Industrial Organization
ECNS 406
Fall 2012

Exam #: 2

Thursday November 8, 2012

Name: ____________________________________________

Instructions:
You must answer exactly 4 of the following 5 questions. Be sure to clearly indicate which questions you are answering. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. You may not use notes or receive any assistance. There is to be no talking during the exam.

Clearly print your name on your blue book. You must turn in your blue book(s) You do not need to turn in the questions, only your blue book(s).
1. Consider a linear city Hotelling model. There are at most two firms, A and B, located at the ends of the product space. The length of the product space is 5 and transportation costs are 1 times the distance traveled. Each consumer has a baseline valuation of 10 and each firm has a constant marginal cost of 2.

(a) If Firm A is a monopolist and located to the very left of the product space, characterize the indifferent consumer.

\textbf{Solution:} The utility a consumer located at \( x \) receives from buying from A is: \( u_A = 10 - p_A - x \). Set \( u_A = 0 \) to find the indifferent consumer’s location.

\[ 10 - p_A - x = 0 \rightarrow x = 10 - p_A. \]

(b) If Firm A is a monopolist and located to the very left of the product space, what is the equilibrium price, quantity and profit?

\textbf{Solution:} Firm A’s profit maximization problem is as follows.

\[ \max_{p_A} \pi_A = (p_A - 2)(10 - p_A) \]

\[ \frac{\partial \pi_A}{\partial p_A} = 10 - p_A - p_A + 2 = 0 \]

\[ 2p_A = 12 \]

\[ p_A = 6 \]

\[ x = 10 - 6 = 4 = q_A \]

\[ \pi_A = (6 - 2)(4) = 16 \]

(c) If Firm B enters the market and is located to the very right of the product space, what is the new competitive equilibrium price?

\textbf{Solution:} The utility a consumer located at \( x \) receives from buying from B is: \( u_B = 10 - p_B - 5 + x \). Set \( u_A = u_B \) to find the indifferent consumer’s location.

\[ u_A = u_B \]

\[ 10 - p_A - x = 10 - p_B - 5 + x \]

\[ p_B - p_A + 5 = 2x \]

\[ x = \frac{p_B - p_A}{2} + \frac{5}{2} \]
Firm A’s profit maximization problem is as follows.

$$\max_{p_A} \pi_A = (p_A - 2) \left( \frac{p_B - p_A}{2} + \frac{5}{2} \right)$$

$$\frac{\partial \pi_A}{\partial p_A} = \frac{p_B - p_A}{2} + \frac{5}{2} - p_A + 1 = 0$$

$$p_B - p_A + 5 - p_A + 2 = 0$$

$$2p_A = p_B + 7$$

$$2p^* = p^* + 7$$

$$p^* = 7$$

(d) Compare the monopoly outcome to the duopoly outcome.

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Note that the duopoly price is higher. This is an example of where competition leads to an increase in price.

2. Consider a Salop circular model of product differentiation with two equally spaced firms. The circumference of the circle is 6. Each consumer has transportation costs of 1 times the distance traveled and a baseline valuation of 9. Both firms have a constant marginal cost of 4. Answer the following questions for the competitive equilibrium if it exists.

(a) What is each firm’s best response function?

| Solution: | As discussed in class there are three possible types of equilibrium: competitive, touching, or local monopoly. The local monopoly case (with one firm) was explored in the question above. The outcome had each firm (there was only one) supplying over half of the market. With two firms the local monopoly outcome is not an equilibrium because both firms can’t supply over half the market each. Thus, we can rule out the possibility of a local monopoly.

To determine the competitive and touching outcome, we start by finding the locations of the indifferent consumers. First there is the consumer located at $x$ who is indifferent from buying from firm A or from firm B. Set $u_A(x) = u_B(x)$ |
and solve for $x$ to get the location of the indifferent consumer.

$$u_A(x) = u_B(x)$$
$$9 - p_A - x = 9 - p_B - (3 - x)$$
$$2x = p_B - p_A + 3$$
$$x = \frac{p_B - p_A}{2} + \frac{3}{2}$$

Next, there’s the indifferent consumer located at $y$ who is indifferent to purchasing from firm A and not purchasing at all.

$$u_A(y) = 0$$
$$9 - p_A - 1(y) = 0$$
$$y = 9 - p_A$$

Finally, there’s the indifferent consumer located at $z$ who is indifferent to purchasing from firm B and not purchasing at all.

$$u_B(z) = 0$$
$$9 - p_B - 1(3 - z) = 0$$
$$z = p_B - 6$$

With the indifferent consumers we define the demand for each firm. The demand each firm faces depends on whether their at a competitive equilibrium or the touching equilibrium. Firm A is at a competitive equilibrium if $x < y$ and a touching equilibrium if $x = y$. Firm B is at a competitive equilibrium if $x > z$ and a touching equilibrium if $x = z$. We reduce each of these conditions as follows.

\begin{align*}
\frac{(p_B - p_A)}{2} + \frac{3}{2} &< 9 - p_A \\
p_B - p_A + 3 &< 18 - 2p_A \\
p_A &< 15 - p_B \\
x &> z
\end{align*}

\begin{align*}
\frac{(p_B - p_A)}{2} + \frac{3}{2} &> p_B - 6 \\
p_B - p_A + 3 &> 2p_B - 12 \\
p_B &< 15 - p_A
\end{align*}
For a touching equilibrium, the relevant conditions are as follows.

\[ \begin{align*}
  x &= y \\
  p_A &= 15 - p_B & \text{from above} \\
  x &= z \\
  p_B &= 15 - p_A & \text{from above}
\end{align*} \]

Note that these are best response functions for the touching equilibrium.

Now just focus on the potential competitive equilibrium. Throughout this part we assume that \( p_A + p_B < 15 \) as required for a competitive equilibrium. Firm A’s demand comes from both sides of the circle, so \( q_A = 2x = p_B - p_A + 3 \). Firm A’s profits are \( \pi_A = (p_A - 4)(p_B - p_A + 3) \). For Firm B, we get \( q_B = 2(3 - x) = p_A - p_B + 3 \). Firm B’s profits are \( \pi_B = (p_B - 4)(p_A - p_B + 3) \). Differentiating profits with respect to that firm’s price we get

\[ \frac{d\pi_A}{dp_A} = p_B - p_A + 3 - (p_A - 4) \]
\[ \frac{d\pi_B}{dp_B} = p_A - p_B + 3 - (p_B - 4) \]

Setting each first order condition equal to zero and solving for that firm’s price gives best response functions of

\[ \begin{align*}
  p_A &= \frac{p_B}{2} + \frac{7}{2} \\
  p_B &= \frac{p_A}{2} + \frac{7}{2}
\end{align*} \]

Combining both best response functions we get the following.

\[ \begin{align*}
  p_A &= \begin{cases} 
    \frac{p_B}{2} + \frac{7}{2} & \text{if } p_A + p_B < 15 \\
    15 - p_B & \text{if } p_A + p_B = 15
  \end{cases} \\
  p_B &= \begin{cases} 
    \frac{p_A}{2} + \frac{7}{2} & \text{if } p_A + p_B < 15 \\
    15 - p_A & \text{if } p_A + p_B = 15
  \end{cases}
\]
(b) Find the equilibrium prices and quantities.

**Solution:** We could either impose symmetry conditions of $p_A = p_B = p^*$ or substitute $p_B$ into the equation for $p_A$. Either way, we get the following prices.

$$p^* = \begin{cases} 
7 & \text{if } p^* < \frac{15}{2} \\
\frac{15}{2} & \text{if } p^* = \frac{15}{2} 
\end{cases}$$

$$p^* = \begin{cases} 
7 & \text{if } 7 < \frac{15}{2} \\
\frac{15}{2} & \text{if } \frac{15}{2} = \frac{15}{2} 
\end{cases}$$

It must be that $p^* = 7$, $q^* = 3$ and we have a competitive equilibrium (read below to find out why).

Why a competitive equilibrium and not a touching equilibrium? Both seem to satisfy the conditions above. Note that if both firms choose the touching equilibrium price of $\frac{15}{2}$, they would split the market and their profits would be higher than under the competitive equilibrium. To figure out which price is relevant, construct a 2x2 strategic form game where each firm can either choose a price of 7 or a price of $\frac{15}{2}$. The payoffs are the firms’ profits. You’ll find that a price of 7 is a best response and that when both firms charge a price of $\frac{15}{2}$, each firm has an incentive to deviate from that price. This is another example of the prisoner’s dilemma from the perspective of the firms.

(c) Find the profits of each firm and represent profits graphically in the preference space.

**Solution:** Firm profits are $\pi_i = (p^* - 4) (3) = (7 - 4) (3) = 9$

(d) Find the consumer surplus and represent this measure graphically in the preference space.

**Solution:** Consumer surplus is as follows.

$$CS = 4 \int_0^\frac{3}{2} (9 - 7 - x) \, dx$$

$$CS = 4 \left[ 2x - \frac{x^2}{2} \right]_0^{\frac{3}{2}}$$

$$CS = 4 \left( 3 - \frac{9}{8} \right)$$

$$CS = \frac{15}{2}$$
3. The inverse demand function is \( P = 120 - 2Q \). Two firms compete in quantities and each firm has a cost of \( C(q_i) = 2q_i^2 \). The interest rate is 25%.

(a) If a competitive equilibrium is maintained over time, what is the discounted sum of profits?

**Solution:** Firm A’s problem is as follows.

\[
\max_{q_A} \pi_A = (120 - 2q_A - 2q_B)q_A - 2q_A^2
\]

The first order condition is

\[
\frac{d\pi_A}{dq_A} = 120 - 2q_B - 8q_A
\]

and setting this equal to zero and solving for \( q_A \) yields the following.

\[
q_A = 15 - \frac{q_B}{4}
\]

The above is a firm’s best response which will be used later.

Imposing symmetry conditions and solving for the equilibrium quantity yields \( q^* = 12 \), \( Q^* = 24 \), and \( P^* = 72 \). The per period competitive profits are

\[
\pi^C = 72(12) - 2(12)^2 = 576
\]

The discounted sum of profits are

\[
\frac{1}{r}\pi^C = 4(576) = 2304.
\]

(b) If firms can perfectly collude and share the profits evenly, what would be the per period profits of each firm?

**Solution:** For a collusive equilibrium, \( Q = 2q \) and a firms profit is

\[
\max_q \pi = (120 - 2(2q))q - 2q^2
\]

The first order condition is

\[
\frac{d\pi}{dq} = 120 - 12q
\]

and setting this equal to zero and solving for \( q \) yields the following.

\[
q^* = 10
\]
\[
Q^* = 20
\]
\[
P^* = 120 - 2(20) = 80
\]
The per period collusive profits are

\[ \pi^{TC} = 80(10) - 2(10)^2 = 600 \]

(c) Consider a trigger strategy where if a firm deviates from a collusive equilibrium, the other firm behaves competitively forever after. What is the discounted sum of profits obtained by deviating from a collusive equilibrium and is it possible to maintain a collusive equilibrium if the other firm plays the trigger strategy?

**Solution:** Assume firm A deviates from the collusive equilibrium. We first need to find firm A’s optimal deviation. In a collusive equilibrium, \( q^* = 10 \), so assume \( q_B = 10 \) and firm A plays their best response. Firm A’s best response is \( q_A = 15 - \frac{9}{4} = \frac{25}{2} \). The market quantity is then \( Q = \frac{45}{2} \) and the price is \( P = 120 - 2 \frac{25}{2} = 75 \). Firm A’s profits from the deviation are

\[ \pi^D_A = 75 \left( \frac{25}{2} \right) - 2 \left( \frac{25}{2} \right)^2 = 625 \]

The discounted sum of profits would then be as follows.

\[ \frac{1}{1 + r} \pi^D_A + \frac{1}{1 + r} \pi^C \]

\[ \frac{4}{5} (625) + \frac{4}{5} (4)(576) = \frac{11716}{5} \]

It’s possible to maintain a collusive equilibrium if the following condition holds.

\[ 2400 > \frac{11716}{5} \]

which is does, so yes it is possible to maintain a collusive equilibrium if the other firm plays the trigger strategy.

(d) Consider another type of trigger strategy where one firm tries to establish a collusive equilibrium from the competitive equilibrium. If one firm deviates from the competitive quantity to the collusive quantity, then the other firm will play along forever after (unless one firm deviates from the collusive equilibrium). What is the discounted sum of profits obtained by deviating from a competitive equilibrium and is it possible to switch from a competitive equilibrium to a collusive one?

**Solution:** Assume firm A deviates from the competitive equilibrium. From the question, we know that firm B plays the competitive outcome of \( q_B = 12 \) and firm A plays the collusive outcome of \( q_A = 10 \). The market quantity is then
Q = 22 and the price is P = 120 − 2(22) = 76. Firm A’s profits from the deviation are

$$\pi^D_A = 76(10) - 2(10)^2 = 560$$

Note that this is not a profitable deviation for firm A, but if they can then get firm B to play along, the discounted sum of profits for firm A is as follows.

$$\frac{1}{1 + r} \pi^D_A + \frac{1}{1 + r} \pi^{TC}$$

$$\frac{4}{5}(560) + \frac{4}{5}(4)(600) = 2368$$

It’s possible to switch from a competitive equilibrium to a collusive one if 2368 > 2304, so yes it is possible.

4. Answer the following questions:

(a) What is a factor that influences the expected punishment received from the formation of a cartel?

**Solution:** The probability of detection, the probability of conviction, the fine/punishment.

(b) What are some pricing practices that facilitate non-cooperative collusion?

**Solution:** Uniform prices, price discount penalties, competitor price match guarantee, advance notice of price changes, delivered pricing instead of FOB pricing, informational exchanges/pricing books

(c) Why don’t anti-trust authorities ban the specific pricing practices you mentioned in part b?

**Solution:** If the case of Uniform prices, price discount penalties, competitor price match guarantee, advance notice of price changes, and informational exchanges/pricing books, these policies most likely benefit consumers in the absence of tacit collusion. If these policies can benefit consumers, then there’s no clear justification for an outright ban of these policies.

(d) How does the size and frequency of transactions influence cartel stability?

**Solution:** If the size of transactions are large, the incentives to deviate from the cartel agreement are greater. A one period deviation would bring a larger windfall before the cartel could do anything about the deviation. If transactions
are infrequent, the incentives to deviate from the cartel agreement are greater. Infrequent transactions give the cartel fewer opportunities to punish defectors.

5. The inverse market demand is \( P = 120 - 2Q \), costs for Firm \( i \) are \( C(q_i) = 20q_i \) and there are four firms.

(a) What is the competitive equilibrium price and the profit of each firm?

Solution: Setup Firm A’s problem and then solve for the symmetric equilibrium. Firm A’s quantity is \( q_A \) and the quantity of all other firms other than Firm A is \( q_{-A} \).

\[
\max_{q_A} \pi_A = (120 - 2q_A - 2q_{-A})q_A - 20q_A \\
\frac{\partial \pi_A}{\partial q_A} = 120 - 4q_A - 2q_{-A} - 20 = 0 \\
100 - 2q_{-A} = 4q_A \\
100 - 2(3q^*) = 4q^* \\
q^* = 10 \\
Q^* = 40 \\
P^* = 120 - 2(40) = 40 \\
\pi_i^* = 40(10) - 20(10) = 200
\]

(b) If all firms perfectly collude, what is the collusive equilibrium price and the profit of each firm?

Solution: If all firms perfectly collude, assume that \( Q = 4q \) so that they split the market evenly. One firm’s problem is as follows

\[
\max_{q} \pi = (120 - 2(4q))q - 20q \\
\frac{\partial \pi}{\partial q} = 120 - 16q - 20 = 0 \\
16q = 100 \\
q^* = \frac{25}{4} \\
Q^* = 25 \\
P^* = 120 - 2(25) = 70 \\
\pi_i^* = 70 \left( \frac{25}{4} \right) - 20 \left( \frac{25}{4} \right) = 312.5
\]
(c) Assume the four firms perfectly collude. A policy maker asks you to come up with a reason why they should do something about collusion and support your explanation with numerical evidence from this problem. How do you respond?

**Solution:** You could argue that collusion reduces consumer surplus. In the absence of collusion

\[ CS = \frac{1}{2} (120 - 40) \times 40 = 1600 \]

and with collusion

\[ CS = \frac{1}{2} (120 - 70) \times 25 = 625 \]

so the change in consumer surplus is

\[ \Delta CS = -975. \]

Collusion makes consumers worse off by $975.

You could also make an argument concerning the total surplus or overall efficiency. In the absence of collusion, industry profits are $800 and the overall efficiency is $2,400. With collusion, industry profits are $1,250 and the overall efficiency is $1,875. Collusion makes the firms better off, the consumers worse off and leads to a lower total surplus.

(d) Another policy maker argues that all policy makers have perfect information/foresight and that they have the ability to construct optimal transfers. Why might policy makers still want to do something about collusion and support your explanation with numerical evidence from this problem?

**Solution:** Transfers would allow you to take the gains of the producers and distribute them to the consumers. Since total surplus decreases by $525, the gains of the producers don’t cover the loss of the consumers. Overall efficiency would increase if policy makers did something about collusion.