Instructions:
There are 16 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
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Math for Math’s Sake

The following is meant to be a review of the math you’re supposed to know but might need some refreshing.

1. Find $f'(x)$ or $\frac{df(x)}{dx}$ for the following functions:
   
   (a) $f(x) = Ax^n$ where $n$ and $A$ are constants
   (b) $f(x) = x^{0.5}$
   (c) $f(x) = 10x^{-3}$
   (d) $f(x) = 5x^4 + 6x^2 + 7x + 3 + \frac{22}{x}$
   (e) $f(x) = g(x)h(x)$
   (f) $f(x) = (x^2)(x^3 - 4)$
   (g) $f(x) = (3x^3)(\ln(x))$
   (h) $f(x) = \frac{g(x)}{h(x)}$
   (i) $f(x) = \frac{\ln(x)}{5x^2}$
   (j) $f(x) = g(h(x))$
   (k) $f(x) = (x^5 - 10x)^a$ where $a$ is a constant
   (l) $f(x) = \ln(2x)$

2. In question 1.c above, what is the slope of $f(x)$ when $x = 2$?

3. Find all the local extrema for the following functions and indicate whether the point is a local maximum or a local minimum.
   
   (a) $f(x) = 5x^2 + 30x + 3$
   (b) $f(x) = -8x^2 + 4x - 50$

4. Find the inverse of the following functions:
   
   (a) $y = f(x) \rightarrow y = x^2 + 4$ with $x \geq 0$
   (b) $p = f(Q) \rightarrow p = 3 - 6Q$
   (c) $p = f(Q) \rightarrow p = a - bQ$

5. Let $f(x)$ be defined as follows where $x \in [0, 5]$ means $0 \leq x \leq 5$:

   $$f(x) = \begin{cases} 
   -2x + 15 & \text{for } x \in [0, 5], \\
   -4x + 25 & \text{for } x \in [5, 6].
   \end{cases}$$

   (a) What is the slope of $f(x)$ when $x = 4$?
   (b) What is the slope of $f(x)$ when $x = 5.5$?
   (c) What is the slope of $f(x)$ when $x = 5$?
6. Solve the following integrals: 
   (a) \( \int_0^5 2x \, dx \)
   (b) \( \int_1^4 \frac{1}{x} \, dx \)

7. Simplify the following: 
   \( \frac{x^{\frac{1}{3}} y^{-\frac{2}{3}}}{x^{-\frac{2}{3}} y^{\frac{1}{3}}} \)

8. Solve the following for \( x \): 
   \( \frac{5}{2} = \frac{8}{2^x} \)

9. Solve the system of two equations and two unknowns and graph the two equations:
   \[ Q = 2p - 40 \]
   \[ Q = 200 - p. \]

10. Solve the system of two equations and two unknowns:
    \[ q_1 + 3q_2 = 10 \]
    \[ 4q_1 + 8q_2 = 4. \]

11. Take the partial derivative of the following with respect to \( x_1 \) and \( x_2 \) where \( a, b \) and \( c \) are constants:
    (a) \( U = 2x_1 + 3x_2 \)
    (b) \( U = x_1^{\frac{1}{3}} + x_2 \)
    (c) \( U = x_1 + \frac{1}{x_2} \)
    (d) \( U = x_1^a + x_2^b \)
    (e) \( U = x_1^{0.3} + x_2^{0.7} \)
    (f) \( U = x_1^2 x_2^3 \)
    (g) \( U = cx_1^a x_2^b \)
Cost Analysis

These questions review the basic properties of cost functions using calculus.

12. Given a linear cost curve of $C(Q) = 5Q + 8$:
   (a) Find the Fixed Cost (FC).
   (b) Find the Variable Cost (VC).
   (c) Find the Average Cost (AC).
   (d) Find the Average Fixed Cost (AFC).
   (e) Find the Average Variable Cost (AVC).
   (f) Find the Marginal Cost (MC).
   (g) Plot the cost curve with the FC and VC curves on the same plot.
   (h) Plot the AC, AFC, AVC and MC curves on the same plot.

13. Given a quadratic cost curve of $C(Q) = 2Q^2 + 5Q + 8$:
   (a) Find the Fixed Cost (FC).
   (b) Find the Variable Cost (VC).
   (c) Find the Average Cost (AC).
   (d) Find the Average Fixed Cost (AFC).
   (e) Find the Average Variable Cost (AVC).
   (f) Find the Marginal Cost (MC).
   (g) What is the minimum of average cost and at what quantity does this occur?
   (h) Show that marginal cost equals average cost at the minimum of average cost.
Revenue Analysis

These questions review the basic connections between demand, revenue and the price elasticity of demand using calculus. Denote the price elasticity of demand as $\xi$, and for those of you who don’t remember:

$$\xi = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}}$$

$$\xi = \frac{\% \Delta \text{ in } Q}{\% \Delta \text{ in } p}$$

$$\xi = \left( \frac{dQ}{dp} \right) \left( \frac{p}{Q} \right)$$

14. Starting with a generic linear inverse demand function $p = a - bQ$ where $a$ and $b$ are parameters:

(a) Find the demand function, $Q(p)$ (the inverse of the inverse demand)

(b) Find $\frac{dQ}{dp}$

(c) Find $\xi$ as a function of $Q$ and the parameters $a$ and $b$ (not $p$)

15. Given an inverse demand function of $p = 20 - 4Q$:

(a) Find the revenue function $R(Q)$.

(b) Find the marginal revenue function $MR(Q)$.

(c) Find the price elasticity of demand $\xi(Q)$.

(d) Show that revenue is maximized when the price elasticity of demand is unit elastic.

Profit Analysis

Combining the cost analysis and the revenue analysis we get the following questions.

16. Given an inverse demand function of $p = 20 - 4Q$ find a firm’s maximum profits and the profit maximizing quantity when the cost function is:

(a) $C(Q) = 5Q + 8$

(b) $C(Q) = 2Q^2 + 5Q + 8$