Instructions:
You must answer exactly 4 of the following 5 questions. Each question is worth the same amount. You have the class period to complete the exam.
Answer each question clearly and concisely. You must show your work to receive credit.
This exam is given under the rules of the Montana State University. You may not use notes or receive any assistance. There is to be no talking during the exam.
Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet.
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Hint: Given two demand curves

\[ q_1 = \frac{(a_1 - P)}{b_1} \]
\[ q_2 = \frac{(a_2 - P)}{b_2} \]

Horizontal summation results in the following inverse demand curve.

\[ P = \frac{(b_2a_1 + b_1a_2)}{(b_1 + b_2)} - \frac{(b_1b_2)}{(b_1 + b_2)}Q \]

1. There are two firms, A and B, with a constant marginal cost of $10. There are two markets, 1 and 2, with different goods. Firm A has a monopoly in market 1 so \( Q_1 = q_{1A} \), and both firms compete in quantities in the second market so \( Q_2 = q_{2A} + q_{2B} \). There is a representative consumer who's utility maximization problem is as follows.

\[
\max_{m, Q_1, Q_2} U = m + 100Q_1 - \frac{1}{2}Q_1^2 + 100Q_2 - \frac{1}{2}Q_2^2
\]

subject to

\[ Y = p_1Q_1 + p_2Q_2 + m \]

\( m \) is a composite good with a price normalized to 1. \( Y \) is the representative consumer’s income and \( p_1 \) and \( p_2 \) are the prices of their respective goods.

(a) Find the profits of each firm.

(b) Find the profits of each firm when firm A offers a pure bundle of \( b_A = q_{1A} + q_{2A} \). Is firm A better off?

(c) Find the profits of each firm when firm A offers a mixed bundle where firm A offers two products: \( b_A \) and \( q_{1A} \) where \( b_A = q_{1A} + q_{2A} \). (Note that firm A does not offer \( q_{2A} \) separately in this mixed bundle.) Is firm A better off with no bundling, a pure bundle or the mixed bundle?

(d) Are consumers better off with no bundling, a pure bundle or the mixed bundle?

2. Given an inverse demand function of \( P = 100 - 2Q \) and costs for Firm \( i \) of \( C(q_i) = 20q_i \), find the Stackelberg duopoly equilibrium:

(a) Firm Quantities

(b) Market Price

(c) Firm Profits

(d) Consumer Surplus

(e) How do your answers compare with the Cournot Duopoly equilibrium?
3. There are two types of representative consumers, 1 and 2, with the following inverse demand curves.

\[ P_1 = 186 - Q_1 \]
\[ P_2 = 186 - 2Q_2 \]

All firms have a constant marginal cost of $6.

(a) Find the equilibrium price and quantity with a monopolist who cannot price discriminate.

(b) Find the equilibrium price and quantity with a monopolist who can price discriminate between the two groups.

(c) Find the equilibrium price and quantity with a Cournot duopoly and no price discrimination.

(d) Find the equilibrium price and quantity with a Cournot duopoly with price discrimination between the two groups.

(e) Is it better to have a monopolist who can price discriminate or a Cournot duopoly that cannot price discriminate? Explain your answer.

4. Consider a linear city Hotelling model. There are two firms, A and B, located at the ends of the product space. The length of the product space is 3 and transportation costs are 1 times the distance traveled. Each consumer has a baseline valuation of 9 and each firm has a constant marginal cost of 4. Answer the following questions for the competitive equilibrium if it exists.

(a) What is each firm’s best response function?

(b) Are the two goods strategic complements or strategic substitutes?

(c) What are the equilibrium prices and quantities?

(d) Find the profits of each firm.

(e) Find the consumer surplus.

5. Given an inverse demand function of \( P = 120 - 2Q \) and costs for Firm \( i \) of \( C(q_i) = 18q_i \), find the Cournot duopoly equilibrium:

(a) Firm Quantity

(b) Price

(c) Consumer Surplus

(d) Deadweight Loss