Instructions:
You must answer exactly 4 of the following 5 questions. Each question is worth the same amount. You have the class period to complete the exam.
Answer each question clearly and concisely. You must show your work to receive credit.
This exam is given under the rules of the Montana State University. You may not use notes or receive any assistance. There is to be no talking during the exam.
Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet.
1. For the following questions, refer to the first part of the attached article, “In Retreat, Bank of America Cancels Debit Card Fee” by Bernard in the 11/1/2011 edition of the New York Times.

(a) Describe a simplistic version of the game that Bank of America is trying to play. Your description should explicitly list all the elements of your game.

**Solution:** You need to describe the game in your own words and list the elements of the game. The elements of the game include the players, rules and payoffs.

(b) Describe the dynamic features of your game and describe why they are important.

**Solution:** The dynamic features would be the features of the game that occur over time. You could have discussed the sequential nature of the game or that it is a repeated game.

(c) Give two economic reasons why the nation’s 2nd largest bank might have trouble establishing an equilibrium with debit card fees.

**Solution:** Refer to the handout I gave you in class regarding cartels.

(d) Assuming that an equilibrium with debit card fees is established, what are two factors that are important for maintaining this equilibrium?

**Solution:** Refer to the handout I gave you in class regarding cartels.

2. A durable goods monopolist faces an inverse demand function of \( P = 20 - Q \) and has a constant marginal cost of production equal to 4. The monopolist can choose two prices before the good becomes obsolete: a price today \((P_1)\) and a price tomorrow \((P_2)\). Tomorrow’s profits are discounted at an interest rate of 25%.

(a) What is the quantity demanded in the second period taking into consideration the consumers who purchased the durable product in the first period?

**Solution:** Rearrange the inverse demand to get the demand: \( Q = 20 - P \). In the second period, the quantity demanded is \( Q_2 = 20 - P_2 \), but \( Q_1 = 20 - P_1 \) of those consumers bought in the first period and will not buy again. This leaves \( Q_2 - Q_1 \) consumers in the second period. \( Q_2 - Q_1 = 20 - P_2 - 20 + P_1 = P_1 - P_2 \).
(b) Find the firm’s period 2 price expressed as a best response to their first period price.

**Solution:** The firm’s profit maximization problem in the second period is as follows.

\[
\max_{P_2} \pi_2 = (P_2 - 4)(P_1 - P_2)
\]

\[
\frac{\partial \pi_2}{\partial P_2} = P_1 - P_2 - P_2 + 4 = 0
\]

\[2P_2 = P_1 + 4\]

\[P_2 = \frac{P_1}{2} + 2\]

(c) Setup the firm’s first period profit maximization problem.

**Solution:**

\[
\max_{P_1} \pi = (P_1 - 4)(20 - P_1) + \frac{4}{5}(P_2 - 4)(P_1 - P_2)
\]

where \(P_2 = \frac{P_1}{2} + 2\).

Note that \(\frac{1}{1+r} = \frac{4}{5}\). Also note that \(P_1 - P_2 = P_1 - \frac{P_1}{2} - 2 = \frac{P_1}{2} - 2\) and \(P_2 - 4 = \frac{P_1}{2} + 2 - 4 = \frac{P_1}{2} - 2\). Substituting this into the profit maximization problem yields the following.

\[
\max_{P_1} \pi = (P_1 - 4)(20 - P_1) + \frac{4}{5}\left(\frac{P_1}{2} - 2\right)^2
\]

(d) What are the equilibrium prices in each period?

**Solution:**

\[
\max_{P_1} \pi = (P_1 - 4)(20 - P_1) + \frac{4}{5}\left(\frac{P_1}{2} - 2\right)^2
\]

\[
\frac{\partial \pi}{\partial P_1} = 20 - P_1 - P_1 + 4 + \left(\frac{4}{5}\right)2\left(\frac{1}{2}\right)\left(\frac{P_1}{2} - 2\right) = 0
\]

\[200 - 20P_1 + 40 + 4P_1 - 16 = 0\]

\[224 = 16P_1\]

\[P_1^* = 14\]

\[P_2^* = 7 + 2 = 9\]
3. Two firms, firm A and firm B, compete in prices and both firms have the same constant marginal cost. When both firms collude, they do so in an optimal fashion and the profits of each firm is $\pi$. In each period, Firm B only chooses between two strategies: the price that results from optimal collusion and the price that results from a non-cooperative equilibrium with no collusion. Firm B’s dynamic strategy is to play a type of tit-for-tat. With this strategy, Firm B plays the price associated with optimal collusion only when Firm A plays this price in the previous period. If Firm A played any other price in the previous period, then Firm B plays the price that results from a non-cooperative equilibrium with no collusion. The interest rate is $r$.

(a) If Firm A makes a profitable deviation from a collusive equilibrium, what is the price and profits associated with an optimal deviation?

**Solution:** Let $p$ be the optimal price with collusion so that when each firm plays $p$, the payoff to each firm is $\pi$. Since the firms compete in prices, the price associated with an optimal deviation from a collusive equilibrium is $p^* = p - \varepsilon$ where $\varepsilon$ is a small positive number. The profits associated with the optimal deviation are $2\pi - \sigma$ where $\sigma$ is a small positive number.

(b) What are Firm A’s profits one period after Firm A makes a profitable deviation?

**Solution:** One period after the deviation, Firm B plays their marginal cost and both firm have zero profits.

(c) What are the discounted sum of profits associated with a collusive equilibrium?

**Solution:** Each firm gets $\pi$ every period, so the discounted sum of profits is $\frac{1}{r}\pi$.

(d) If both firms are in a collusive equilibrium and Firm A makes an optimal deviation in one period but then plays the collusive strategy from then on out, what are the discounted sum of profits for Firm A associated with the one time deviation?

**Solution:** In the deviating period, the firm gets $2\pi - \sigma$. In the period after, they get 0. Every period after that, they get $\pi$. The discounted sum of profits is

$$
\left( \frac{1}{(1 + r)} \right) (2\pi - \sigma) + \left( \frac{1}{(1 + r)} \right)^2 (0) + \left( \frac{1}{(1 + r)} \right)^2 \left( \frac{1}{r} \right) \pi
$$
(e) In this case, what should the interest rate be so that a collusive equilibrium is maintained? Why?

**Solution:** To maintain a collusive equilibrium, the following condition must hold.

\[
\frac{1}{r^2} \pi \geq \left( \frac{1}{1 + r} \right)^2 (2\pi - \sigma) + \left( \frac{1}{1 + r} \right) \left( \frac{1}{r} \right) \pi
\]

Since \( \sigma \) is small, take the limit as \( \sigma \) goes to zero.

\[
\frac{1}{r^2} \pi \geq \left( \frac{1}{1 + r} \right)^2 2\pi + \left( \frac{1}{r} \right) \pi
\]

\[
(1 + r)^2 \pi \geq r(1 + r)2\pi + \pi
\]

\[
(1 + r)((1 + r)\pi - 2r\pi) \geq \pi
\]

\[
(1 + r)(1 - r) \geq 1
\]

\[
1 - r^2 \geq 1
\]

\[
0 \geq r^2
\]

We normally expect that the interest rate is non-negative, so the condition only holds when \( r = 0 \). When \( r = 0 \), you don’t discount the future at all. Collusion is only maintained without collusion because the firm can deviate in one period and get \( 2\pi \) now, nothing tomorrow, and \( \pi \) every period after that. If you discount the future, you’d rather have more today than a constant stream over time.

4. Consider a linear city Hotelling model. There are at most two firms, A and B, located at the ends of the product space. The length of the product space is 5 and transportation costs are 1 times the distance traveled. Each consumer has a baseline valuation of 10 and each firm has a constant marginal cost of 2.

(a) If Firm A is a monopolist and located to the very left of the product space, characterize the indifferent consumer.

**Solution:** The utility a consumer located at \( x \) receives from buying from A is: \( u_A = 10 - p_A - x \). Set \( u_A = 0 \) to find the indifferent consumer’s location. \( 10 - p_A - x = 0 \rightarrow x = 10 - p_A \).
(b) If Firm A is a monopolist and located to the very left of the product space, what is the equilibrium price, quantity and profit?

**Solution:** Firm A’s profit maximization problem is as follows.

\[
\max_{p_A} \pi_A = (p_A - 2)(10 - p_A)
\]

\[
\frac{\partial \pi_A}{\partial p_A} = 10 - p_A - p_A + 2 = 0
\]

\[2p_A = 12\]

\[p_A = 6\]

\[x = 10 - 6 = 4 = q_A\]

\[\pi_A = (6 - 2)(4) = 16\]

(c) If Firm B enters the market and is located to the very right of the product space, what is the new competitive equilibrium price?

**Solution:** The utility a consumer located at \(x\) receives from buying from B is:

\[u_B = 10 - p_B - 5 + x\]

Set \(u_A = u_B\) to find the indifferent consumer’s location.

\[u_A = u_B\]

\[10 - p_A - x = 10 - p_B - 5 + x\]

\[p_B - p_A + 5 = 2x\]

\[x = \frac{p_B - p_A}{2} + \frac{5}{2}\]

Firm A’s profit maximization problem is as follows.

\[
\max_{p_A} \pi_A = (p_A - 2) \left( \frac{p_B - p_A}{2} + \frac{5}{2} \right)
\]

\[
\frac{\partial \pi_A}{\partial p_A} = \frac{p_B - p_A}{2} + \frac{5}{2} - \frac{p_A}{2} + 1 = 0
\]

\[p_B - p_A + 5 - p_A + 2 = 0\]

\[2p_A = p_B + 7\]

\[2p^* = p^* + 7\]

\[p^* = 7\]
(d) Compare the monopoly outcome to the duopoly outcome.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>Primitive</th>
<th>Monopoly</th>
<th>Duopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ((p))</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Firm quantity ((q))</td>
<td>4</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Market quantity ((Q))</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Firm Profits ((\pi))</td>
<td>16</td>
<td>12.5</td>
<td></td>
</tr>
<tr>
<td>Industry Profits</td>
<td>16</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>8</td>
<td>7.5</td>
<td></td>
</tr>
</tbody>
</table>

Note that the duopoly price is higher. This is an example of where competition leads to an increase in price.

5. Consider a Salop circular model of product differentiation with equally spaced firms. The circumference of the circle is 10. Each consumer has transportation costs of 1 times the distance traveled and a baseline valuation of 100. Firms have constant marginal costs of 5. There are \(N\) firms. Answer the following question for a competitive equilibrium assuming that it exists.

(a) Express a firm’s demand as a function of the neighboring firm’s prices and the number of firms. What will a firm’s demand be in equilibrium?

**Solution:** Let Firm \(A\) be located at zero. The length between each firm is \(10/N\). The indifferent consumer between Firm \(A\) and Firm \(B\) to the right is as follows.

\[
100 - p_A - x = 100 - p_B - \frac{10}{N} + x
\]

\[
x = \frac{p_B - p_A}{2} + \frac{5}{N}
\]

The quantity demanded from the consumers to the right of Firm \(A\) is \(\frac{p_B - p_A}{2} + \frac{5}{N}\). Likewise the quantity demanded from the consumers to the left of Firm \(A\) (with Firm \(C\) being the closest to Firm \(A\)) is \(\frac{p_C - p_A}{2} + \frac{5}{N}\). The firms demand is \(\frac{p_B - p_A}{2} + \frac{p_C - p_A}{2} + \frac{10}{N}\). In equilibrium, all the prices are the same \((p_A = p_B = p_C = \ldots = p^*)\) and equilibrium demand is \(\frac{10}{N}\).
(b) Setup a firm’s profit maximization problem and determine a firm’s best response as a function of the number of firms and neighboring firm’s prices.

**Solution:**

\[
\begin{align*}
\max_{p_A} \pi &= (p_A - 5) \left( \frac{p_B - p_A}{2} + \frac{p_C - p_A}{2} + \frac{10}{N} \right) \\
\frac{\partial \pi}{\partial p_A} &= \frac{p_B - p_A}{2} + \frac{p_C - p_A}{2} + \frac{10}{N} - p_A + 5 = 0 \\\np_B + p_C - 2p_A + \frac{20}{N} - 2p_A + 10 = 0 \\\n4p_A &= p_B + p_C + \frac{20}{N} + 10 \\\np_A &= \frac{p_B + p_C}{4} + \frac{5}{N} + \frac{5}{2}
\end{align*}
\]

(c) What is the equilibrium price as a function of the number of firms?

**Solution:** Impose price symmetry on the best response found above.

\[
\begin{align*}
p_A &= \frac{p_B + p_C}{4} + \frac{5}{N} + \frac{5}{2} \\
p^* &= \frac{p^*}{2} + \frac{5}{N} + \frac{5}{2} \\
p^* &= \frac{5}{N} + \frac{5}{2} \\
p^* &= \frac{10}{N} + 5
\end{align*}
\]

(d) Determine the following comparative static: \( \frac{\partial \pi}{\partial N} \).

**Solution:** From above, \( \pi = (p_A - 5) \left( \frac{10}{N} \right) \), and \( p_A - 5 = \frac{10}{N} \). The equilibrium profits are \( \pi = \left( \frac{10}{N} \right)^2 \). \( \frac{\partial \pi}{\partial N} = -2 \frac{10^2}{N^2} < 0 \).