Arriving in Time: Estimation of English Auctions with a Stochastic Number of Bidders

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Abstract

We develop a new econometric approach for estimation of second-price ascending-bid auctions with a stochastic number of bidders. Our empirical framework considers the arrival process of new bidders as well as the distribution of bidders' valuations of objects being auctioned. By observing the timing of bidder arrival the model is identified even when the number of potential bidders is stochastic and unknown. The relevance of our approach is illustrated with an empirical application using a unique dataset of art auctions on eBay. Our results suggest a higher impact of sellers' reputation on bidders' valuations than previously reported in cross-sectional studies but the impact of reputation on bidder arrival is largely insignificant. Interestingly, a seller's reputation impacts not only the actions of the bidders but the actions of the seller as well. In particular, experience and a good reputation increase the probability of a seller posting items for sale on longer-lasting auctions which we find increases the expected revenue for the seller.

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1 Introduction

We develop a new econometric approach for estimation of second-price ascending-bid auctions with a stochastic number of bidders. This approach is relevant for eBay auctions and is also of value for other auction settings. Our empirical framework considers the arrival process of new bidders as well as the distribution of bidders’ valuations for objects being auctioned. A fully parametric partial likelihood model is developed allowing for unobserved heterogeneity and flexible bidder behavior. The partial likelihood is based on observing the arrival of new bidders and the transaction price. We assume that the timing of a bidder’s first bid represents her time of arrival to the auction. Our model of bidder behavior is based on the assumptions made by Haile and Tamer (2003). These assumptions, along with eBay’s proxy bidding system with low bid increments, imply that the highest bid from the second highest bidder (the transaction price) equals that bidder’s valuation. By observing the timing of bidder arrival in addition to the submitted bids, the model is identified even when the number of potential bidders is stochastic and unknown. The relevance of our approach is illustrated with an empirical application using a unique dataset of art auctions on eBay.

This research contributes to the growing literature on structural estimation of auctions while paying special attention to the specific features of the eBay marketplace. We view our approach as complementary to the work of others as our approach is based on a non-overlapping set of assumptions required for identification and a novel estimation approach. Our research also contributes to the growing literature on online markets (Lucking-Reiley, 2000).

A significant body of work has emerged which uses a structural approach, especially for first-price auctions, but also for second-price ascending auctions and other types of auction formats. Surveys include Hendricks and Paarsch (1995), Perrigne and Vuong (1999), and Hendricks and Porter (2000). Researchers have used the structural approach for many purposes: to provide estimates of optimal reserve prices (Paarsch 1997), to analyze the effects of bidder collusion (Baldwin, Marshall and Richard, 1997), to measure the extent of the winner’s curse and simulate seller revenue under different reserve prices (Bajari and Hortaçsu 2003), to analyze the effects of congestion (Canals-Cerdá, 2005) or advertising in an online auctions market (Canals-Cerdá, 2006).

Most of the existing structural econometric models of second-price ascending auctions have
been inspired by the button auction model of Milgrom and Weber (1982). This model has a unique dominant-strategy equilibrium where each bidder submits a bid equal to their valuation of the object being auctioned. A prevalent approach, proposed by Donald and Paarsch (1996), assumes that the highest bid of each one of the auction participants, except the winner of the auction, equals their true valuation (see also Paarsch, 1997; Hong and Shum, 2003; Bajari and Hortacsu, 2003). Alternatively, some papers require only that the winning bid equals the valuation of the second-highest bidder and do not use the information from losing bids (Paarsch, 1992; Baldwin et al., 1997; Haile, 2001). In a remarkable paper, Haile and Tamer (2003) use information on the highest bid from each bidder, but they do not require any bid to equal the bidder’s true valuation of the object being auctioned. We use the same assumptions regarding bidder behavior as Haile and Tamer (2003) and show that these assumptions along with eBay’s proxy bidding result in the transaction price being equal to the valuation of the secondhighest bidder when there are two or more bidders. Information regarding other bids is used to the extent to which these bids change the standing price which in turn influences observed bidder arrival.

Two main features distinguish eBay/Internet auctions from the button auction model. First, not all bidders necessarily bid their true valuation and secondly, the actual number of bidders is observed but the potential number of bidders is stochastic and unknown. (Athey and Haile (2007) distinguish potential bidders from actual bidders and discuss the two deviations from the button auction model mentioned here. See Section 4 and Section 6.3 of their paper.) Applying button auction methods to estimate Internet auction models is problematic because identification results for almost all ascending auction environments require the observation of the potential number of bidders.

Numerous studies make use of eBay data. Song (2004), Bajari and Hortacsu (2003; 2004), Lucking-Reiley (2000) and others provide descriptions of the eBay auction format. Similar to us, Song (2004) uses a structural framework to estimate an independent private value model with eBay data. Both this paper and Song’s paper are primarily interested in the estimation of the distribution of private values, \( F(v) \), when the number of potential bidders is unknown. Song estimates her model using semi-parametric techniques, while the model presented in this paper is fully parametric. While it’s possible to develop a semi-parametric version of our model, we leave this for future work. The primary difference between our approach and Song’s is a non-overlapping
set of assumptions for identification.

Song identifies the distribution of private values from the observation of two order statistics independent of the potential number of bidders. Because on eBay the highest bid is not observed, Song relies on the highest bid from the second and third highest bidders. In addition, Song places restrictions on the data in order to increase the chances of satisfying the identification assumptions in her paper. These restrictions require the exclusion of a significant proportion of auctions. In fact, if we had to restrict our data to auctions with three or more bidders in which a bid greater than the third-highest bid was submitted not earlier than two days before the end of the auction, as is done in Song (2004), we would need to disregard more than 85% of auctions.

Table 1 presents three stylized examples of eBay bidding behavior and illustrates the drawbacks to the traditional button auction model and Song’s (2004) approach. Each auction contains four potential bidders with increasing valuations. Each bidder arrives at a different time and submits a proxy bid which influences the current price. In each auction, the number of actual bidders is less than the potential number of bidders. This happens because potential bidders with lower valuations arrive to the auction at a time when the current minimum acceptable bid is above their valuation. The first auction is consistent with the button auction for actual bidders, but inconsistent if one considers the set of potential bidders.

Another study similar to ours is by Ackerberg, Hirano and Shahriar (2006). Their study evaluates eBay’s buy-it-now feature and uses a structural framework to estimate a model using eBay data. The process of bidder arrival as well as the distribution of bidders’ valuations is modeled using similar behavioral assumptions. Their estimation technique employs a partial likelihood approach combined with moment simulation while our estimation technique is based only on a partial likelihood approach.

The rest of the paper proceeds as follows. Section 2 presents a structural model of eBay auctions and describes the econometric methodology. In Section 3 we consider an empirical application of our methodology utilizing a unique dataset of art auctions on eBay. A model is estimated to evaluate the impact of sellers’ reputation on bidder arrival and on bidders’ valuations. Our results suggest a higher impact of sellers’ reputation on bidders’ valuations than previously reported in cross-sectional studies but the impact of reputation on bidder arrival is largely insignificant. We also analyze which factors influence the sellers choice of auction length. Interestingly, a seller’s reputation impacts not
only the actions of the bidders but the actions of the seller as well. In particular, experience and a good reputation increase the probability of a seller posting items for sale on longer-lasting auctions. Finally, we conduct a counter-factual analysis simulating what would happen if seven-day auctions were instead listed as ten-day auctions. This exercise highlights the value of our structural approach. Our analysis indicates that if seven-day auctions were instead listed as ten-day auctions sellers’ overall revenue would increase by 9%. Section 4 concludes. More descriptive information about the dataset and eBay auctions are included in a supplementary appendix.

2 A Structural Model of eBay Auctions

A detailed description of eBay auctions is found in Song (2004), Bajari and Hortacsu (2003; 2004), Lucking-Reiley (2000) and others, and is not repeated here. We model eBay auctions as independent private value (IPV), ascending-bid, second-price auctions subject to some specific rules. The seller of the object being auctioned sets the duration of the auction, $T$, and the starting value, $s_0$. Each potential bidder, $n$, assigns a value, $v_n$, to the object being auctioned. Each value, $v_n$, is an independent realization from a distribution $F(v)$ with density $f(v)$. Bidders know and care only about their own valuation.

A proxy bidding system is used where bids, $b$, are submitted electronically at any time $t$ within the $[0, T]$ time interval. The bid history at time $t$, denoted $B(t)$, includes previous bid amounts with the corresponding time of bid submission. At each time $t$, the price of the auction, denoted $s(t)$, is set at the current second highest bid plus a minimum increment. New bids arrive sequentially at any time during the $[0, T]$ interval. The auction begins with $s(0) = s_0$ and ends with a transaction price of $s(T)$.

Any new bid has to surpass $s(t)$ by a minimum increment in order to be recorded. The value of the minimum increment in eBay auctions varies with the standing bid, $s(t)$. The minimum increment is $0.05$ for standing bids under $1.00$ and increases up to $100.00$ for standing bids above $5000.00$. The average increment in our empirical application is $1$ which is slightly more than 1% of the transaction price. Throughout the paper we assume that the minimum increment is negligible and do not mention it to avoid additional notation. Bid increments are discussed in the Supplementary Appendix.
For a particular auction $ij$ (artist $i$ auctioning object $j$), $N_{ij}$ is the number of potential bidders which is the number of bidders who would have chosen to bid in an otherwise identical sealed bid auction for which any bid amount is acceptable. $M_{ij}$ represents the total number of bids from observed bidders, and $K_{ij}$ represents the observed number of bidders. $N_{ij} \geq K_{ij}$ as some potential bidders are not observed and $M_{ij} \geq K_{ij}$ as bidders are free to bid as many times as they want. Denote the bids for an auction as $b_m$ for $m = 1, \ldots, M_{ij}$ and the timing of bids as $t_m$, for $m = 1, \ldots, M_{ij}$. Also of importance is the timing of the first bid by bidder $k$ denoted as $t^1_k$, for $k = 1, \ldots, K_{ij}$. $B_{ij}(t)$ represents the bidding history at time $t$ which includes all the $(b_m, t_m, k)$ pairs previously submitted over $[0, t)$ where $k$ identifies the bidder. Note that while the timing of the highest proxy bid in $B_{ij}(t)$ is observed the bid amount is not. Knowledge of the bid history allows one to identify the auction price, $s_{ij}(t)$, throughout the auction as well as the timing at which $s_{ij}(t)$ changes. In what follows, we avoid mentioning auction-specific heterogeneity ($ij$) except when absolutely necessary.

2.1 The Distribution of Bidders’ Valuations

Bidders’ behavior is modeled in terms of the underlying distribution of bidders’ valuations and the following two behavioral assumptions.

**Assumption 1.** Bidders do not bid more than they are willing to pay.

**Assumption 2.** Bidders do not allow an opponent to win at a price they are willing to beat.

Assumptions 1 and 2 are identical to those in Haile and Tamer (2003). These assumptions combined with eBay’s proxy bidding system with sufficiently small increments guarantees that with two or more active bidders the transaction price, $s(T)$, will equal the highest bid from the second highest bidder and will also equal that bidder’s valuation. We evaluate the robustness of this statement in a Supplementary Appendix.

Our assumptions are consistent with many different types of bidding behaviors. Bidders are free to bid as many times as they want. We only require that the bidder with the second highest valuation bids her true valuation before the end of the auction and that the bidder with the highest valuation wins the auction.
We account for the presence of heterogeneity across auctions. Denote by \((x_{ij}, \eta_{ij})\) the vector of relevant characteristics describing object \(j\) being auctioned by seller \(i\). \(x_{ij}\) represents the object characteristics observed by the econometrician and \(\eta_{ij}\) is a real valued index summarizing unobserved object characteristics which is treated as a random effect from a distribution \(H(\cdot)\). Denote by \(v_{ijk}\) bidder \(k\)'s valuation of object \(j\) being auctioned by artist \(i\). We impose the following structure:

\[
\log(v_{ijk}) = x_{ij}' \beta + \delta \eta_{ij} + w_{ijk},
\]

where \(w_{ijk}\) represents the residual, bidder specific (log-) valuation of the object being auctioned and is distributed \(N(0, \sigma^2)\).

### 2.2 The Bidders’ Arrival Process

We model only the timing of the first bid by any new bidder and make the following assumption to identify their arrival.

**Assumption 3.** *Any potential bidder that arrives at an auction at time \(t\) immediately places a bid if their valuation, \(v\), is such that \(v \geq s(t)\).*

The process we envision is one in which a bidder arrives to a particular auction, incurs a cost to learn about the object being auctioned, and then places a bid. This cost may include reading the auction’s description, looking at available pictures, examining the seller’s reputation and past auctions, and e-mailing the seller with any questions. After the first bid, we assume the bidder can continue bidding as many times as she wants at no additional cost.

We model the arrival of new bidders to an auction similar to the arrival of job offers in a structural job search model (Flinn and Heckman, 1982) and also similar to how others have modeled the arrival process in an auction environment (Wang, 1993). Denote the instantaneous probability of arrival of new potential bidders at any time \(t \in [0, T]\) as \(\lambda_{ij}(t)\). Only new potential bidders with valuation above \(s_{ij}(t)\) are able to bid which leads to two hazard functions. The instantaneous probability of arrival of new bidders and the instantaneous probability of arrival of potential, unrealized bids are

\[
\alpha_{ij}(t) = \lambda_{ij}(t)(1 - F(s_{ij}(t))) \quad \gamma_{ij}(t) = \lambda_{ij}(t)F(s_{ij}(t))
\]

respectively. The above hazard functions are the product of the arrival rate of potential bidders and the probability that their valuations are above or below the minimum acceptable bid.
Accounting for heterogeneity across auctions, we model the arrival hazard for new potential bidders as \( \log(\lambda_{ij}(t)) = \log(\lambda(t)) + x_{ij}'\theta + \eta_{ij} \). Our specification of \( \lambda_{ij}(t) \) includes a baseline hazard \( \lambda(t) \), an index function \( x_{ij}'\theta \) accounting for observed heterogeneity, and an unobserved heterogeneity component \( \eta_{ij} \), which is treated as a random effect from a distribution \( H(\cdot) \). This specification resembles a mixed proportional hazard model. Mixing with our specification of unobserved heterogeneity accommodates overdispersion and excess zeros (Cameron and Trivedi, 1998).

The likelihood of bidder arrival may not be uniform over the duration of the auction \([0, T]\). Impatient bidders may choose to concentrate their attention on auctions close to ending, which the available search options on eBay facilitates. We address this point by allowing the baseline hazard \( \lambda(t) \) to take on different values over the duration of the auction. More precisely, we divide \([0, T]\) into \( R \) mutually exclusive intervals, \( \{I_r\}_{r=1}^R \) with \( I_r = [i_{r-1}, i_r] \), and allow \( \lambda(t) \) to take on different values across subintervals while remaining constant within subintervals, \( \log(\lambda(t)) = \lambda_r \) for \( t \in I_r \).

### 2.3 The Partial Likelihood

The model outlined thus far does not completely specify bidder behavior. Bidders are allowed to bid and rebid any amount (below their valuation and above the standing bid) as often as they want. Assumptions 1 and 2 only imply that with two or more bidders the highest bid from the second highest bidder is that bidder’s valuation, which at the end of the auction will equal the transaction price. The restriction imposed by Assumption 3 just pertains to the initial time of arrival of any active bidder.

Estimation of the full likelihood function based on the observed bids and the timing of bids is not possible given the stated assumptions, as there is an incomplete mapping between bidders’ valuations and all bids, and between bidders’ arrival and the timing of all bids. (We are grateful to Keisuke Hirano, the Co-Editor, for pointing this out.) Rather than imposing additional restrictions on bidder behavior, we consider a partial likelihood similar to the approach by Ackerberg, Hirano, and Shahriar (2006) and noted as an alternative in Villas-Boas (2006). A key difference between our partial likelihood approach and that by Ackerberg, Hirano, and Shahriar (2006) is that the probability of observing a particular transaction price is included into our partial likelihood, while Ackerberg, Hirano, and Shahriar (2006) include this probability via a simulated moment condition.

The partial likelihood for a particular auction in our sample is constructed based on the trans-
action price, \( s(T) \) and the timing of a bidders’ first bid, \( t^1_k \), (their time of arrival). While the partial likelihood is a function of all observed bids and the timing of these bids, given our model assumptions, only the transaction price and the bidders’ time of arrival provide relevant statistical information. Observed bids and the timing of bids not indicative of arrival play a role in the proper specification of the bidder arrival process and for this reason are part of the bidding history included in the conditioning sets.

The partial likelihood function for a particular auction is defined as,

\[
PL(\Theta) = P(s(T) | B(t_M), K, t_M; \Theta) \cdot \prod_{k=1}^{K} P(t^1_k | B(t^1_k); \Theta)
\]

with \( \Theta \) representing the vector of parameters associated to the arrival and bidding processes described in prior subsections, and \( K \) is the number of observed bidders. The bidding history, \( B(t) \), includes information from \([0, t]\) and \( t_M \) is the time the last bid is placed. For notational simplicity, let \( PL(\Theta) = P_s(\Theta) \cdot P_t(\Theta) \) and let \( B^* = (B(t_M), K, t_M) \). The first component of the partial likelihood is the probability of the transaction price being \( s(T) \) and the second part of the partial likelihood represents the probability of bidders’ arrival. Observe that \( PL(\Theta) \) is not a standard likelihood function because it does not represent the joint density for the observed data, which would also include the complete vector of realized bids and the timing of additional subsequent bids.

The second component of the partial likelihood function, \( P_t(\Theta) \), is defined within the framework of a multistate duration model, (see, for example, Heckman and Walker (1990)). The multistate duration model is a generalization of a duration model allowing us to consider several durations or spells. The probability structure of any duration model can be characterized by its hazard function (Lancaster, 1990). For a hazard, \( \lambda'(t) \), the integrated hazard \( \Lambda(t|\lambda') = \int_0^t \lambda'(z)dz \), survival function \( G(t|\lambda') = \exp(-\Lambda(t|\lambda')) \), distribution function \( G(t|\lambda') = 1 - G(t|\lambda') \), and density function \( g(t|\lambda') = \lambda'(t)G(t|\lambda') \) are obtained. In Section 2.2, we defined the hazard specifications employed in our model (equation 1).

The problem of finding an analytic expression for \( P_t(\Theta) \) is more tractable when the hazard function is constant. Both \( \alpha(t) \) and \( \gamma(t) \), defined in equation (1), vary over the duration of the auction, \([0, T]\), from changes in \( \lambda(t) \) (a baseline hazard term) and \( s(t) \). We divide the duration of
the auction into non-overlapping subintervals such that the hazard functions are constant within each subinterval in the spirit of Meyer (1990). $s(t)$ changes when new bids are submitted at times $\{t_k\}_{k=1}^M \subseteq [0, T]$ and can be divided into $M + 1$ subintervals with nodes $\{0, \{t_k\}_{k=1}^M, T\}$ where $s(t)$ is constant within subintervals. Let $\{T_k\}_{k=1}^{M+1}$ with $T_k = [t_{k-1}, t_k]$ indicate the subintervals where $s(t)$ is constant. The baseline hazard term, $\lambda(t)$, is allowed to take on different values across subintervals $\{I_r\}_{r=1}^R$ in $[0, T]$ while remaining constant within subintervals. Consider the $R + M$ subintervals of the form $D_{kr} \in \{\{T_k \cap I_r\}_{r=1}^{M+1}\}_{k=1}$ where the hazard functions are constant within a subinterval $D_{kr}$.

The probability $P_t(\Theta)$ is determined by analyzing the contribution of each subinterval $D_{kr}$. For each $D_{kr}$, there are two scenarios: $D'_{kr} = [a, i_r]$ or $D''_{kr} = [a, t_k]$ with $a \in \{i_{r-1}, t_{k-1}\}$. In the case of $D'_{kr}$, no new bidder has entered the auction and the contribution to the overall probability of this subinterval is $\bar{G}(i_r - a | \lambda' = \alpha(t))$. For $D''_{kr}$, a new bidder has entered the auction and the contribution is $g(t_k - a | \lambda' = \alpha(t))$. Finally $P_t(\Theta)$ is the product of the corresponding probabilities from each subinterval $D_{kr}$.

The first component of the partial likelihood function, $P_s(\Theta)$, represents the contribution of the transaction price. The conditioning set for this term includes the total number of observed bidders, $K$, in the auction. Two special and simple cases are auctions with zero bidders ($K = 0$) and one bidder ($K = 1$) where the transaction price is $s_0$. For $K = 0$, none of the potential bidders valued the object above its starting bid and $P(s_0|K = 0; \Theta) = 1$ since this is the only possible bid sequence with no actual bidders. Similarly for $K = 1$, $P(s_0|K = 1; \Theta) = 1$ because this is the only possible outcome from the eBay auction environment with one actual bidder.

When the number of active bidders in an auction is greater than one, the transaction price is equal to the second highest valuation, $s(T) = \bar{u}_{N-1}$ (see Section 2.1). In this case, $P_s(\Theta)$ is a function of the potential number of bidders, $N (\geq K)$, which is unknown. Denote by $P(N|B^*; \Theta)$ as the probability that an auction has $N$ potential bidders which depends on the arrival process defined by $\Theta$. We calculate $P_s(\Theta)$ as

$$P(s(T)|B^*; \Theta) = E_{N} (P(s(T)|N; \Theta)|B^*; \Theta)$$

$$= \sum_{N=K}^{\infty} P(s(T)|N; \Theta) P(N|B^*; \Theta). \quad (3)$$
To compute this probability in our empirical application we truncate the infinite sum at the value $N^*$ such that $P(N > N^*|\text{B}^*; \Theta) < \varepsilon$. (In our analysis we choose $\varepsilon = 10^{-6}$ and then repeated the estimation with $\varepsilon = 10^{-9}$ with no significant changes observed in the results.)

For a particular $N$, the probability $P(s(T)|N; \Theta)$ is derived from the density of the second order statistic obtained from the underlying distribution of bidders’ valuations, $F(v)$, where

$$P(s(T)|N; \Theta) = F_{\Theta(N-1,N)}(s(T)) = N(N-1)f_{\Theta}(s(T)) F_{\Theta}(s(T))^{N-2}(1-F_{\Theta}(s(T))).$$

Furthermore, the probability of $N$ potential bidders, $P(N|\text{B}^*; \Theta)$ in equation (3) is computed from the specific bidder arrival process. At each point in time, $t$, for a given $s(t)$, there are two types of potential bidders: type I bidders with valuations above $s(t)$ and type II bidders with valuations below $s(t)$. We observe the arrival of type I bidders and the total number of type I bidders at the end of the auction is the total number of observed bidders $K$. The total number of potential bidders at any time $t$ is the sum of type I and type II bidders and the total number of potential bidders at the end of the auction is denoted by $N$. The rate of arrival of type I bidders is governed by the hazard $\alpha(t)$ and the rate of arrival of type II bidders is governed by the hazard $\gamma(t)$ both defined in equation (1). The process that generates $N$ is defined from knowledge of the history of arrival of type I agents and knowledge of the rate of arrival of type II agents.

More precisely, consider the $R + M$ subintervals $D_{kr}$ in $\{\{T_k \cap I_r\}_{r=1}^{R} \}_{k=1}^{M+1}$ where the hazard functions are constant within subintervals. Within an interval $D_{kr}$ the arrival of potential type II bidders, say $n_{kr}$, follows a simple Poisson process with associated hazard $\gamma(t)$. When computing $P(N|\text{B}^*; \Theta)$ for a particular $(N, K)$ we need to take into account that this pair imposes a restriction on the number of type II potential bidders, which must equal $N - K$. Thus for each $R + M$ vector of $n_{kr}$’s, $(n_1, \ldots, n_{R+M})$, satisfying the restriction $n_1 + \cdots + n_{R+M} = N - K$ we can compute the probability $P((n_1, \ldots, n_{R+M})|N, \text{B}^*; \Theta)$ as the product of Poisson probabilities $P(n_{kr} | \gamma(t)) = t\gamma(t)^{n_{kr}} \exp(-t\gamma(t))/(n_{kr}!)$ within each subinterval $D_{kr}$. The sum of all the $P((n_1, \ldots, n_{R+M})|N, \text{B}^*; \Theta)$’s over the set of all possible vectors of $(n_1, \ldots, n_{R+M})$ defines $P(N|\text{B}^*; \Theta)$.

Finally, it is also conceptually straightforward within our framework to account for the presence of an unobserved heterogeneity component, $\eta$, with distribution $H(\eta)$. Unobserved heterogeneity
is included in the arrival process and in the bidding process. For each auction, we assume that the same \( \eta_{ij} \) is associated with both processes. In this sense, we adopt the view that the unobserved heterogeneity enters the empirical specification like any other variable (size, reputation) and that the differential impact on the arrival and bidding process is achieved through the multiplicative parameters.

We incorporate unobserved heterogeneity using the popular semi-parametric approach described in Heckman and Singer (1984). Subject to the standard assumptions of a random effects model, this approach hypothesizes a discrete distribution with finite support as a good approximation to the true distribution of the unobserved heterogeneity component. In this framework, the contribution to the partial likelihood of an observation can be computed as

\[
PL(\Theta) = \sum_{h=1}^{H} q_h PL(\Theta, \eta_h),
\]

with \( PL(\Theta, \eta_h) \) defined in equation (2). \( H \) represents the number of points of support of the discrete distribution. \( q_h \geq 0 \) is the probability associated to a realization \( \eta_h \), with \( \sum_{h=1}^{H} q_h = 1 \). Identification requires a mean restriction of \( \sum_{h=1}^{H} q_h \eta_h = 0 \).

The partial likelihood for the overall sample can be computed as the product the auction specific partial likelihoods derived throughout this section. The parameter vector, \( \Theta \), contains parameters describing bidders’ valuations and bidders’ arrival process where \( \Theta = (\beta, \delta, \lambda, \theta, \varphi, \sigma, H(\cdot)) \). After defining the partial likelihood function for the overall sample, standard maximum likelihood methods can be applied for estimation (Cox 1975, Ackerberg, Hirano, and Shahriar 2006).

## 3 Empirical Application

### 3.1 Data Description

From July 13, 2001 to November 14, 2001 we collected all auction data (approximately ten thousand auctions) from a group of artists, self-denominated EBSQ, who sell their own artwork through eBay. EBSQ is an abbreviation for “e-basquiat” after the artist Jean-Michel Basquiat. The artists on this group organize online activities, like art contests. (For more information visit www.ebsqart.com.) In most cases, the item for sale was an original painting, but other forms of art like collages, ceramic
tiles, or sculptures, were also offered. Between August 1, 2004 and December 25, 2004 we conducted a second round of data collection from a subgroup in the original group of artists consisting of the artists who listed auctions regularly during the original data collection period. In this study, our overall sample consists only of original paintings for this subgroup of artists in the years 2001 and 2004. Also, we do not include in our analysis approximately 2% of the auctions in which the ‘buy it now’ option was exercised. (Before any bids have been submitted, a potential buyer can terminate the auction and purchase the item being auctioned by paying a ‘buy it now’ price set by the seller. After the first bid has been submitted, this option disappears.)

In our empirical application we focus our attention on the artists’ choice of auction length and the effect of this choice on market outcomes. Sellers can choose the auction length from four possible alternatives: three, five, seven, or ten days. Ten-day auctions require an extra fifty cents fee. We further restrict our sample to include only seven and ten-day auctions and only artists that list regularly in both seven and ten-day auctions because we intend to control for artists’ specific fixed effects. The selected subsample includes auctions by 10 artists from the original group of 37. Even after our restrictions, the selected subsample is still significantly larger than sample sizes observed in most studies using eBay data. We also employ the overall dataset when empirically relevant: in particular to conduct tests of the independent private value assumption or to provide additional support to certain findings from the structural estimation.

For each auction we collected information on item characteristics, auction characteristics, artists’ reputation history, and a complete bidding history, except for the highest bid which is not reported. Item characteristics include information on the height, width, style (abstract, pop, whimsical, etc.), medium (acrylic, oil, etc.), and ground (stretch canvas, paper, wood, etc.) of the painting. Auction characteristics include the length of the auction, the opening bid, the shipping and handling fees, and the eBay category in which the object was being listed. Each transaction in eBay can be rated by the buyer and the seller as positive (1), neutral (0) or negative (-1). We collect reputation history from the artists’ transactions and define two measures of artists’ reputation history: a reputation index used by eBay representing the percentage of positive feedback, and another measure of artists’ reputation equal to the total number of unique buyers from past transactions. This second measure of an artist’s reputation is used to proxy the effect of having a large customer base as well as a potential ‘word of mouth’ effect.
Since we model auctions as independent private value second-price ascending-bid auctions, one might question whether this is the right framework for modeling art auctions. In this regard, the value of a painting up for auction from the bidder’s perspective is likely to be a combination of several factors: (1) painting and artist specific characteristics; (2) bidder’s private information about the painting’s intrinsic value and (3) bidder specific tastes. In our framework the IPV paradigm is justified if (1) is known a priori by all bidders, (2) plays an immaterial role and (3) is not correlated across buyers. That is, after controlling for (1) buyers do not value a painting more just because other potential buyers do. We believe that these conditions hold in our empirical application.

Because the artwork in our sample is auctioned directly from the artist, all bidders have equal access to a wealth of information about the art auctions. Besides the current auction’s characteristics, bidders also observe the artist’s feedback history as well as information about recently completed auctions for which links remain active for several weeks after the auction has ended, including auction characteristics and complete bidding histories. Thus, in general we can discount a scenario in which one bidder may have private information about the item being auctioned that could be relevant to other bidders.

We also think it is reasonable to assume that a bidder’s valuation is not affected by other bidders’ willingness to pay because our data concerns auctions from lesser known artists and this type of artwork is most likely purchased for personal enjoyment rather than as an investment. In our view, bidders’ valuations are more likely to be correlated in auctions for artworks by well-known artists in which case resale can be an attractive option. In this case, bids from other bidders may convey useful information about the potential future resale value of the artwork. In contrast, most of the buyers in our sample are unlikely to be very concerned about the resale value of the artwork and if they choose to resell the artwork on their own they are likely to incur substantial losses because most buyers in this market would rather buy directly from the artist. It is also highly unlikely that the artwork in our sample will appreciate in value sufficiently to justify viewing its purchase as a long term investment. In the highly unlikely case that the artwork does appreciate substantially over time, the eBay bidding information is unlikely to be relevant. In Section A of the Appendix we present tests which indicate that the IPV approach is appropriate and further discuss the relevance of the dataset to our application in the supplementary appendix.

Table 2 presents descriptive statistics for relevant variables from the original sample and the
selected subsample. The average number of seven and ten-day auctions among the 10 artists is 59 and 64, respectively. The artists in our subsample experience a higher average probability of sale, receive bids from a larger number of bidders, and have a more extensive eBay history, as indicated by the ‘unique feedback’ variable. Auctions with zero bidders represent about 35% of auctions in the selected subsample, auctions with two, three, or four bidders are not uncommon. Six percent of the auctions in the seven-day auctions category and 18% in the ten-day auctions had more than four active bidders. Multiple bids from single bidders are not uncommon. The average size of a painting is close to two square-feet but there is a large variation in size. All artists’ in our sample enjoy an excellent eBay feedback rating, with an average rating of about 99.8. The average sale price is about $52 for seven-day auctions and about $57 for ten-day auctions, including shipping costs. The observed average number of bidders is 2.3 for seven-day auctions and 3.4 for ten-day auctions. Other relevant variables not included in this table are style (abstract, contemporary, etc.), medium (acrylic, pen and ink, etc.), and ground (stretch canvas, canvas panel, etc.). Descriptive statistics for these additional variables are available from the authors.

3.2 Empirical Framework

We apply the methodology described in Section 2 to the analysis of auction outcomes and to the artists’ choice of auction length. This application underscores the relevance of a methodology which considers the process of bidder arrival as well as the distribution of bidders’ valuations. It also underscores the flexibility of our methodology which can easily be embedded into a more complex empirical framework.

We model the artists’ choice of a seven-day or ten-day auction and determine how auction characteristics influence the process of bidder arrival and the distribution of bidders’ valuations. The artist’s choice of auction length is a potentially complex decision that may depend on the artist’s expectations about the probability of a sale, the sale price, and the problem of inventory management, among others. We model this problem using a simple discrete choice framework where \( I_{ij} = 1 \) represents the ten-day auction choice on the part of the artist and is characterized as a standard logit model with associated probability \( p_{ij} = L(\Psi_{ij}) \), where \( L \) is the logistic CDF, and with \( \Psi_{ij} \) representing an index function which accounts for artists’ specific fixed effects and unobserved heterogeneity. (In our empirical specification \( \Psi_{ij} \) takes the form \( x_{ij}'\rho + \eta_{ij} \).)
incorporate the choice of auction length into the model described in Section 2 and the contribution
to the partial likelihood function of a certain auction \( j \) from artist \( i \) is described as

\[
I_{ij} p_{ij} PL_{ij}^{10} + (1 - I_{ij}) (1 - p_{ij}) PL_{ij}^{7},
\]

\( PL_{ij}^{10} \) and \( PL_{ij}^{7} \) are the partial likelihood components defined by equation (2) and are specific to
ten-day and seven-day auctions.

Our econometric framework postulates an empirical specification governed by four equations:
the artist’s choice of auction length, the process of bidders’ arrival (modeled as two separate Poisson
processes, one for ten-day auctions and one for seven-day auctions), and the bidders’ valuation
of the painting being auctioned, (modeled as a single common equation for seven and ten-day
auctions). We experimented with other model specification structures, like a structure with a single
bidder arrival equation, but this specification was selected based on a likelihood-ratio-test criteria.
Differences across auctions are captured by a vector of covariates as described in Section 3.1. We
also include a dummy for year 2001 that captures any potential overall change in the market across
years, biweekly dummies intended to capture seasonal effects on demand that are common across
years, and a dummy variable intended to capture the potential impact of the terrorists attacks
of 9/11 on auctions that took place in the weeks after this date. The empirical specification also
includes artists’ specific fixed effects to account for unobserved differences across artists, in addition
to those captured by the vector of covariates.

An unobserved heterogeneity component, as described in Section 2, is incorporated into our
empirical specification. We consider a distribution of unobserved heterogeneity with two points of
support and with mean zero and variance one restrictions. Baker and Melino (2000) show that this
specification works well. (The estimated empirical distribution takes values 0.905 and -1.357 with
associated probabilities 0.628 and 0.372.) Estimation results are very similar for models with and
without unobserved heterogeneity, and the coefficients associated to the unobserved heterogeneity
are insignificant in all equations. However, a likelihood ratio test cannot reject the possible presence
of unobserved heterogeneity.

In addition, the process of new bidders’ arrival includes a baseline hazard, \( \lambda(\cdot) \), consisting of
a constant and four additional baseline parameters. The first baseline parameter accounts for
differences in the arrival of bidders between the second day and the day before the end of the auction; the second, third, and fourth baselines account for differences in the rate of arrival on the last day, the last hour, and the last ten minutes of the auction, respectively. The arrival process also includes dummies for the day of the week in which the auction ends, allowing for the possible changes in the flow of potential buyers at different days of the week. Some variables are included in the bidder-arrival equation but not the bidder-valuation equation. We hypothesize that bidders' valuations are independent of day-of-the-week effects and other calendar dummies, which are more likely to affect the process of bidders’ arrival and have no direct effect on bidders’ valuation. These variables are insignificant when included in the bidders’ valuation equation.

3.3 Estimation Results

Table 3 summarizes the most relevant estimation results. The results from the bidder-valuation indicate that the most significant determinant of bidder-valuation is painting size, after controlling for other factors including artists’ fixed effects. An increase in one square foot in size increases a bidder’s valuation by 30% on average. Interestingly, both measures of seller’s feedback are significant. An increase in 100 unique previous buyers increases bidders’ valuation by 21%. Possible interpretations are that this represents a reputation effect or that this measures the increase in sellers’ intrinsic value as a result of learning-by-doing. Two possible learning-by-doing channels are improvements in technique or gaining a better understanding of what characteristics of the artwork bidders’ value the most. In particular, we observe an increase in the average painting size overtime which is consistent with the artists learning about bidders’ tastes. The average impact of a decrease in eBay feedback rating of 0.1, for example from 99.8 to 99.7, will result in a relatively large decrease in average bidders’ valuation of 10.1%.

Our results contrast with reported findings of the cross-sectional literature in this area. Bajari and Hortacșu (2003) find that a negative reputation does not significantly impact the final auction price. Melnik and Alm (2002) estimate that the impact of negative feedback is significant but very small in magnitude, and the same holds for the impact of the overall rating. Houser and Wooders (2005) estimate that the average cost to sellers stemming from neutral or negative reputation scores is less than one percent (0.93%) of the final sales price. Lucking-Reiley, Bryan, Prasad, and Reeves (2007) estimate that a one percent increase in positive/negative feedback increase/reduce
sale price by only 0.03% and 0.11% respectively. Bajari and Hortaçsu (2004) review this evidence and conclude: “We believe that these results are likely to significantly understate the returns from having a good reputation...Since getting positive feedback requires effort on the part of sellers, it appears that sellers are making efforts to avoid negative feedback...” The estimated effect in our case is much larger than what has been reported in existing cross-sectional studies.

With regards to the bidder-arrival process, we observe again that size has a significant positive effect. Interestingly, reputation measures have an insignificant impact on bidders’ arrival rates, with the exception of the ‘number of feedbacks’ variable that is significant for the seven-day bidder-arrival process. Thus, reputation does not seem to play a significant role in the arrival of bidders for the group of sellers considered in our analysis. The baseline hazard coefficients associated to the bidder-arrival equations are similar across seven and ten-day auctions. The baseline hazard coefficients, ‘Baseline 3’ and ‘Baseline 4’, are larger and more significant indicating that bidders are more likely to arrive later in the auction. The first and second baselines are somewhat larger for the seven-day auction arrival process, indicating a lower bidder arrival rate between the first and last day of the auction in ten-day auctions.

Looking at the equation that models the choice of auction length, both the painting size as well as the seller reputation measures are significant. We view this as an interesting novel result which indicates that the measures of seller reputation not only have a significant effect on the valuations of bidders but also on the actions of the sellers. However, while bidder arrival and valuation equations are identified from the choices of a large number of independent bidders, the choice of ten-day auctions is identified from the actions of the sub-group of ten different sellers. Thus, it is warranted to analyze to what extent the findings about the choice of auction length also holds in the larger population of artists in the larger dataset. With this purpose in mind we estimate a conditional logit model in the larger sample with the same specification and controlling for artist’s specific fixed effects. The results from this exercise are comparable to those from our structural estimation. The coefficients associated with the first and second measure of reputation are 3.92 and 0.59, respectively and are significant with associated t-values 2.7 and 4.3 respectively. These values represent relatively large effects taking into account that the proportion of ten-day auctions is 27% in the large sample.
3.4 Counter-factual Analysis

In addition to examining what factors influence the choice of auction length, we conduct a counter-factual analysis simulating what would happen if seven-day auctions were instead listed as ten-day auctions. (We thank an anonymous referee for suggesting this application.) This analysis highlights the value of our structural model and estimates the impact of auction length on market outcomes such as sellers’ revenue, buyers’ surplus and the revenue of the market intermediary.

The bidders’ arrival component of our structural model allows us to determine the impact of changes in auction length on the number of potential bidders in an auction. Similarly, the bidders’ valuation component of our structural model is applied, along with the implied changes in the number of potential bidders, to ascertain the impact of a change in auction length in the final transaction price. The final transaction price has an impact on sellers’ revenue, the revenue of the market intermediary and the buyers’ surplus. Changes in the value of these factors as a result of changes in the auction length can be analyzed within the proposed structural framework.

We conduct our experiment assuming the estimated structural model represents an accurate description of the data generating process. For each seven-day auction in our selected subsample we generate 100 realizations of potential outcomes resulting from posting an item with the same characteristics in an auction of a specific length and consider the average outcome from the simulation.

Simulation results for the average outcome for seven and ten day auctions are presented in Table 4. The first two columns present results for seven-day auctions, the next two columns present results under an alternative ten-day scenario, and in the final column we report the difference in overall outcomes. We observe that the number of potential bidders is similar across auction formats but about 3% higher for ten-day auctions. This is consistent with our prior results concerning the bidder’s arrival process in seven and ten day auctions which suggests a higher intensity of bidder arrival in seven day auctions, especially towards the end of the auction. Average seller revenues are $52.7 and $57.3 for seven and ten day auctions. The simulated revenues for seven-day auctions are in line with what we observe in the data but the simulations predict a higher sales rate for seven day auctions than what we observe in the data (76% predicted vs. 65% observed from Table 2). (In our analysis we employed several parametric assumptions for simplicity, but our general approach is
not reliant on theses assumptions. Thus, the model fit could potentially improve by experimenting with different semi-parametric techniques and model specifications. This is potentially an avenue for further research, but beyond the current scope of the paper.) Comparing simulation results for the second highest and highest bids we compute the average buyer surplus. Buyer surplus is $50 and $52 for seven and ten day auctions, respectively. Finally, examining the list of fees applied by eBay we approximately compute eBay’s revenues for seven and ten day auctions. Our results indicate that eBay revenues are $128 (6%) higher for ten-day auctions.

4 Conclusions

This paper presents a novel structural approach for the estimation of independent private value, ascending-bid, second-price auctions, the most popular type of auction mechanism used on Internet markets. Unlike previous studies, our estimation approach makes use of information on the time of arrival of bids, which is readily available on eBay, along with information on bids from all auction participants. Both the arrival of bidders and the distribution of bidders’ valuations are modeled explicitly. In addition, we also take into account the specific features of eBay auctions: the number of potential bidders is a stochastic function of the auction characteristics, and not all potential bids and bidders are observed. Our approach avoids potential problems of selection bias present in existing studies.

Estimation is performed by maximizing a partial likelihood function. While our estimated model includes a number of parametric functional form assumptions, it is possible to use semi-parametric model specifications instead. We leave this for future work. We apply our methodology to the analysis of the artist’s choice of auction length (seven or ten-day auctions) and to the determination of auction outcomes in general. This application underscores the flexibility of our methodology which can be easily embedded into a more complex empirical framework.

Our analysis indicates that painting size is an important determinant of bidder’s valuation and bidder’s participation in an auction. We also consider the effect of two measures of seller’s reputation and find that reputation has a significant effect on bidders valuations. The estimated effect is much larger than what has been reported in existing cross-sectional studies. In contrast, the impact of a seller’s reputation on bidder’s auction participation is to a large extent insignificant.
Interestingly when we consider the seller’s choice of auction length, our analysis indicates that a seller’s reputation has a significant effect on this choice. We find that a seller with a better reputation is more likely to choose an auction with a longer duration, thus a seller’s reputation impacts not only the choices of bidders but the choices of sellers as well. Our counter-factual analysis indicates that if sellers were to list seven day auctions as ten day auctions, the number of potential bidders would slightly increase and seller revenues would increase by 9%.
References


Table 1: Three Stylized Examples of eBay Auctions

<table>
<thead>
<tr>
<th>t</th>
<th>j – bk</th>
<th>s(t) – CWB</th>
<th>j – bk</th>
<th>s(t) – CWB</th>
<th>j – bk</th>
<th>s(t) – CWB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>C – $75</td>
<td>$0.01 – C</td>
<td>B – $30</td>
<td>$0.01 – B</td>
<td>D – $100</td>
<td>$0.01 – D</td>
</tr>
<tr>
<td>1</td>
<td>D – $100</td>
<td>$76 – D</td>
<td>C – $25</td>
<td>$26 – B</td>
<td>A – $25</td>
<td>$26 – D</td>
</tr>
</tbody>
</table>

NOTE: Each auction has 4 potential bidders \( j \in \{A, B, C, D\} \), with respective valuations $25, $50, $75 and $100. Bidders submit bids \( b_k \) at various points in time \( t \). The current price, \( s(t) \), depends on the 2\textsuperscript{nd} highest bid and the bid increment. Each auction starts at $0.01. The current winning bidder (CWB) is shown at each point in the auction. A bid of ? indicates that \( s(t) \) is above that bidders valuation. In Auction 2, Bidder A arrives after Bidder C and is unable to bid.

Table 2: Descriptive Statistics for Relevant Variables

<table>
<thead>
<tr>
<th>Selected Sample</th>
<th>Seven-Day Auctions</th>
<th>Ten-Day Auctions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Min.</td>
</tr>
<tr>
<td>Square Feet</td>
<td>1.77</td>
<td>0.01</td>
</tr>
<tr>
<td>eBay Feedback</td>
<td>99.85</td>
<td>99.24</td>
</tr>
<tr>
<td>Unique Feedbacks*</td>
<td>3.31</td>
<td>0.00</td>
</tr>
<tr>
<td>Sale Price**</td>
<td>51.87</td>
<td>2.23</td>
</tr>
<tr>
<td>Bidders**</td>
<td>2.30</td>
<td>1</td>
</tr>
<tr>
<td>Bids**</td>
<td>3.9</td>
<td>1</td>
</tr>
<tr>
<td>Auctions per Artist</td>
<td>58.9</td>
<td>27</td>
</tr>
<tr>
<td>N. Obs. (% Sold)</td>
<td>589 (65%)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Bidders</th>
<th>Number of Bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>% Frequency</td>
<td>35.0</td>
</tr>
<tr>
<td>Overall Sample</td>
<td>Average</td>
</tr>
<tr>
<td>Square Feet</td>
<td>2.15</td>
</tr>
<tr>
<td>eBay Feedback</td>
<td>99.89</td>
</tr>
<tr>
<td>Unique Feedbacks*</td>
<td>2.65</td>
</tr>
<tr>
<td>Sale Price**</td>
<td>41.01</td>
</tr>
<tr>
<td>Bidders**</td>
<td>2.13</td>
</tr>
<tr>
<td>Bids**</td>
<td>3.48</td>
</tr>
<tr>
<td>N. Obs. (% Sold)</td>
<td>2523 (51%)</td>
</tr>
</tbody>
</table>

NOTE: Sale price includes shipping costs. (*) Measured in hundreds of feedbacks by unique buyers. (**) Refers to sold paintings.
Table 3: Estimation Results from Model with Unobserved Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>Ten-Day Auction Choice</th>
<th>Bidder Arrival Ten-Day Auction</th>
<th>Bidder Arrival Seven-Day Auction</th>
<th>Bidder Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-10.39</td>
<td>5.75</td>
<td>-11.73</td>
<td>8.94</td>
</tr>
<tr>
<td>Baseline 1</td>
<td>-0.648</td>
<td>8.75</td>
<td>-0.426</td>
<td>4.03</td>
</tr>
<tr>
<td>Baseline 2</td>
<td>0.5749</td>
<td>5.53</td>
<td>0.7760</td>
<td>6.15</td>
</tr>
<tr>
<td>Baseline 3</td>
<td>2.5117</td>
<td>15.81</td>
<td>2.9360</td>
<td>16.69</td>
</tr>
<tr>
<td>Baseline 4</td>
<td>5.3731</td>
<td>45.28</td>
<td>4.9595</td>
<td>29.70</td>
</tr>
<tr>
<td>eBay Feedback</td>
<td>3.9270</td>
<td>2.66</td>
<td>-0.013</td>
<td>0.01</td>
</tr>
<tr>
<td>Unique Feedbacks</td>
<td>0.5942</td>
<td>4.25</td>
<td>-0.077</td>
<td>0.50</td>
</tr>
<tr>
<td>Square Feet</td>
<td>0.3426</td>
<td>2.48</td>
<td>0.1920</td>
<td>3.37</td>
</tr>
<tr>
<td>Square Feet²</td>
<td>-0.023</td>
<td>2.05</td>
<td>-0.011</td>
<td>3.11</td>
</tr>
<tr>
<td>U. Heterogeneity</td>
<td>-0.110</td>
<td>0.00</td>
<td>0.2010</td>
<td>0.00</td>
</tr>
<tr>
<td>Item Dummies</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Artists' FE</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Calendar Dummies</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Number of Observations</td>
<td>1233</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-34473.21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Item Dummies indicate item specific dummies for style, medium and ground types.

Table 4: Simulated Market Outcomes For Two Auction Lengths

<table>
<thead>
<tr>
<th></th>
<th>Seven-Day Auctions</th>
<th>Ten-Day Auctions</th>
<th>Difference Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>eBay Revenues</td>
<td>3.39</td>
<td>1997</td>
<td>2125</td>
</tr>
<tr>
<td>Buyers' Surplus</td>
<td>50.32</td>
<td>20952</td>
<td>22835</td>
</tr>
<tr>
<td>Sellers' Revenues</td>
<td>52.73</td>
<td>21957</td>
<td>25292</td>
</tr>
<tr>
<td>Potential Bidders (% sold)</td>
<td>3.47 (76%)</td>
<td>3.59 (77%)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Characteristics of each seven-day auction in our selected subsample are used for simulation. Simulated values are generated using average results from 100 draws from the estimated model. Seller revenues represent gross values before fees to eBay.

Table 5: Regression of Bid Value on the Number of Bidders

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>IV</th>
<th>Bidders' FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bidders</td>
<td>1.2665</td>
<td>4.02</td>
<td>2.6112</td>
</tr>
<tr>
<td>eBay Feedback</td>
<td>15.719</td>
<td>2.15</td>
<td>12.090</td>
</tr>
<tr>
<td>Unique Feedbacks</td>
<td>0.3908</td>
<td>0.81</td>
<td>0.3797</td>
</tr>
<tr>
<td>Square Feet</td>
<td>15.718</td>
<td>22.17</td>
<td>15.036</td>
</tr>
<tr>
<td>Square Feet²</td>
<td>-0.4281</td>
<td>8.88</td>
<td>-0.3879</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>4695</td>
<td></td>
<td>4695</td>
</tr>
<tr>
<td>R Squared</td>
<td>0.5177</td>
<td></td>
<td>0.5159</td>
</tr>
</tbody>
</table>

NOTE: All models include controls for style, medium, ground and artists' fixed effects. The last model also controls for bidders' fixed effects.
A Tests of Independent Private Values

In the main body of the paper we present qualitative arguments supportive of the IPV assumption while in this appendix we formally test the IPV vs. common value (CV) assumption. We rely on tests that have been applied by researchers in prior studies. Existing tests to distinguish IPV models from CV models are based on the winner’s curse (Paarsch 1992; Haile, Hong and Shum 2003; Bajari and Hortacsu 2003). In CV auctions, bidders lower their bids in expectation of the winner’s curse and the amount bids are lowered is a function of the number of observed bidders. The winner’s curse does not apply to the IPV paradigm where there is no signaling component to additional observed bidders.

We use IPV/CV tests similar to those used by Bajari and Hortacsu (2003) and present a novel test taking advantage of the unique panel structure of our dataset. The tests we use regress bidders’ highest bids on the number of bidders in the auction and other auction characteristics. Under the CV (IPV) paradigm we would expect to observe a negative (positive) coefficient associated to the number of bidders variable. (The coefficient for the number of bidders is expected to be positive for IPV due to the truncation from the eBay auction format. See Footnote 16 of Bajari and Hortacsu (2003) and Theorem 9 of Athey and Haile (2002).)

The tests we use distinguish a pure common value model from an IPV model. These tests do not distinguish between IPV and an interdependent private values model. Haile, Hong and Shum (2003) develop a test to distinguish between IPV and interdependent values with unobserved heterogeneity and endogenous entry, but this test is only relevant for first price auctions rather than the second price auctions considered here. Additionally, in practice it may be difficult to distinguish an interdependent private values model from a model of unobserved heterogeneity. Given the very limited role that unobserved heterogeneity plays in the results from our structural model, we feel that the IPV assumption is not inappropriate for our application.

Table 5 presents results from our IPV/CV tests for the overall sample of auctions with two or more bidders. The first model presents results from a simple OLS regression that includes controls for relevant painting characteristics (i.e. style, medium and ground) as well as artist’ reputation and artists’ fixed effects. Observable characteristics in our sample explain a large proportion of the variation in final auction prices. Furthermore, our structural analysis suggests that unobserved heterogeneity plays a very limited role. Thus, we feel confident that the OLS specification of the test provides relevant information. Furthermore, a second model considers an IV approach that includes a battery of calendar dummies (i.e. day of the week and month) as instruments for the number of bidders, which should account for changes in demand but should not affect bidders’ valuations directly, after controlling for the number of bidders’ in an auction. These variables are insignificant when included in our structural analysis in the bidders’ valuation equation. One potential problem with the previous two tests is that they do not take into account the potential effect of differences in bidders’ tastes. To control for this confounding effect one needs a panel data that includes repeated observations of bids for the same bidders, which we have in our data. The coefficients associated with seller’s reputation and with painting size have the expected sign for all models with the exception of the last one which does not include controls for seller’s reputation because there is no sufficient remaining variation to identify these coefficients with any level of precision. The coefficient associated to the number of bidders is positive in all models and ranges from $1.2 to $2.6. This coefficient is significant at the usual level for the first and third model, while it is significant at the 90% level for the second model. These results are supportive of the IPV paradigm.