Name: ____________________________________________________________

Instructions:
There are 5 questions worth a total of 100 points. Answer each question clearly and concisely. You must show your work to receive credit. You are allowed to work with others, but all work must be your own.

Clearly print your name above and in the space provided on the next page. You must turn in both sides of this cover sheet along with your responses. You do not need to turn in the questions, only your responses with the cover sheet. All pages must be stapled to be graded.
Product Differentiation

1. Consider a Salop circular model of product differentiation with equally spaced firms. The circumference of the circle is 4. Each consumer has transportation costs of 2 times the distance traveled and a baseline valuation of 10. Firms have a constant marginal cost of 2. There are 2 firms. Answer the following questions for the competitive equilibrium if it exists.

   (a) What is each firm’s best response function?
   (b) Find the equilibrium prices and quantities.
   (c) Find the profits of each firm.
   (d) Find the consumer surplus and represent this measure graphically in the preference space.

2. Consider a Salop circular model of product differentiation with equally spaced firms. The circumference of the circle is 4. Each consumer has transportation costs of 2 times the distance traveled and a baseline valuation of 10. Firms have a constant marginal cost of 2. There are 3 firms. Answer the following questions for the competitive equilibrium if it exists.

   (a) What is firm A’s best response function?
   (b) Find the equilibrium prices and quantities.
   (c) Find the profits of each firm.
   (d) Find the consumer surplus and represent this measure graphically in the preference space.

3. Using your results from Question 1 and Question 2 how does the Salop circular model change with more firms?

4. Consider a Salop circular model of product differentiation with equally spaced firms. The circumference of the circle is 8. Each consumer has transportation costs of 1 times the distance traveled and a baseline valuation of 15. There are two firms: A and B. Firm A has a constant marginal cost of 3 and firm B has a constant marginal cost of $c_B > 0$. Answer the following questions for the competitive equilibrium if it exists.

   (a) What is each firm’s best response function?
   (b) Find the equilibrium prices and quantities.
   (c) Find the profits of each firm.
   (d) Find the consumer surplus.
   (e) What are the comparative statics with respect to $c_B$ i.e. how does the equilibrium change as $c_B$ increases?
Collusion

5. The inverse demand function is \( P = 120 - 2Q \). Two firms compete in quantities and each firm has a cost of \( C(q_i) = 2q_i^2 \). The interest rate is 25%.

(a) What are the per period profits of one firm in a competitive equilibrium?

(b) If firms can perfectly collude and share the profits evenly, what would be the per period profits of each firm?

(c) If a competitive equilibrium is maintained over time, what is the discounted sum of profits?

(d) If firms perfectly collude over time, what is the discounted sum of profits for a firm?

(e) Consider a trigger strategy where if a firm deviates from a collusive equilibrium, the other firm behaves competitively forever after. What is the discounted sum of profits obtained by deviating from a collusive equilibrium and is it possible to maintain a collusive equilibrium if the other firm plays the trigger strategy?

(f) Consider another type of trigger strategy where one firm tries to establish a collusive equilibrium from the competitive equilibrium. If one firm deviates from the competitive quantity to the collusive quantity, then the other firm will play along forever after (unless one firm deviates from the collusive equilibrium). What is the discounted sum of profits obtained by deviating from a competitive equilibrium and is it possible to switch from a competitive equilibrium to a collusive one?