Reflector Telescope Design

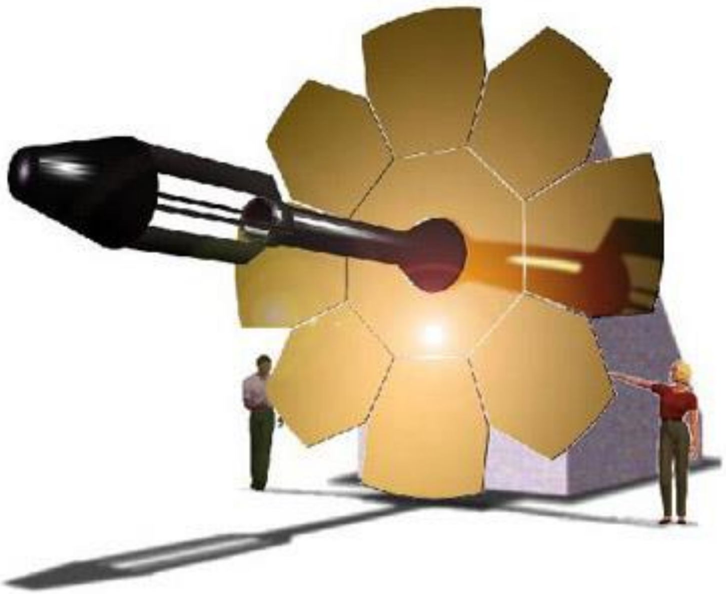
Objective: Describe the optical imaging performance of reflector telescopes. Telescope eyepieces are not discussed here, but must be designed carefully to not destroy the imaging capabilities of a telescope.

+ mirrors have no chromatic aberration (same telescope useful in uv-vis-IR)

+ mirrors have high reflectance over a very broad wavelength range

+ large mirrors can be built stronger and lighter than large lenses

- mirrors easily get in the way of each other



http://ngst.gsfc.nasa.gov/about.html

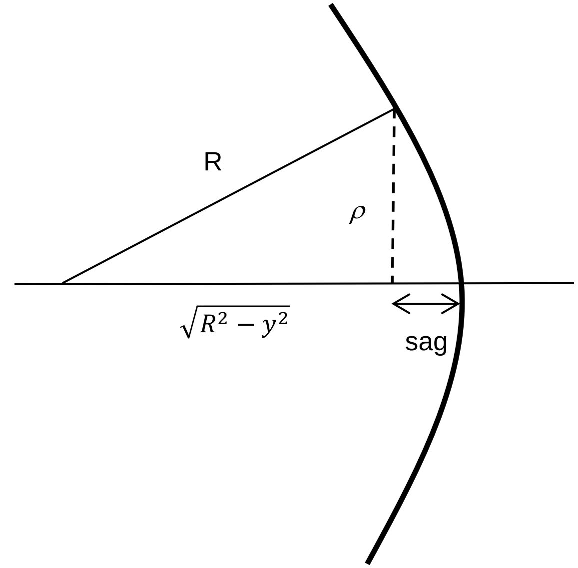
Surface sag

"sag" is the optical term for the shape of a surface that deviates from flat.

$$\text{Sag} = R - \sqrt{R^2 - \rho^2} = R - R\left[1 - \left(\frac{\rho}{R}\right)^2\right]^2$$

$$\text{Sag} \cong R - R\left[1 - \frac{\rho^2}{2R^2}\right] = \frac{\rho^2}{2R}$$

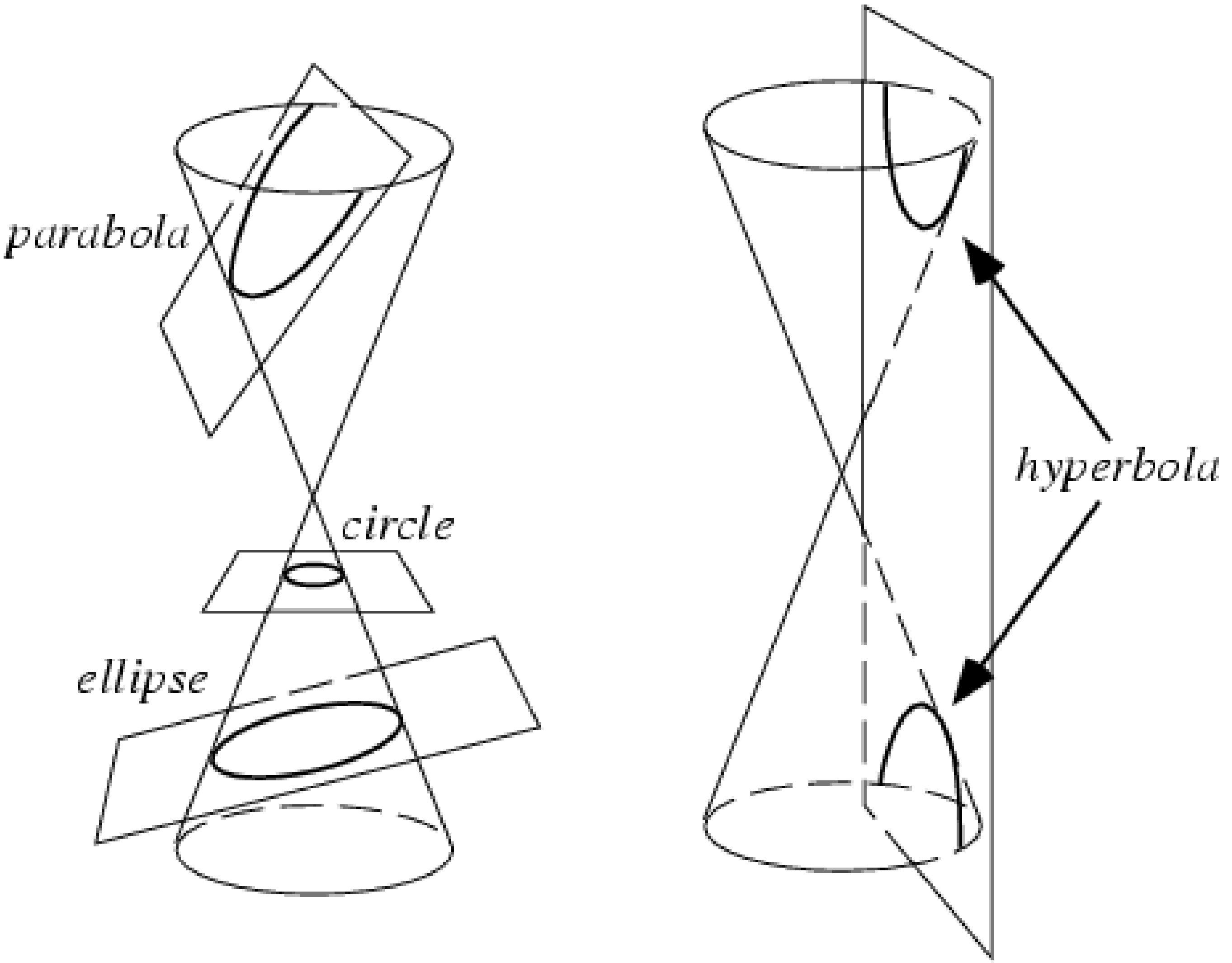
$$\text{Sag} \cong \frac{\rho^2}{2R}$$



…parabolic approximation of the sag of a spherical surface…

Refs: J. M. Geary, Intro to Lens Design with practical Zemax examples, pp. 22-23.

Conic Sections



Conics

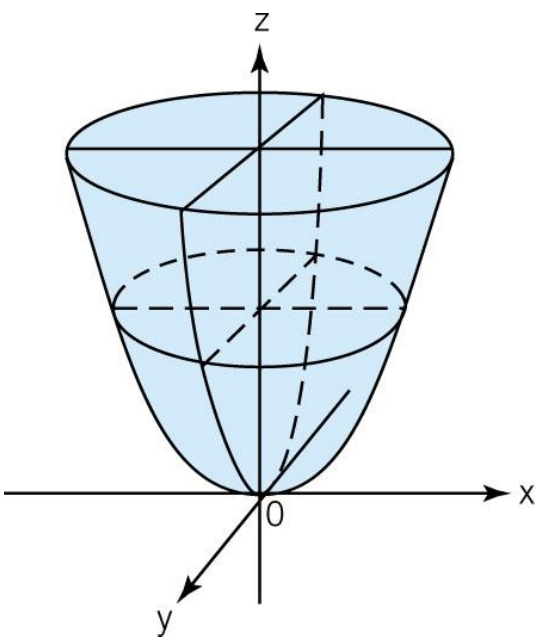
A conic is a surface of revolution formed by spinning a conic section around the axis.

Equation for a conic centered on the *z* axis: $$\rho^2 - 2Rz + (k+1)z^2 = 0$$

ρ = radial coordinate ($\rho^2 = x^2 + y^2$)

R = vertex radius of curvature,

k = conic constant.



All conics satisfy Fermat's principle (that is, have perfect imaging) at 2 conjugate points.

http://www.yahooligans.com/reference/dictionary/illus/pbloid.html

Conic constants

conic constant = *K* Eccentricity = *e*

$$k = -e^2$$

Shape	eccentricity	conic constant	focal length
sphere	$e = 0$	$k = 0$	$f < R/2$
paraboloid	$e = 1$	$k = -1$	$f = R/2$
Hyperboloid	$e > 1$	$k < -1$	$f > R/2$
Prolate ellipsoid	$0 < e < 1$	$-1 < k < 0$	$f < R/2$
Oblate ellipsoid		$k > 0$	

Sag of aspheric surface

The aspheric sag equation is found simply by solving the conic equation for *z*.

Sag for a conic:

$$z = \frac{\dfrac{\rho^2}{R}}{1+\sqrt{1-(k+1)\left(\rho/R\right)^2}}$$

Sag for even asphere:

$$z = \frac{\dfrac{\rho^2}{R}}{1+\sqrt{1-(k+1)\left(\rho/R\right)^2}} + \alpha_4\rho^4 + \alpha_6\rho^6 + \ldots$$

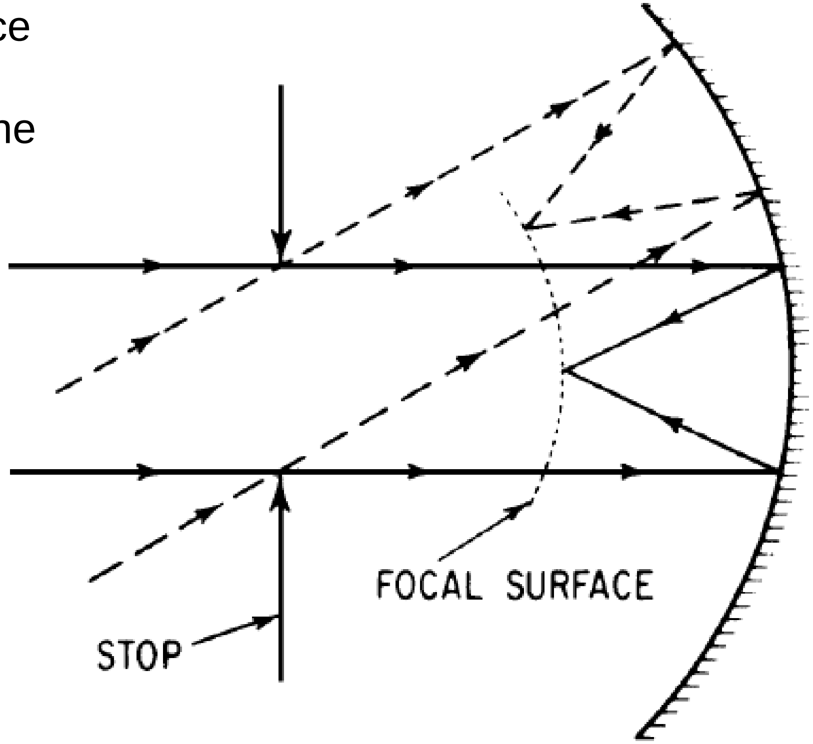
α's = high-order aspheric coefficients

R = vertex radius of curvature

Refs: Zemax manual (ver 13, 2013), p. 329
D. Malacara, Optical Shop Testing, 2nd ed., Appendix A
R. Shannon, Art and Science of Optical Design, sec. 7.1.6

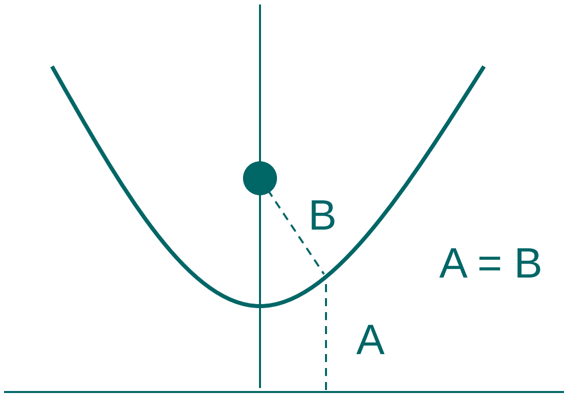
Spherical Mirror with stop at center of curvature

When the aperture stop is at the mirror's center of curvature, there are no "off-axis" rays and the image plane is a curved surface with radius equal to the mirror focal length. Coma and astigmatism are both zero, but the mirror still has spherical aberration.



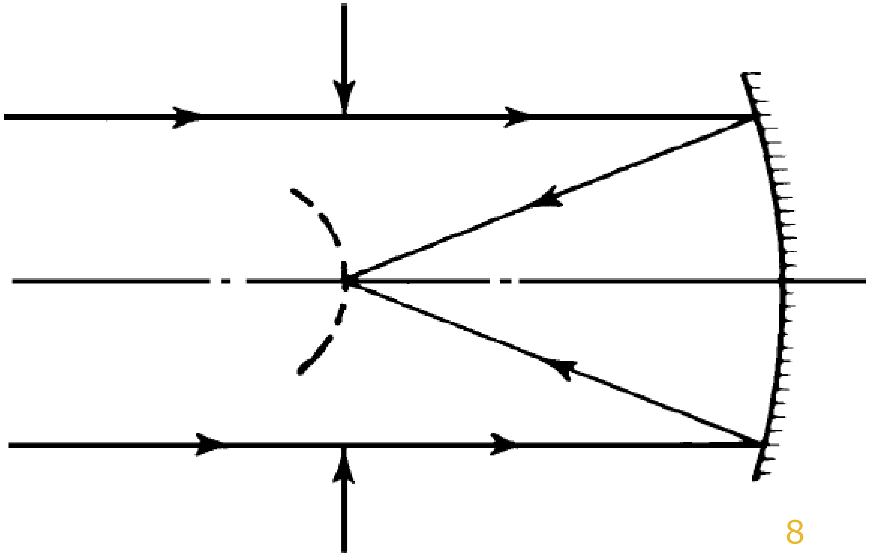
Paraboloidal Mirror

Parabola defined as the locus of points in a plane whose distance to the focus equals the distance to a fixed line (called the directrix).



A parabolic mirror has zero astigmatism when the aperture stop is located at the focus. But, because spherical is also zero, stop shifts do not affect coma.

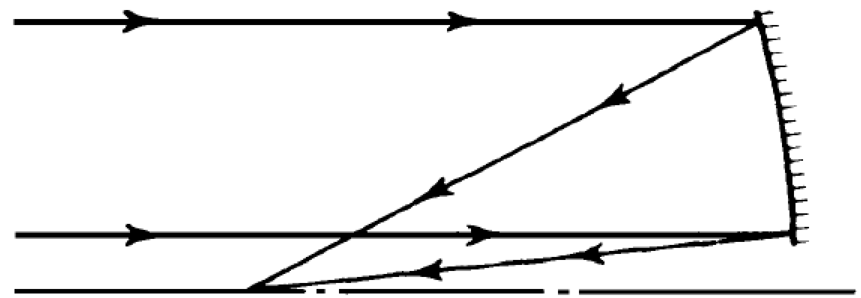
Stigmatic imaging for parallel light (i.e., the "other" focus is at infinity)



Paraboloidal one-mirror telescopes

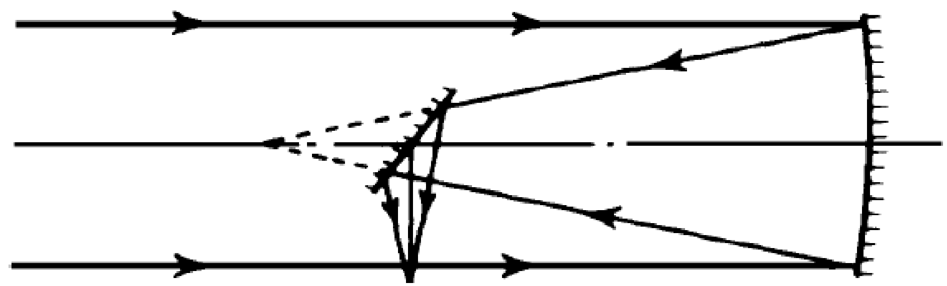
Herschellian

(off-axis parabola)



Newtonian

(high obscuration ratio)

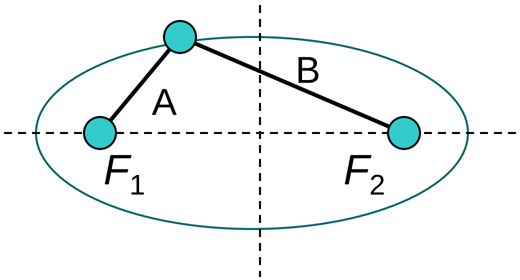


Field always limited by coma for parabolas.

(arc minutes at best)

Ellipsoidal Mirror

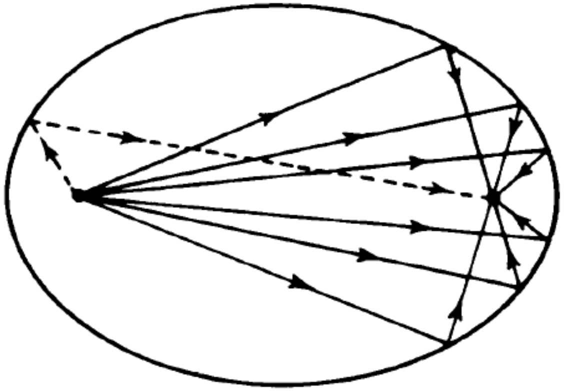
Ellipse defined as the locus of points from which the sum of distances to two foci (F_1 and F_2) is a constant.



A + B = constant

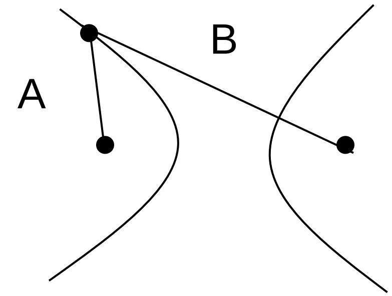
Optical property:

A ray passing through one focus is reflected to pass through the other focus.



Hyperboloid Mirror

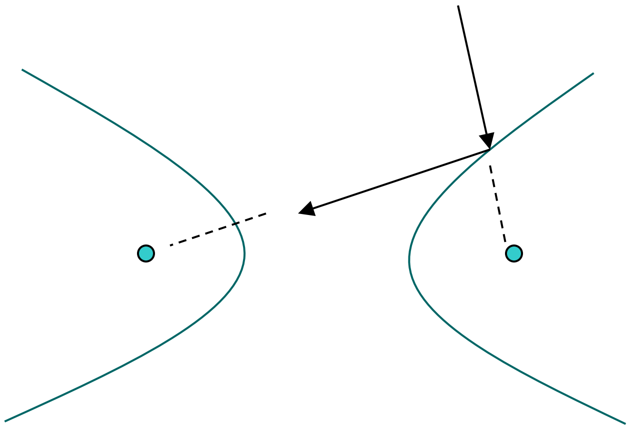
Hyperbola is defined as the locus of points for which the |difference of distances| to two foci is constant (points & foci in a common plane)



|A-B| = constant

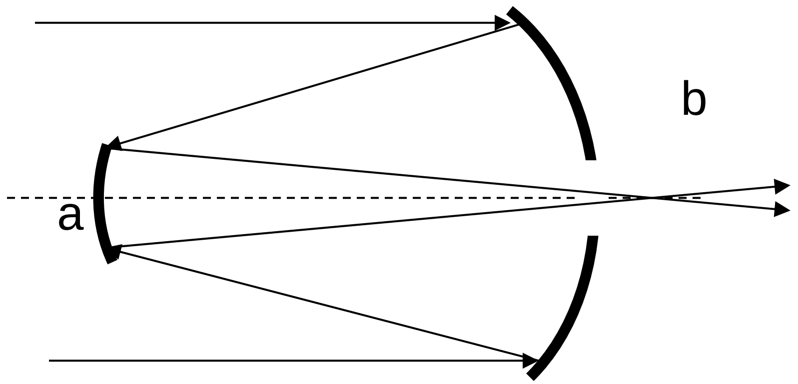
Optical property:

A ray that approaches a hyperbola from the side opposite a focus and pointing toward that focus reflects toward the other focus.



Two-mirror telescopes

$$\Phi = \Phi_a + \Phi_b - \frac{\Phi_a \Phi_b d}{n}$$



Relationship between primary-mirror and secondary-mirror conic constants *(k)*, radius of curvature *(R)*, marginal ray heights *(y)*, and magnifications *(m)* for <u>zero spherical aberration</u>.

$$k_1 + 1 = \frac{y_2^4}{y_1^4}\frac{R_1^3}{R_2^3}\left[k_2 + \left(\frac{m-1}{m+1}\right)^2\right]$$

D. Schroeder, *Astro. Optics*, p. 54 with *m* sign convention changed to match Smith's.

Ex: choose k_2 = 0 (sphere), solve for k_1 = -0.7696 (ellipse) … *Dall-Kirkham*

(see references for many aberration equations)

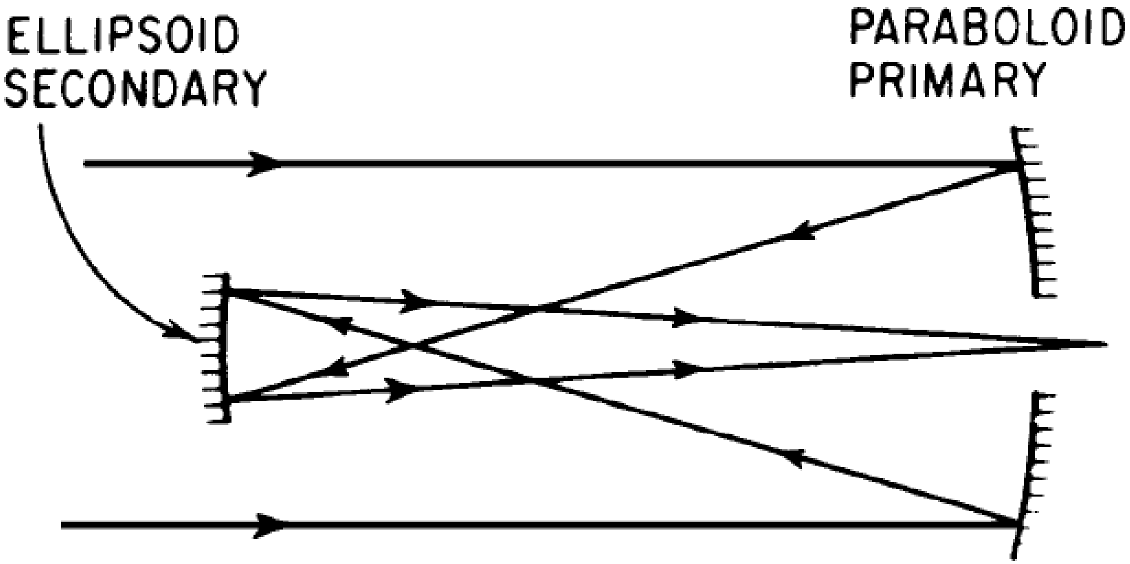
Gregorian

Primary mirror = parabola

Secondary = ellipsoid

Spherical = 0

Field limited by …



Cassegrain

Primary mirror = parabola   *(k* = -1*)*

Secondary = hyberbola

Spherical = 0

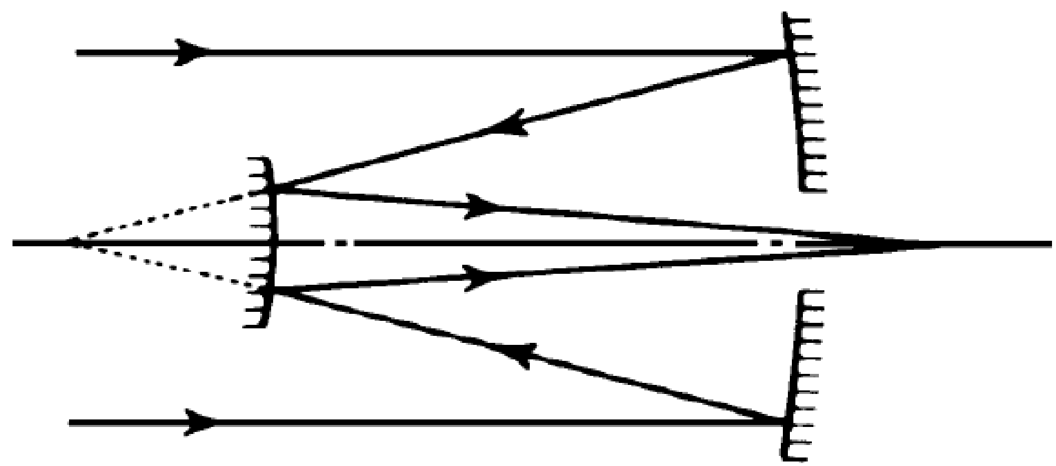
Astigmatism larger than for parabola alone,
but field usually limited by coma.

+ very compact

- convex secondary
  difficult to test

Shorter than optically
equivalent Gregorian

Cassegrain – France 1672



Dall-Kirkham

- Primary mirror = ellipse
- Secondary = sphere
- Spherical = 0 (on axis, obj at inf)
- Spherical secondary generates additional aberrations …
- Field limited by coma, to about $\frac{1}{2}(m_2^2+1)$ times smaller than for classical cassegrain.

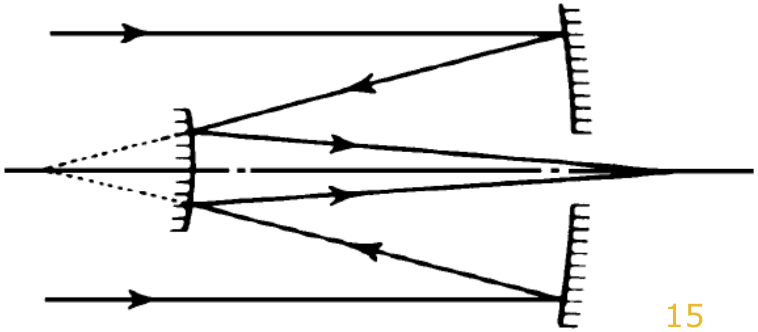
"poor man's cassegrain"

(spherical 2ndary cheaper)

+ much easier to build and test than cassegrain (cheaper)

+ less sensitive alignment

- larger aberrations



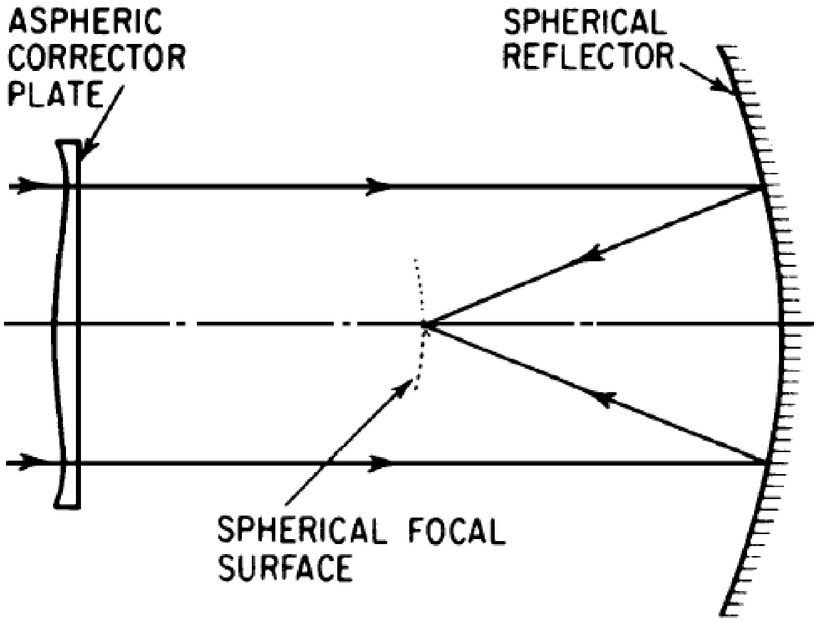
Schmidt

- **“Catadioptric”** system (combined reflector and refractor)
- Spherical mirror + aspheric refracting (corrector) plate at center of curvature
- Corrector plate at center leads to low off-axis aberrations
- Corrector plate minimizes spherical

+ very compact

+ corrector plate easier to build than parabolic primary

- images limited by spherochromatism and high-order astigmatism & spherical



Schmidt – Estonia 1930

Schmidt-Cassegrain

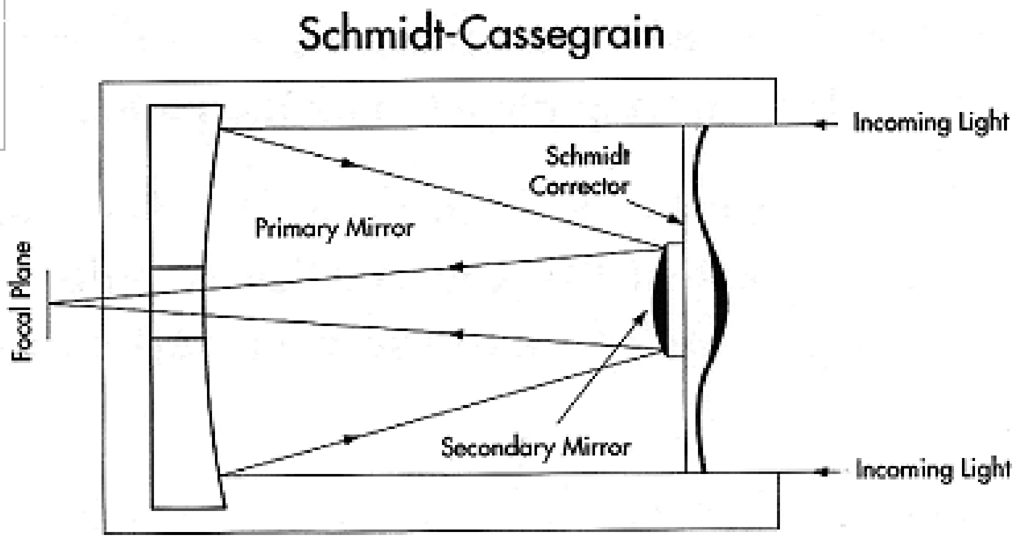
Spherical primary and secondary plus aspheric corrector plate

+ very compact

+ easy to build and test

- images contain some coma, astigmatism, spherochromatism …

Celestron C8 Photo



Maksutov

Spherical primary and secondary plus <u>meniscus</u> corrector plate
(sometimes elliptical primary … ex. AstroPhysics)

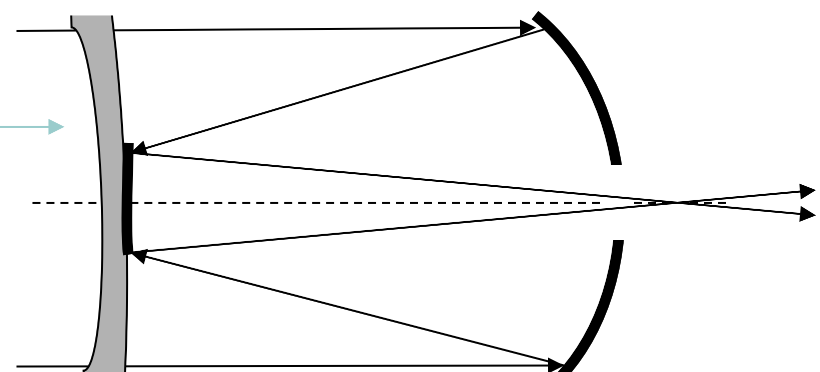
+ very compact

+ easy to build and test

- images contain some coma,
  astigmatism, spherochromatism …

Maksutov – Russia 1941

correcting plate
& secondary



Ritchey-Cretian

"Aplanatic Cassegrain"

~ Cassegrain with

primary mirror = hyperboloid

secondary = hyperboloid
(conic constant more negative
than for classical cassegrain)

Spherical = coma = 0 ("aplanat")

Slightly more astigmatism than cc,
but zero coma allows larger FOV

Field limited by astigmatism
(larger than coma-limit for cc
because aberration symmetry)

+ wider field of view than cc

+ spherical = coma = 0

- expensive to build

- more difficult to test than
other cassegrain types
(Hubble = f/24 2.4 m R-C !)

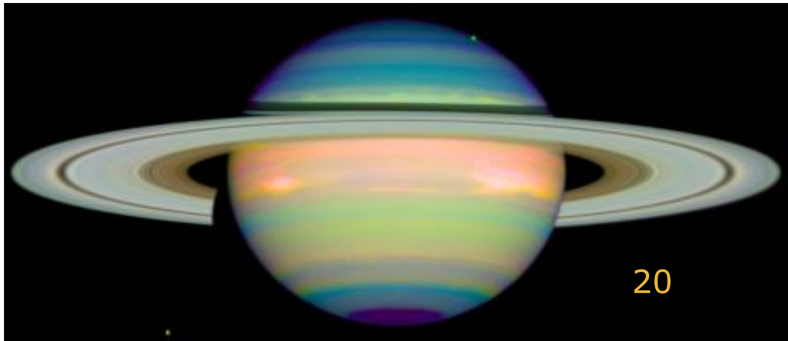
Hubble Space Telescope

Ritchey-Cretian with 2.4-m primary and 0.3-m secondary (both hyberboloids)

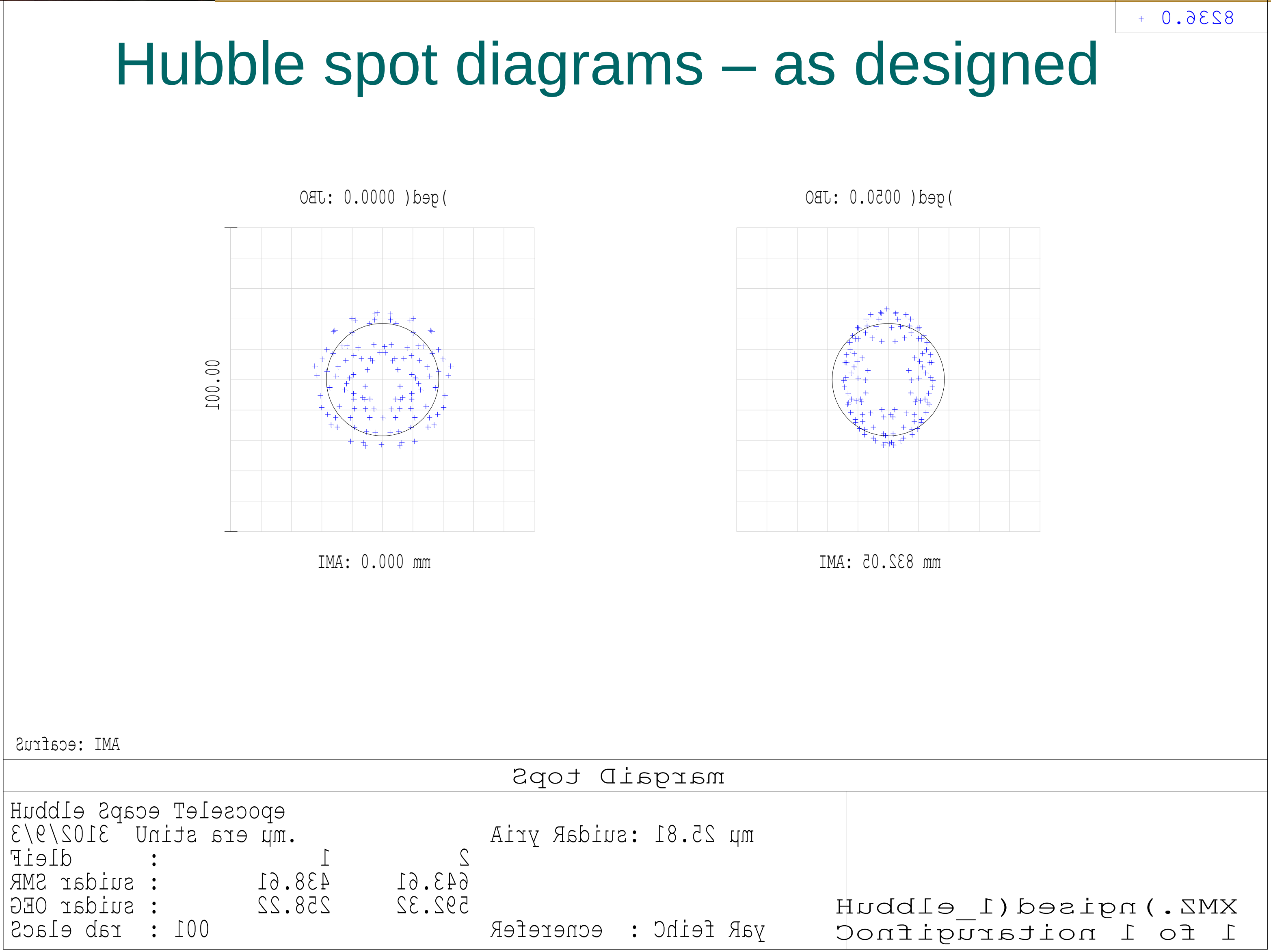
Before COSTAR After

Hubble primary was ground to the wrong figure because of a slightly misplaced null lens during interferometric testing.

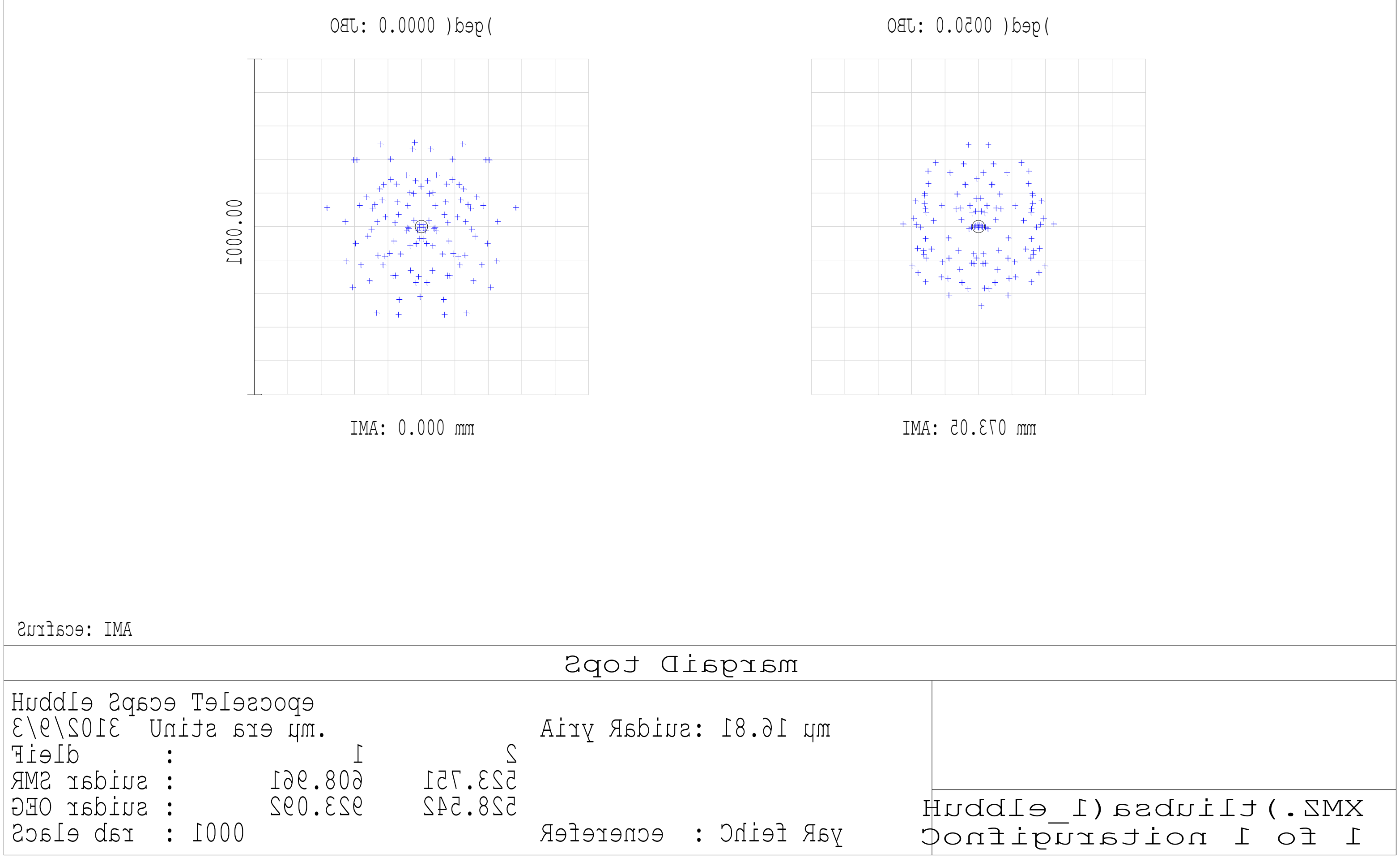
The result was large Spherical aberration.



Hubble spot diagrams – as designed



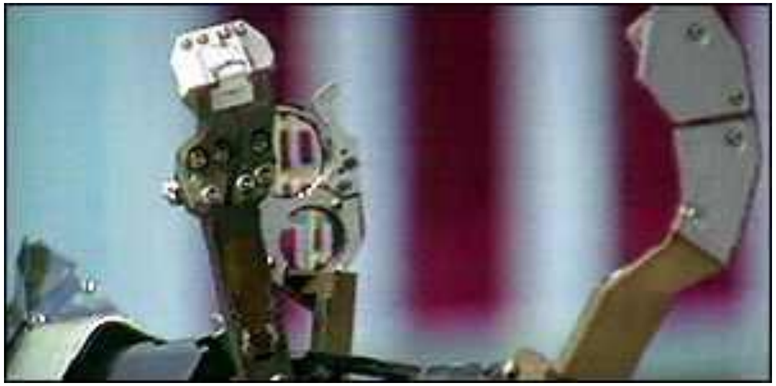
Hubble spot diagrams – as built



Hubble testing

The test interferometer was accidentally focused on a metal cover over the null corrector spacing rod instead of on the rod itself, as intended (this happened because a black paint chip fell off, revealing a metal glint point).

interferometric null-lens test



COSTAR corrective optics installed in 1993

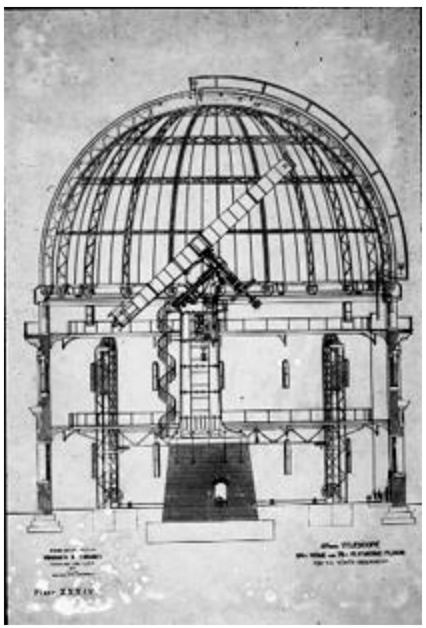
See Jeff Hecht, "Saving Hubble," Optics and Photonics News, pp. 42-49, March 2013.

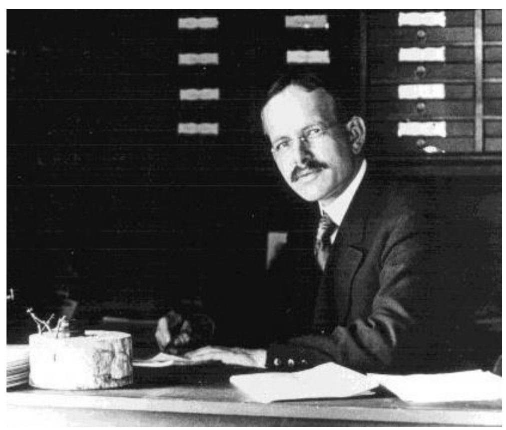
Images: http://news.bbc.co.uk/1/hi/sci/tech/638187.stm

Large Telescopes

Modern mirrors can be made much larger than lenses, which would break or sag severely under their own weight.

- Largest refractor in the world is the 40-inch (1-m) telescope installed at Yerkes Observatory in Wisconsin by George Ellery Hale in 1897.





G. Hale 1928

http://astro.uchicago.edu/vtour/

5-m Hale Telescope - Mt. Palomar

http://www.astro.caltech.edu/palomar/images/

University of Arizona Mirror Lab

Pioneered technique of making huge mirrors on honeycomb substrates

Largest casting to date = 8.4 m

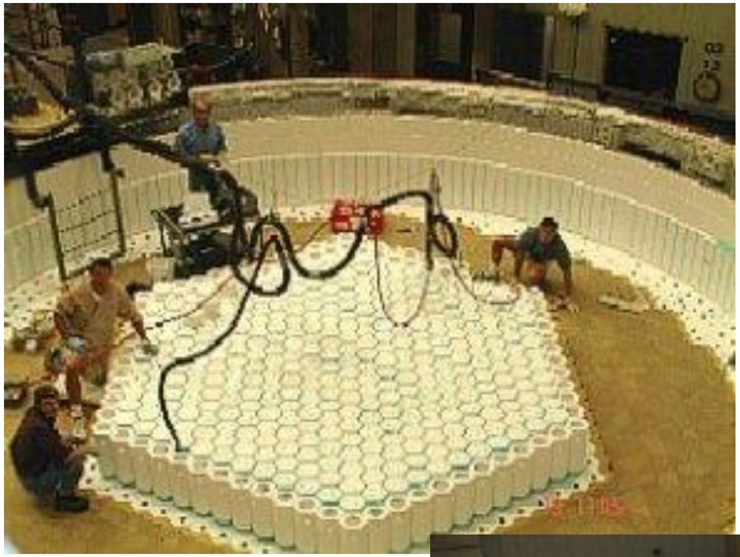
- Borosilicate glass (used in making cooking dishes)
- honeycomb substrate provides strength with light weight
- First tried by Dr. Roger Angel in his backyard

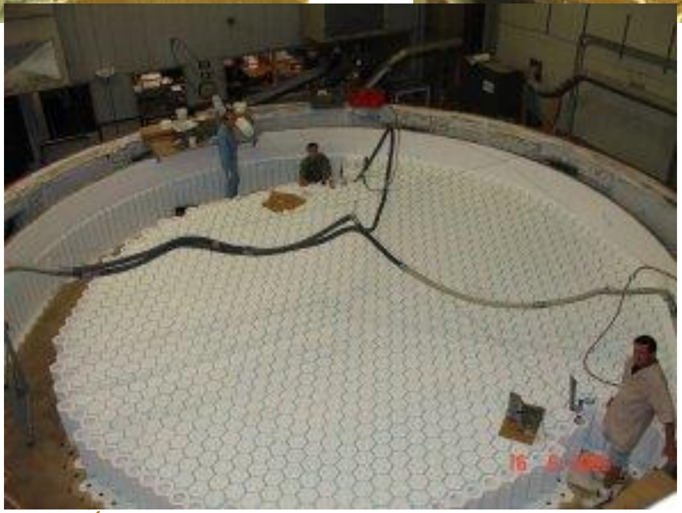
University of Arizona 8.4-m mirror

http://mirrorlab.as.arizona.edu/index.php

University of Arizona 8.4-m mirror

Inserting hexagonal honeycomb structures for strength in a lightweight structure





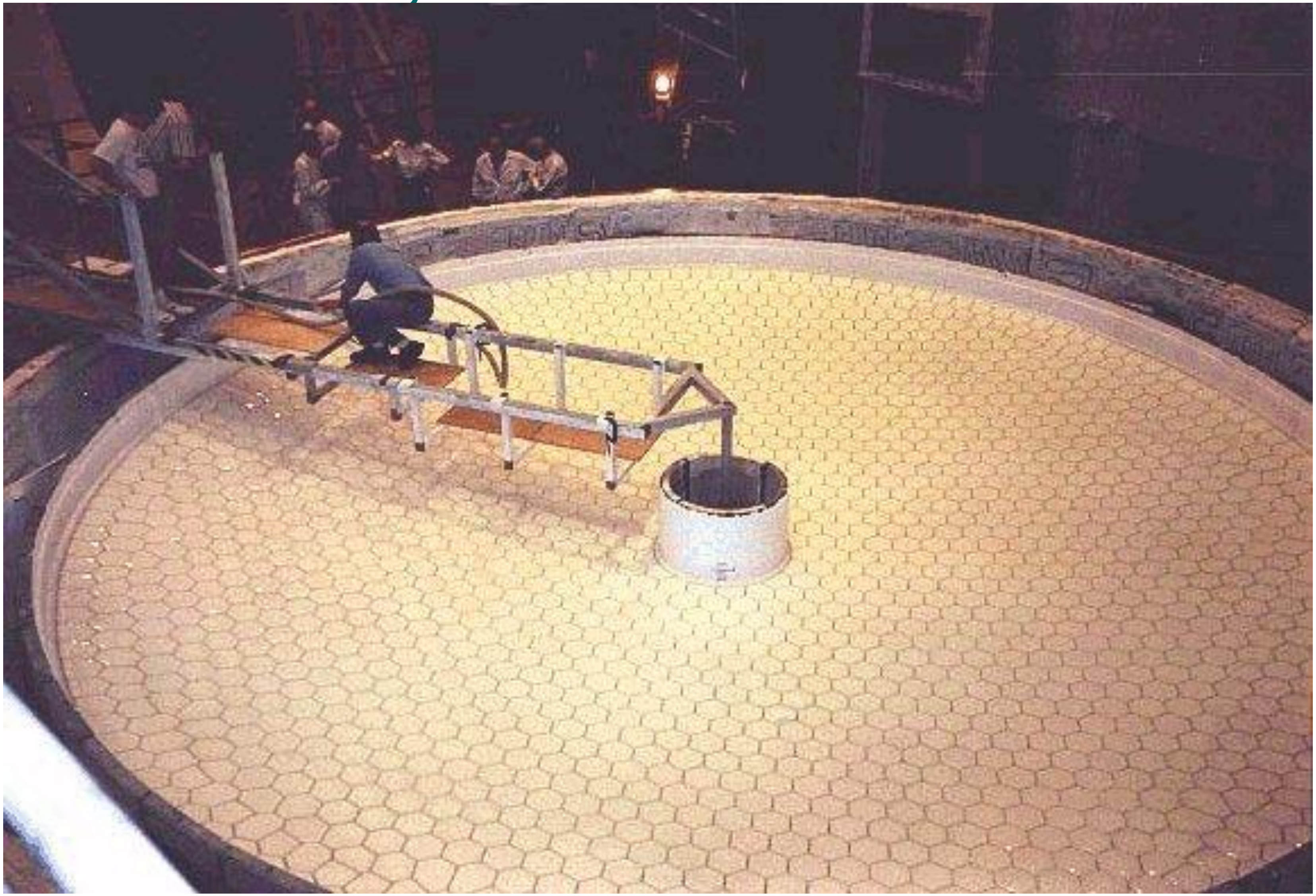
http://mirrorlab.as.arizona.edu/GMT.php

University of Arizona 8.4-m mirror

Grinding – part of a 6-year+ process to make the mirror

http://mirrorlab.as.arizona.edu/index.php

University of Arizona 8.4-m mirror



http://mirrorlab.as.arizona.edu/index.php

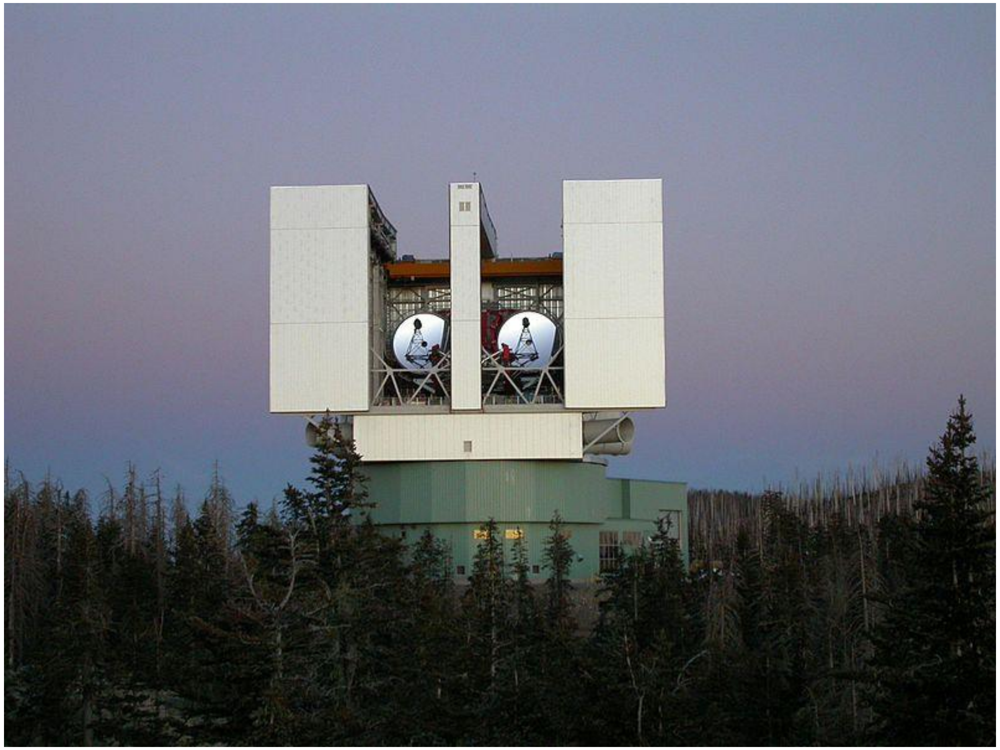
University of Arizona 8.4-m mirror !

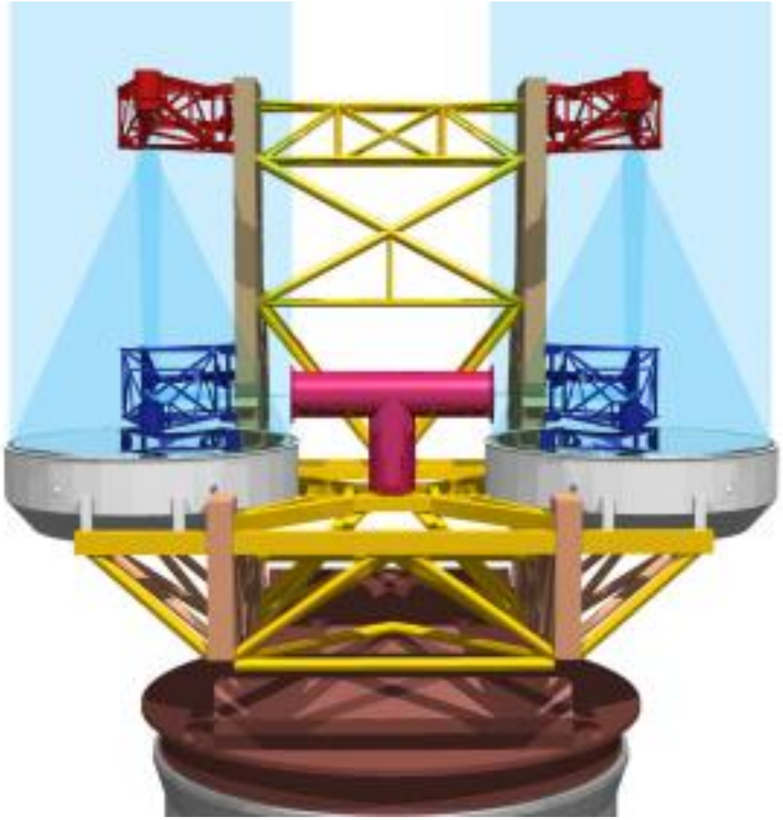
D = 8.408 m   F/1.142

http://mirrorlab.as.arizona.edu/index.php

Large Binocular Telescope

Mt. Graham – southeast of Tucson, Arizona (first light 12 Oct. 2005)





Two f/1.142 mirrors:
8.4-m diameter, obscuration 0.889 m, center spacing 14.417 m
Collecting area equivalent to 11.8-m mirror
(diffraction-limited resolution equiv. to 22.8 m)

http://mirrorlab.as.arizona.edu/index.php
http://medusa.as.arizona.edu/lbto/

Giant Magellan Telescope

Las Campanas Observatory – NNE of La Serena, Chile in Atacama Desert (2018)

Three 8.4-m mirrors casted as of 2013

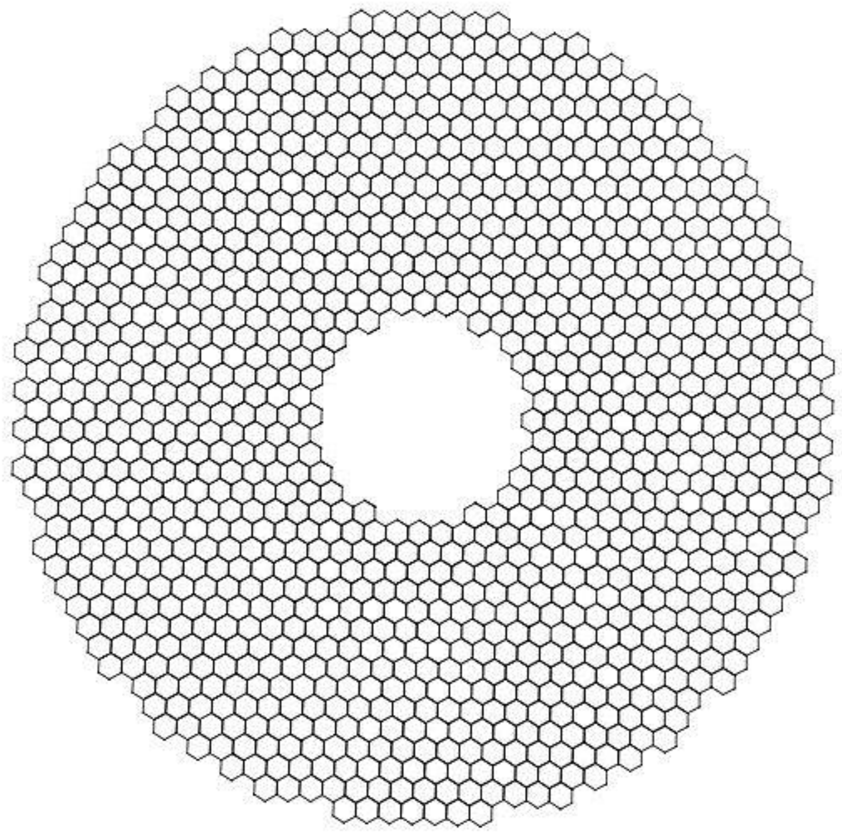
Aplanatic Gregorian

SEVEN off-axis 8.4-m primary mirrors
Collecting area equivalent to 21.4-m mirror
Diffraction-limited resolution equiv. to 24.5 m mirror

Seven off-axis adaptive secondary mirrors on hexapod mounts

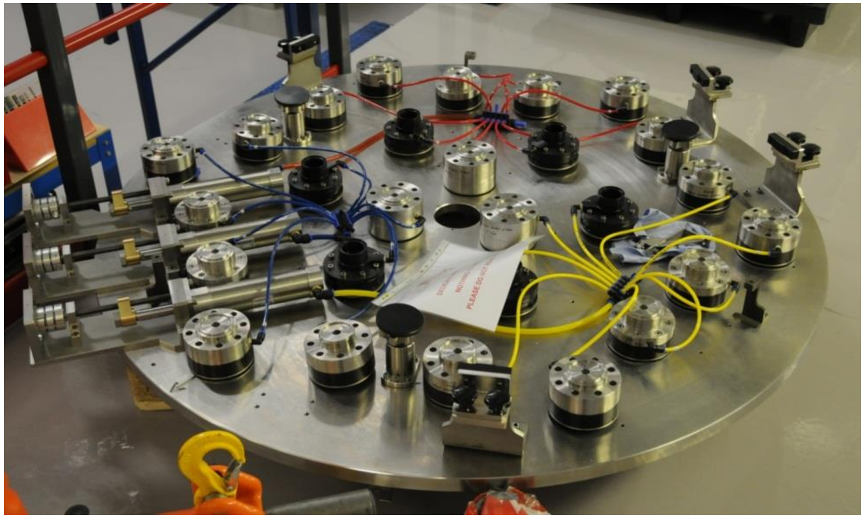
European Extremely Large Telescope

Cerro Armazones, Chile in Atacama Desert (2018)



http://www.gmto.org/

~42-m primary mirror made of
~1.4-m adaptive segments



J. Shaw photo

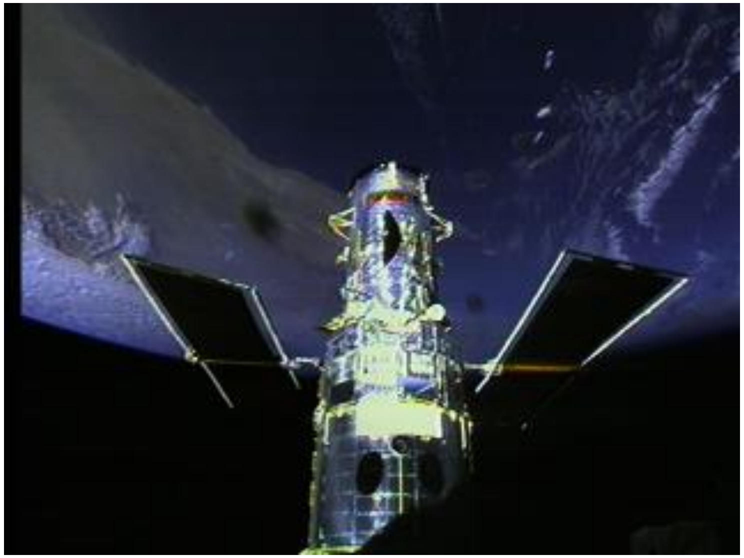
References

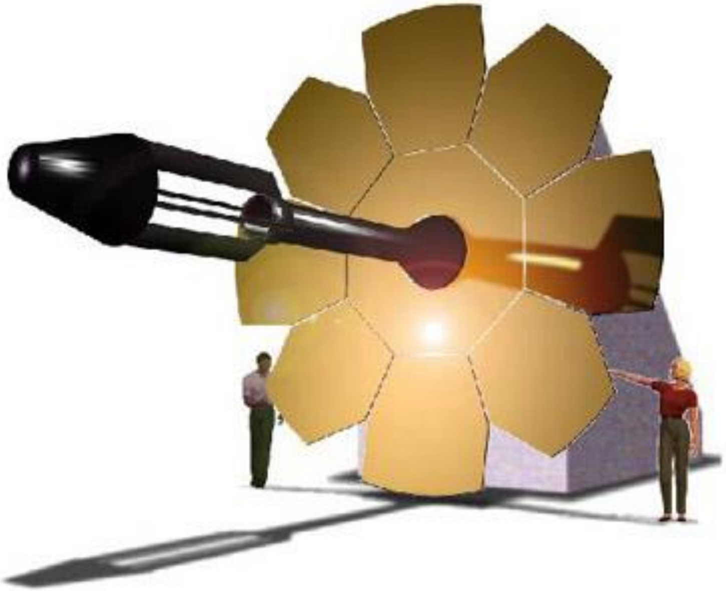
Daniel J. Schroeder, *Astronomical Optics*, Academic Press, 1987.

William L. Wolfe, *Introduction to Infrared System Design*, SPIE Press, 1996.

Warren J. Smith, *Modern Optical Engineering*, 3rd ed., McGraw-Hill, 2000.

W. Welford, *Aberrations of the Symmetrical Optical System,* Adam-Hilger, 1986.





http://ngst.gsfc.nasa.gov/about.html

Appendix

Additional information for further study

Spherical Mirror

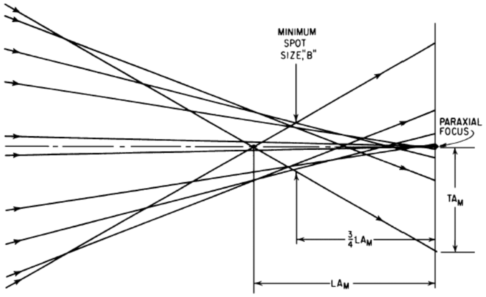
- Surface = section of circle (k = 0)
- Simplest reflector
- highest local radius of curvature
- Perfect imaging at 1:1 conjugates (m = 1)
- Undercorrected spherical and off-axis aberrations at non-unity conjugates
- For distant objects, spherical aberration is ~ 1/8 that of an equivalent lens bent at ‘minimum bending’

Tx 3rd-order spherical for spherical mirror with magnification m, at paraxial focus (Smith p. 475 & Schroeder p. 47)

$$TSA3 = \frac{(m-1)^2}{m+1}\frac{\rho^3}{2R^2}$$

Spherical aberration for a spherical mirror

For 3rd-order spherical, the minimum geometrical spot size is at ¾ of the distance from the paraxial focus to the marginal focus.



At this location, the minimum blur spot diameter is equal to ½ the minimum transverse spherical:

$$B = \frac{(m-1)^2}{(m+1)} \frac{\rho^3}{4R^2}$$

Divide by $f = R/2$ to obtain the angular blur in radians:

$$\beta = \frac{(m-1)^2}{(m+1)} \frac{\rho^3}{2R^3}$$

Use $F\# = f/(2\rho)$, $f = R/2$, and $m = 0$ to rewrite this as

$$\beta = \frac{1}{128(F\#)^3} = \frac{NA^3}{16}$$

(need to add high-order aberrations below about f/2)

(Smith, p. 475)

Spherical mirror off-axis aberrations (stop not at center)

For distant object, the 3rd-order coma and astigmatism contributions are found by using *n'* = - *n* in the surface-aberration equations from Smith's chapter 10 (which are just scaled versions of the Seidel sums) …

Sagittal 3rd-order coma

$$CC^* = \frac{\rho^2 \left(R - \bar{l}\right)\bar{u}}{2R^2} = \frac{\left(R - \bar{l}\right)\bar{u}}{32\left(F\#\right)^2}$$

Longitudinal 3rd-order astigmatism (1/2 separation of *S* and *T* fields)

$$AC^* = \frac{\left(\bar{l} - R\right)^2 \bar{u}^2}{4R}$$

Longitudinal Petzval

$$PC = \frac{\bar{u}^2 R}{4} = \frac{h_i^2}{2f}$$

Note: when the Stop is at distance *R*, CC* = AC* = 0.

In these equations ρ is the normalized aperture radial coordinate, *R* is the mirror radius of curvature, h_i is the image height, *f* is the focal length, $\bar{l}$ is the distance from vertex to aperture stop, and $\bar{u}$ is the chief-ray angle at the image.

Spherical mirror with stop at the mirror

When the aperture stop is located at the mirror, the minimum angular blur sizes are:

3rd-order sagittal coma $$\beta_c = \frac{-\bar{u}}{16(F\#)^2}$$ radians

3rd-order astigmatism at best focus $$\beta_a = \frac{\bar{u}^2}{2(F\#)}$$ radians

Spherical mirror with stop in between

When the aperture stop is located somewhere between the mirror and the center of curvature, we can simply add the minimum blur spot sizes from The equations for spherical blur, coma blur, and astigmatism blur, adjusted for the following scaling:

1. coma = astigmatism = 0 when stop is at center of curvature

2. coma varies linearly with distance of stop from center of curvature

3. astigmatism varies quadratically with distance of stop from center…

Keep in mind that the appropriately scaled β sum is still only approximate (uses only 3rd-order aberrations) and is <u>angular</u> blur in units of radians.