Intermediate Microeconomics
ECNS 301
Fall 2015

Homework #: 9

Due by the beginning of class on: Thursday December 3, 2015

Name: ________________________________

Instructions:
There are 2 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
Monopoly

1. Cigna is a patent holder on a key anti-retroviral drug in the AIDS cocktail, and therefore is a monopolist. Assume the following demand and cost functions:

\[ Q = 100 - 4P \quad \text{and} \quad C(Q) = 20 + Q^2. \]

(a) Calculate the profit maximizing price, output level, and profits.

Solution:

\[
\begin{align*}
Q &= 100 - 4P \\
P &= 25 - \frac{1}{4}Q \\
R(Q) &= \left(25 - \frac{1}{4}Q\right)Q \\
MR(Q) &= 25 - \frac{1}{2}Q \\
C(Q) &= 20 + Q^2 \\
MC(Q) &= 2Q \\
AC(Q) &= \frac{20}{Q} + Q \\
\end{align*}
\]

The firm’s profit maximization problem is

\[
\max_Q \pi(Q) = R(Q) - C(Q)
\]

and the first order condition is

\[
\frac{d\pi(Q)}{dQ} = MR(Q) - MC(Q).
\]

Setting the first order condition equal to zero, we get that to maximize profits the marginal revenue must equal the marginal cost.

\[
\begin{align*}
MR(Q) &= MC(Q) \\
25 - \frac{1}{2}Q &= 2Q \\
25 &= \frac{5}{2}Q \\
Q^* &= 10 \\
P^* &= 25 - \frac{1}{4}Q^* = 22.5 \\
\pi(Q^*) &= R(Q^*) - C(Q^*) = (22.5)(10) - 20 - (10)^2 = 105
\end{align*}
\]
(b) Graph demand, $MR(Q)$, $MC(Q)$, and $AC(Q)$. Indicate the area representing profits.

**Solution:** The graph should have the intercept for demand and $MR$ at 25, $MC$ crossing $MR$ at $Q = 10$, Profits = rectangle, with points on the $P$ axis at 12 (the $ATC$) and 22.5 (the Price), and $Q = 10$. The height of the rectangle should go from the $AC$ curve to the demand curve.

(c) What is the price that maximizes Total Surplus?

**Solution:** The price that maximizes total surplus is where the demand curve crosses the marginal cost curve.

$$P = MC(Q)$$
$$25 - \frac{1}{4}Q = 2Q$$
$$25 = \frac{9}{4}Q$$
$$Q^* = \frac{100}{9} = 11.111$$
$$P^* = 25 - \frac{1}{4}Q^* = 25 - \frac{100}{36} = 22.222.$$ 

(d) What is the deadweight loss due to the monopoly pricing? Assume that the supply curve for the competitive industry would be given by $P = 2Q$.

**Solution:** Deadweight loss is the area of the triangle: $\frac{1}{2}(11.111 - 10)(22.5 - 20) = 1.375$. Deadweight loss can also be expressed by the integral $\int_{10}^{11.111}(25 - \frac{1}{4}Q - 2Q)dQ = 1.375$.

2. Suppose Cigna develops a new drug and is a patent holder. This drug has demand $Q = 16 - 2P$ and cost function $C(Q) = 4Q$. Calculate the monopoly output, market price, and profits.

**Solution:** Profits are maximized by producing the quantity where marginal revenue
equals marginal cost.

\[ Q = 16 - 2P \]
\[ P = 8 - \frac{1}{2}Q \]
\[ R(Q) = \left(8 - \frac{1}{2}Q\right)Q \]
\[ MR = 8 - Q \]
\[ MC = 4 \]
\[ 8 - Q = 4 \]
\[ Q^* = 4 \]
\[ P^* = 8 - \frac{1}{2}Q^* = 6 \]
\[ \pi = R(Q) - C(Q) = (4)(8) - 4(4) = 24 - 16 = 8 \]