\( q_b \)) of the vertical axis and the quantity of apples (\( q_a \)) on the horizontal axis, we can rewrite the equation for the budget constraint as

\[ q_b = \frac{m}{p_b} - \frac{p_a}{p_b} q_a. \]

The slope of this line is \(-\frac{p_a}{p_b} \) which is the marginal rate of transformation. When we move from 6 apples and 14 bananas to 10 apples and 6 bananas, the rise of this change is \(6 - 14 = -8 \) bananas and the run of this change is \(10 - 6 = 4 \) apples. The rise over the run is \(-\frac{8}{4} = -2 \) and is the slope of the budget constraint.

The equation for the budget constraint is now

\[ q_b = \frac{m}{p_b} - 2q_a \]

and our objective is to find the intercept, \( \frac{m}{p_b} \), as this is the amount of bananas you could buy if you spent all of your money on bananas. Take either the first point of 6 apples and 14 bananas or the second point of 10 apples and 6 bananas, substitute these values into the budget constraint for \( q_a \) and \( q_b \), and then solve for the intercept. Using the first point:

\[ q_b = \frac{m}{p_b} - 2q_a \]

\[ 14 = \frac{m}{p_b} - 2(6) \]

\[ 14 = \frac{m}{p_b} - 12 \]

\[ 26 = \frac{m}{p_b} \]
Using the second point:

\[
q_b = \frac{m}{p_b} - 2q_a \\
6 = \frac{m}{p_b} - 2(10) \\
6 = \frac{m}{p_b} - 20 \\
26 = \frac{m}{p_b}
\]

If you spend all your money on bananas, you could buy 26 bananas.

5. The Smith family has $1,000 per month to spend on food and other goods. (The two goods are food and dollars spent on all other goods. If you want, just assume that the prices of food and other goods are both $1.) Graph each budget constraint and write the equation. The units for the budget constraint will be dollars spent on food and dollars spent on all other goods. (Note: Some equations or lines may be defined piecewise/have multiple parts. Writing the equations may be trickier so be careful with the graphs first.)

Solution: For each part, remember that there are three steps for figuring out what the budget constraint looks like.

1. If you spend all your money on food, how much food could you buy?

2. If you spend all your money on other goods, how much of other goods could you buy?

3. Are there any points where the slope changes?

After you put all these points on the graph, all you need to do is connect the dots. The budget constraint for every part is represented by the following equation

\[
1000 = p_f q_f + p_{og} q_{og}.
\]

In the graphs below, I put other goods on the vertical axis and food on the horizontal axis. To represent the budget constraint with an equation, it’s useful to solve for the quantity of other goods.

\[
q_{og} = \frac{1000}{p_{og}} - \frac{p_f}{p_{og}} q_f
\]

Additionally, if you want, you could consider each price to be equal to one so that \( p_f = p_{og} = 1 \). The equation for the basic budget constraint is then \( q_{og} = 1000 - q_f \).
(a) They are given $200 in welfare payments.

**Solution:** If they spent all their money on other goods, they could buy $1,200 worth. If they spent all their money on food, they could buy $1,200 worth. The slope of the budget constraint does not change. The graph of the budget constraint is as follows.

The equation for the budget constraint is as follows.

\[ q_{og} = \frac{1200}{p_{og}} - \frac{p_f}{p_{og}}q_f \]
(b) They are given $200 in food stamps, which cannot be sold.

**Solution:**
If they spent all their money on other goods, they could buy $1,000 worth. If they spent all their money on food, they could buy $1,200 worth. If they wanted to, they could also spend $1,000 on other goods and $200 on food. At this point the slope of the budget constraint changes. The graph of the budget constraint is as follows.

The equation for the budget constraint is as follows.

\[ q_{og} = \begin{cases} 
1000 & \text{for } 0 \leq p_f q_f \leq 200 \\
\frac{1200}{p_{og}} - \frac{p_f}{p_{og}} q_f & \text{for } 200 \leq p_f q_f \leq 1200 
\end{cases} \]
(c) They are given $100 in food stamps and the option to buy another $100 worth of food stamps at the price of $0.50 per $1 of food stamps. (This was the actual government policy until 1979.)

**Solution:**

If they spent all their money on other goods, they could buy $1,000 worth. If they spent all their money on food, they could buy $1,150 worth. The first $100 worth of food they get in food stamps, the second $100 worth of food in food stamps costs them $50, and then they have $950 left to spend on food. Add that up to get $1,150 worth of food. The budget constraint will change slopes at $(100, 1000)$ and at $(200, 950)$ where the numbers represent (dollar amount of food, dollar amount of other goods). The graph of the budget constraint is as follows.

The equation for the budget constraint is as follows.

\[
q_{og} = \begin{cases} 
1000 & \text{for } 0 \leq p_f q_f \leq 100 \\
\frac{1050}{p_{og}} - \frac{p_f}{2p_{og}} q_f & \text{for } 100 \leq p_f q_f \leq 200 \\
\frac{1150}{p_{og}} - \frac{p_f}{p_{og}} q_f & \text{for } 200 \leq p_f q_f \leq 1150
\end{cases}
\]
(d) They are given $200 in food stamps, but each $1 of food stamps can be sold on the black market for $0.50.

**Solution:**

If they spent all their money on other goods, they could buy $1,100 worth. This assumes they convert their $200 in food stamps to $100 worth of other goods. If they spent all their money on food, they could buy $1,200 worth. If they wanted to, they could also spend $1,000 on other goods and $200 on food. At this point the slope of the budget constraint changes. The graph of the budget constraint is as follows.

![Graph of budget constraint](image)

The equation for the budget constraint is as follows.

\[
q_{og} = \begin{cases} 
1100 - \frac{p_f}{2p_{og}} q_f & \text{for } 0 \leq p_f q_f \leq 200 \\
1200 - \frac{p_f}{p_{og}} q_f & \text{for } 200 \leq p_f q_f \leq 1200 
\end{cases}
\]

**The Consumer’s Problem**

6. Use indifference curves to show that the Smith family’s (question 5) preferences determine if they will take advantage of the option to purchase food stamps in part c. (3)
**Solution:** Whether or not they take advantage of this option depends on where their highest indifference curve is tangent to the budget constraint.

If their preferences are represented by the red indifference curve, then they don’t take full advantage of the option. If their preferences are represented by the blue indifference curve, then they do take full advantage of the option.
7. Charlie has a utility function $U = x_A^2 x_B$. He has $100 to spend on apples and bananas. Apples cost $1 and bananas cost $2.

(a) What is the equation of his budget constraint? Draw a graph of it. What is its slope?

**Solution:** Let’s put the quantity of bananas, $x_B$, on the vertical axis and the quantity of apples, $x_A$ on the horizontal axis. The budget constraint is

$$100 = x_A + 2x_B.$$ 

Solving for $x_B$ we get

$$x_B = 50 - \frac{x_A}{2}$$

so the slope of the budget constraint is $-\frac{1}{2}$.

Here’s the graph:
(b) What is his marginal utility of apples?

**Solution:** The marginal utility of apples is the partial derivative of the utility function with respect to a change in the quantity of apples.

\[
\frac{\partial U}{\partial x_A} = 2x_A x_B
\]

(c) What is his marginal utility of bananas?

**Solution:** The marginal utility of bananas is the partial derivative of the utility function with respect to a change in the quantity of bananas.

\[
\frac{\partial U}{\partial x_B} = x_A^2
\]

(d) What is a mathematical expression for the slope of his indifference curves?

**Solution:** The utility function is

\[
U = x_A^2 x_B
\]

and if the utility level is \( \bar{u} \) then we have

\[
\bar{u} = x_A^2 x_B.
\]

The equation for any indifference curve is

\[
x_B = \frac{\bar{u}}{x_A^2}
\]

because we put \( x_B \) on the vertical axis. You would get higher or lower indifference curves by increasing or decreasing \( \bar{u} \). The slope of the indifference curve is found by taking the derivative of the indifference curve equation.

\[
\frac{dx_B}{dx_A} = -\frac{2\bar{u}}{x_A^3}
\]

We could simplify this by substituting in \( \bar{u} = x_A^2 x_B \).

\[
\frac{dx_B}{dx_A} = -\frac{2x_A^2 x_B}{x_A^3}
\]

\[
\frac{dx_B}{dx_A} = -\frac{2x_B}{x_A}
\]

You also get the same thing when you take the ratio of the marginal utilities.
(e) How many apples and bananas should Charlie buy?

**Solution:** This is the solution to the consumer’s problem. Charlie’s optimization problem is

$$\max_{x_A, x_B} U = x_A^2 x_B$$

subject to $100 = x_A + 2x_B$.

Write the Lagrangian as

$$\max_{x_A, x_B, \lambda} L(x_A, x_B, \lambda) = x_A^2 x_B + \lambda (100 - x_A - 2x_B)$$

and the first order conditions are as follows.

$$\frac{\partial L}{\partial x_A} = 2x_A x_B - \lambda = 0$$

$$\frac{\partial L}{\partial x_B} = x_A^2 - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 100 - x_A - 2x_B = 0$$

Combine the first two first order conditions to get

$$2x_A x_B = \frac{x_A^2}{2}$$

$$4x_B = x_A.$$  

Now plug this into the last first order condition (the budget constraint) to get

$$100 = x_A + 2x_B$$

$$100 = (4x_B) + 2x_B$$

$$100 = 6x_B$$

$$x_B^* = \frac{50}{3}$$

$$x_A^* = 4(\frac{50}{3}) = \frac{200}{3}$$
8. Consider the standard case in which a consumer’s utility maximizing choice occurs at a tangency between an indifference curve and a budget constraint.

True or False and explain: Any other budget constraint which passes through the original point of tangency leaves the consumer better off. Use graphs to justify your answer.

Solution:
True, any other budget constraint which passes through the original point of tangency would leave the consumer better off. The graphs below illustrate two different scenarios. In one, the budget constraint becomes steeper and in the other the budget constraint is flatter.

Since you can always afford the initial bundle \((x_0^*, y_0^*)\), you can never be worse off. After the budget constraint changes, there are new bundles that are affordable that weren’t affordable before. One of these new bundles will be tangent to a higher indifference curve.

The new budget constraint has a different slope which implies that the price ratio changes. To get the new budget constraint to pass through a point on the old budget constraint when the price ratio changes, you have to increase the income. The increase in income is one reason why the consumer is better off.
9. Consider the following utility function: \( U = x_1^2x_2 \). The consumer has income of \( M \), the price of \( x_1 \) is \( P_1 \), and the price of \( x_2 \) is \( P_2 \).

(a) What is the marginal rate of substitution?

**Solution:** The marginal rate of substitution is the slope of the indifference curve. The utility function is

\[ U = x_1^2x_2 \]

and if the utility level is \( \bar{u} \) then we have

\[ \bar{u} = x_1^2x_2. \]

The equation for any indifference curve is

\[ x_2 = \frac{\bar{u}}{x_1^2} \]

because we put \( x_2 \) on the vertical axis. You would get higher or lower indifference curves by increasing or decreasing \( \bar{u} \). The slope of the indifference curve is found
by taking the derivative of the indifference curve equation.

\[
\frac{dx_2}{dx_1} = -\frac{2\bar{u}}{x_1^3}
\]

We could simplify this by substituting in \( \bar{u} = x_1^2 x_2 \).

\[
\frac{dx_2}{dx_1} = -\frac{2x_1^2 x_2}{x_1^3}
\]

\[
\frac{dx_2}{dx_1} = -\frac{2x_2}{x_1}
\]

You also get the same thing when you take the ratio of the marginal utilities.

(b) What is the equation for the budget constraint?

**Solution:** The budget constraint is

\[
M = P_1 x_1 + P_2 x_2.
\]

Solving for \( x_2 \) we get

\[
x_2 = \frac{M}{P_2} - \frac{P_1}{P_2} x_1.
\]

(c) What is the optimal consumption bundle if \( M = 60, \ P_1 = 1 \) and \( P_2 = 2 \)? What level of utility is achieved?

**Solution:** This is the solution to the consumer’s problem. The optimization problem is

\[
\max_{x_1, x_2} U = x_1^2 x_2
\]

subject to \( M = P_1 x_1 + P_2 x_2 \).

Write the Lagrangian as

\[
\max_{x_1, x_2, \lambda} \mathcal{L}(x_1, x_2, \lambda) = x_1^2 x_2 + \lambda (M - P_1 x_1 - P_2 x_2)
\]
and the first order conditions are as follows.

\[ \frac{\partial L}{\partial x_1} = 2x_1x_2 - P_1\lambda = 0 \]
\[ \frac{\partial L}{\partial x_2} = x_1^2 - P_2\lambda = 0 \]
\[ \frac{\partial L}{\partial \lambda} = 100 - P_1x_1 - P_2x_2 = 0 \]

Combine the first two first order conditions to get

\[ \frac{2x_1x_2}{P_1} = \frac{x_1^2}{P_2} \]
\[ 2P_2x_2 = P_1x_1. \]

Now plug this into the last first order condition (the budget constraint) to get

\[ M = P_1x_1 + P_2x_2 \]
\[ M = (2P_2x_2) + P_2x_2 \]
\[ M = 3P_2x_2 \]
\[ x_2^* = \frac{M}{3P_2} \]
\[ x_1^* = \frac{2P_2x_2}{P_1} = \frac{2M}{3P_1} \]

If \( M = 60 \), \( P_1 = 1 \) and \( P_2 = 2 \), then

\[ x_2^* = \frac{M}{3P_2} = \frac{60}{3(2)} = 10 \]
\[ x_1^* = \frac{2M}{3P_1} = \frac{2(60)}{3(1)} = 40 \]
\[ U = x_1^2x_2 = (40)^210 = 16000 \]

(d) Derive the demand function for good \( x_1 \) with income of \( M \), the price of \( x_1 \) is \( P_1 \), and the price of \( x_2 \) is \( P_2 \) (general, not specific).

\[ \textbf{Solution:} \text{ We did this in the last part.} \]
\[ x_1^* = \frac{2M}{3P_1} \]

10. Carrie’s utility function is \( U = 3x + y \), where \( x \) is cocoa and \( y \) is tea.
(a) Are cocoa and tea substitutes or complements for Carrie? How many mugs of cocoa are equivalent to a mug of tea in terms of her utility level?

**Solution:** Cocoa and tea are perfect substitutes. Carrie is always willing to substitute 1 cocoa for 3 teas.

(b) If the price of cocoa is $8, the price of tea is $3 and her income is $24, how many units of cocoa and tea will she consume?

**Solution:** Since cocoa and tea are perfect substitutes, Carrie will either spend all her money on cocoa or all her money on tea. If she spends all her money on cocoa, she can buy $24/8 = 3$ cocoas and her utility level is $U = 3(3) + 0 = 9$. If she spends all her money on tea, she can buy $24/3 = 8$ teas and her utility level is $U = 3(0) + 8 = 8$. She’s better off buying 3 cocoas and 0 teas.

(c) What is another utility function that would also represent Carrie’s preferences?

**Solution:** Another utility function representing the same preferences is

$$U = 3x + y + 1000000.$$  

Any monotonic transformation works, so you could add any number or multiply utility by any constant, or any other monotonic transformation.

11. Miss Muffet consumes two units of whey per unit of curds. (Whey and curds are complements for her.) Her utility can be expressed as $U = \min\{w, 2c\}$.

(a) Draw sample indifference curves for Miss Muffet, labeling them carefully.

**Solution:** Her indifference curves form right angled L’s.
(b) If the price of curds is $3, the price of whey is $6 and Miss Muffet’s income is $30, what is her demand for curds?

**Solution:** She’ll always consumer bundles where \( w = 2c \). If this is not the case, then she’s either spending too much on whey or curds. Her budget constraint is

\[
30 = 3c + 6w.
\]

Substitute \( w = 2c \) into this equation to get the following.

\[
\begin{align*}
30 &= 3c + 6(2c) \\
30 &= 3c + 12c \\
30 &= 15c \\
c^* &= 2 \\
w^* &= 2c^* = 4
\end{align*}
\]

Her quantity demanded of curds is 2.