Industrial Organization
ECNS 406
Fall 2015

Homework #: 4

Due by the beginning of class on: Tuesday September 29, 2015

Name: _____________________________________________________________

Instructions:
There are 4 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
Bertrand Model With Asymmetric Costs

Here are some different variations on the Bertrand Model.

1. The inverse demand function is \( P = 120 - 2Q \), costs for Firm 1 are \( C(q_1) = 20 q_1 \), and costs for Firm 2 are \( C(q_2) = c_2 q_2 \). The two firms compete in prices.
   (a) Plot the best response functions assuming \( c_2 > 20 \)
   (b) Describe the equilibrium of the model as \( c_2 \) increases for \( c_2 > 0 \).

Bertrand Model With Capacity Constraints

2. The inverse demand function is \( P = 120 - 2Q \). There are two firms which compete in prices and have constant marginal costs of 0. Each firm has a production capacity of 20.
   (a) Assuming Firm 2 produces \( q_2 = 20 \), what is Firm 1’s residual demand curve?
   (b) Assuming Firm 2 produces \( q_2 = 20 \), what is Firm 1’s marginal revenue from the residual demand curve?
   (c) What is the equilibrium price and quantity?
   (d) How much profit does each firm make?
   (e) How does the equilibrium of this model compare to the equilibrium if firms compete in quantities rather than prices?
   (f) Does the introduction of capacity constraints relax price competition?

3. The inverse demand function is \( P = 120 - 2Q \). There are two firms which compete in prices and have constant marginal costs of 20. Each firm has a production capacity of 20.
   (a) Assuming Firm 2 produces \( q_2 = 20 \), what is Firm 1’s residual demand curve?
   (b) Assuming Firm 2 produces \( q_2 = 20 \), what is Firm 1’s marginal revenue from the residual demand curve?
   (c) What is the equilibrium price and quantity?
Bertrand Model With Imperfect Substitutes

4. The inverse demand functions for Firms 1 and 2 are given by \( p_i = 120 - 2q_i - 2\theta q_{j\neq i} \) with \( \theta = \frac{3}{4} \) so that

\[
\begin{align*}
p_1 &= 120 - 2q_1 - \frac{3}{2}q_2 \\
p_2 &= 120 - 2q_2 - \frac{3}{2}q_1
\end{align*}
\]

which lead to the corresponding demand functions

\[
\begin{align*}
q_1 &= \frac{240}{7} - \frac{8}{7}p_1 + \frac{6}{7}p_2 \\
q_2 &= \frac{240}{7} - \frac{8}{7}p_2 + \frac{6}{7}p_1.
\end{align*}
\]

The two firms compete in prices and have constant marginal costs of 20.

(a) What is the equilibrium price, firm quantity and market quantity?

(b) Are the two products strategic substitutes or strategic compliments?

(c) How does the equilibrium of this model compare to the equilibrium if firms compete in quantities rather than prices and the goods are perfect substitutes (\( \theta = 1 \))?

(d) Is price competition relaxed with the addition of imperfect substitutes?

(e) How does the equilibrium of this model compare to the equilibrium if firms compete in quantities rather than prices and the goods are imperfect substitutes with \( \theta = \frac{3}{4} \)?