Instructions:
You must answer all of the following questions. Each question is worth the same amount. You have the class period to complete the exam.

Answer each question clearly and concisely. You must show your work to receive credit.

This exam is given under the rules of the Montana State University. By printing your name above you acknowledge the University’s Honor Code and agree to comply with the provisions of the Honor Code. You may not use notes or receive any assistance. There is to be no talking during the exam. You may use a calculator, but are never allowed to use device allowing you to take photographs or transmit over a network. No notes, no assistance, no talking, no cell phones, but you can use a calculator.

Clearly print your name above, in the space provided on the next page and in your blue book(s). You must turn in your blue book(s).
1. Consider a linear city Hotelling model. There are two firms, A and B, located at the ends of the product space. The length of the product space is 3 and transportation costs are 1 times the distance traveled. Each consumer has a baseline valuation of 9 and each firm has a constant marginal cost of 4. Answer the following questions for the competitive equilibrium if it exists.

(a) What is each firm’s best response function?
(b) Are the two goods strategic complements or strategic substitutes?
(c) What are the equilibrium prices and quantities?
(d) Find the profits of each firm.
(e) Find the consumer surplus.

2. Consider a Salop circular model of product differentiation with equally spaced firms. The circumference of the circle is 10. Each consumer has transportation costs of 1 times the distance traveled and a baseline valuation of 100. Firms have constant marginal costs of 5. There are \( N \) firms. Answer the following question for a competitive equilibrium assuming that it exists.

(a) Express a firm’s demand as a function of the neighboring firm’s prices and the number of firms. What will a firm’s demand be in equilibrium?
(b) Setup a firm’s profit maximization problem and determine a firm’s best response as a function of the number of firms and neighboring firm’s prices.
(c) What is the equilibrium price as a function of the number of firms?
(d) Determine the following comparative static: \( \frac{\partial \pi}{\partial N} \).
3. The inverse demand function is $P = 120 - 2Q$. Two firms compete in quantities and each firm has a cost of $C(q_i) = 2q_i^2$. The interest rate is 25%.

(a) If a competitive equilibrium is maintained over time, what is the discounted sum of profits?

(b) If firms can perfectly collude and share the profits evenly, what would be the per period profits of each firm?

(c) Consider a trigger strategy where if a firm deviates from a collusive equilibrium, the other firm behaves competitively forever after. What is the discounted sum of profits obtained by deviating from a collusive equilibrium and is it possible to maintain a collusive equilibrium if the other firm plays the trigger strategy?

(d) Consider another type of trigger strategy where one firm tries to establish a collusive equilibrium from the competitive equilibrium. If one firm deviates from the competitive quantity to the collusive quantity, then the other firm will play along forever after (unless one firm deviates from the collusive equilibrium). What is the discounted sum of profits obtained by deviating from a competitive equilibrium and is it possible to switch from a competitive equilibrium to a collusive one?

4. The inverse market demand is $P = 120 - 2Q$, costs for Firm $i$ are $C(q_i) = 20q_i$ and there are four firms. Assume firms compete in quantities.

(a) What is the competitive equilibrium price and the profit of each firm?

(b) If all firms perfectly collude, what is the collusive equilibrium price and the profit of each firm?

(c) Assume the four firms perfectly collude. A policy maker asks you to come up with a reason why they should do something about collusion and support your explanation with numerical evidence from this problem. How do you respond?

(d) Another policy maker argues that all policy makers have perfect information/foresight and that they have the ability to construct optimal transfers. Why might policy makers still want to do something about collusion and support your explanation with numerical evidence from this problem?