

## Modeling infrared lunar radiance

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Subject terms: radiative transfer; infrared; radiometry; atmospheric optics; remote sensing; backgrounds.

Paper 980427 received Nov. 9, 1998;  
accepted for publication May 18, 1999.  
[S0091-3286(99)02110-8]

### 1 Introduction

Recently I encountered two remote sensing problems that required an estimate of direct lunar radiance at thermal IR wavelengths. One was the calculation of signal levels for atmospheric absorption spectroscopy using the moon as a source,<sup>1,2</sup> and the other was the determination of the effect that moon glints have on IR polarization radiometry of water surfaces.<sup>3</sup> Many optical system designers and analysts turn to the popular MODTRAN radiative transfer program<sup>4</sup> in these kinds of situations to quickly estimate atmospheric, solar, and lunar effects on optical and IR system performance. However, the MODTRAN lunar-source model as of the time of this writing does not include thermal emission from the sunlit moon, making estimates of direct lunar radiance inaccurate beyond about 2.5  $\mu\text{m}$ .

Lunar radiance scattered in the atmosphere exceeds thermal atmospheric path emission for wavelengths shorter than about 1.8  $\mu\text{m}$  and is represented adequately by a reflected-solar model alone. Conversely, scattered lunar radiance is overpowered by thermal atmospheric path radiance at longer wavelengths. Therefore, at thermal IR wavelengths lunar radiance can be safely neglected, except for when the sensor sees the moon directly or in direct reflection (e.g., moon glints on water). In these cases, a simple Planck-function calculation simulating lunar thermal emission is superior to a reflected-solar treatment alone. This note collects information from several sources<sup>5-8</sup> to produce a simple alternative model for estimating direct lunar radiance for a full moon. The objective is to provide a simple and practical method of estimating lunar radiance for engineers and scientists who design, analyze, or use IR sensors in the natural earth environment.

### 2 Lunar Radiative Transfer

To first order, solar radiance outside the earth's atmosphere can be modeled<sup>9</sup> by blackbody radiation at 5900 K. The reflected-solar component of lunar radiance can be estimated from the blackbody solar radiance  $L^{\text{bb}}(\lambda, 5900 \text{ K})$  as

$$L_{\text{reflected}}(\lambda) = \frac{L^{\text{bb}}(\lambda, 5900 \text{ K}) \Omega_s R_m(\lambda)}{\pi}, \quad (1)$$

where  $\Omega_s$  is the solid angle subtended by the sun viewed from the earth ( $\sim 6.8 \times 10^{-5}$  sr) and  $R_m$  is the directional-hemispherical reflectivity of the moon at wavelength  $\lambda$ , which is shown in Figure 1 (this curve is a result of fitting a smooth curve through several sets of measurements<sup>5-7</sup>). The blackbody solar model of Eq. (1) ignores a lot of fine structure in the solar spectrum below 1  $\mu\text{m}$ , but provides a good match with the MODTRAN solar calculation at longer wavelengths (the MODTRAN solar-radiance calculation can be used for more accurate modeling of the solar term at short wavelengths). Also, the Lambertian assumption indicated by the  $\pi$  in the denominator of Eq. (1) is not exactly valid for the moon's partially retroreflective surface, but is acceptable in this approximation.

Thermal emission from the full moon can be approximated by blackbody radiation at 390 K multiplied by the moon's spectral emissivity  $\epsilon(\lambda)$ :<sup>5-7</sup>

$$L_{\text{emitted}} = \epsilon(\lambda) L^{\text{bb}}(\lambda, 390 \text{ K}). \quad (2)$$

The total lunar spectral radiance outside the earth's atmosphere is the sum of these reflected and emitted terms. Multiplying the exoatmospheric radiance by the atmospheric spectral transmissivity  $\tau_{\text{atm}}(\lambda)$ , and adding the integrated atmospheric path radiance  $L_{\text{atm}}(\lambda)$ , yields a spectral radiance within the atmosphere,

$$L_{\text{moon}}(\lambda) = \tau_{\text{atm}}(\lambda) [L_{\text{reflected}}(\lambda) + L_{\text{emitted}}(\lambda)] + L_{\text{atm}}(\lambda). \quad (3)$$

The atmospheric spectral transmissivity can be calculated with MODTRAN or a similar atmospheric radiative transfer code.

### 3 Results and Discussion

Figure 2 shows the spectral lunar radiance outside the earth's atmosphere calculated for a full moon with the equations presented here (solid line) and with MODTRAN 3 (dashed line). MODTRAN actually calculates spectral irradiance, which was converted to radiance by dividing by the solid angle that the moon subtends at the earth ( $\sim 6.8 \times 10^{-5}$  sr). The two curves in Figure 2 are similar at wavelengths below 2.5  $\mu\text{m}$ , but MODTRAN massively underestimates the thermal IR radiance by neglecting lunar emission.

These results are valid only for a full moon (zero phase angle). At larger phase angles, the moon's brightness decreases more rapidly than a Lambertian reflector.<sup>7,8</sup> Modeling the moon at phase angles other than zero requires more than a simple geometric correction, however, because the

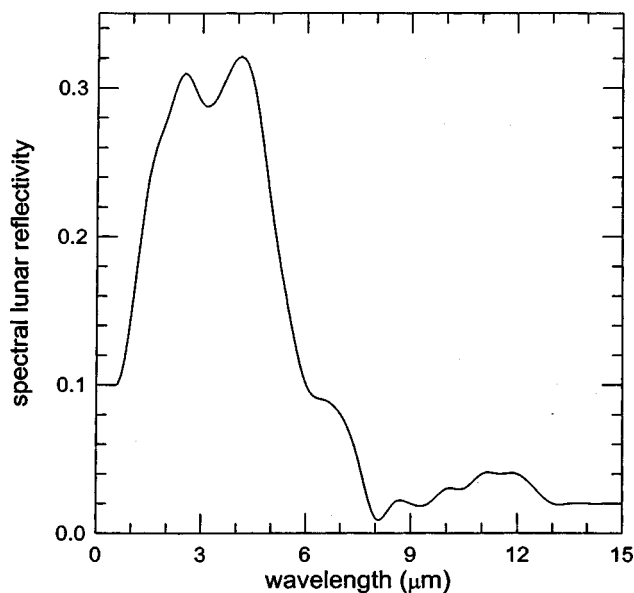


Fig. 1 Average directional-hemispherical reflectivity of the full moon. The emissivity is 1 minus the reflectivity, making the moon a highly efficient emitter beyond about  $6 \mu\text{m}$ .

effective radiating temperature of the moon also changes. The average lunar temperature decreases from about 390 K at full moon to about 90 K at new moon.<sup>10</sup>

Other small corrections could include accounting for the variation of solid angle with earth to moon distance, and

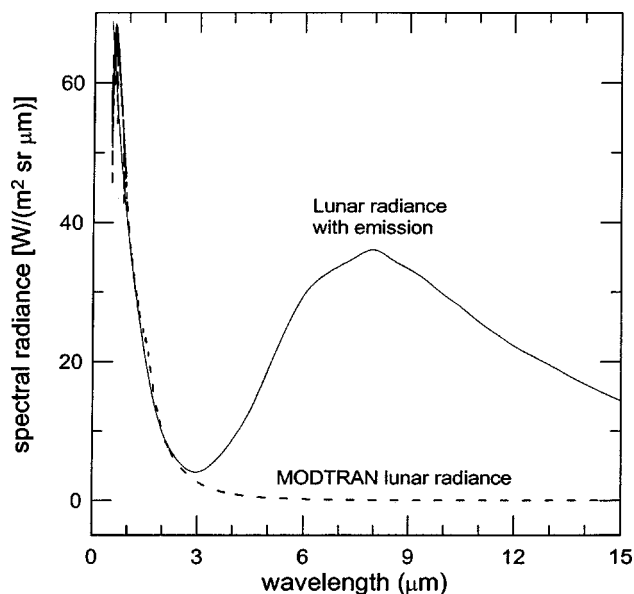


Fig. 2 Spectral radiance of the full moon outside the earth's atmosphere calculated with the model described here, which includes both solar reflection and thermal emission (solid line) and the corresponding radiance calculated with MODTRAN 3 (dashed line).

accounting for the small polarization of reflected sunlight. The solid angle subtended by the moon varies from  $5.7 \times 10^{-5}$  sr at apogee to  $7.5 \times 10^{-5}$  sr at perigee for the lunar orbit around the earth.<sup>11</sup> The moon and the sun both subtend approximately the same solid angle from the earth, so an appropriate value of  $6.8 \times 10^{-5}$  sr has been used here. Polarization has also been neglected because of its very small magnitude ( $<1\%$ ) near zero lunar phase angle (for visible light, the moon's polarization varies between extrema of  $-1.2\%$  and  $+8\%$  at respective phase angles of 12 and 90 deg).<sup>7</sup>

#### Acknowledgments

This document has been generated as part of a joint National Oceanic and Atmospheric Administration/Department of Defense (NOAA/DOD)-Advanced Sensors Applications Program. The Matlab source code for the lunar radiance model described here is available on request from the author (jshaw@etl.noaa.gov). An infrared lunar radiance model has been incorporated recently into version 1.60 of the MODerate Spectral Atmospheric Radiance and Transmittance code<sup>12</sup> (MOSART, available from the Air Force Research Laboratory, Battlespace Environment Division: www-VSB.plh.af.mil). The same lunar radiance model will also be included<sup>13</sup> in a future version of MODTRAN.

#### References

1. J. Notholt, "The moon as a light source for FTIR measurements of stratospheric trace gases during the polar night: Application for  $\text{HNO}_3$  in the Arctic," *J. Geophys. Res.* **99**(D2), 3607-3614 (1994).
2. J. Schreiber, T. Blumenstock, and F. Hase, "Application of a radiometric calibration method to lunar Fourier transform IR spectra by using a liquid-nitrogen-cooled high-emissivity blackbody," *Appl. Opt.* **36**(31), 8168-8172 (1997).
3. J. A. Shaw, "Degree of linear polarization in spectral radiances from water-viewing infrared radiometers," *Appl. Opt.* **38**(15), 3157-3165 (1999).
4. A. Berk, L. S. Bernstein, and D. C. Robertson, "MODTRAN: a moderate resolution model for LOWTRAN7," Air Force Geophysics Laboratory Technical Report GL-TR-89-0122, Hanscom Air Force Base, MA (1989).
5. F. H. Murcray, D. G. Murcray, and W. J. Williams, "Infrared emissivity of lunar surface features I. Balloon-borne observations," *J. Geophys. Res.* **75**(14), 2662-2669 (1970).
6. J. W. Salisbury, A. Basu, and E. M. Fischer, "Thermal infrared spectra of lunar soils," *Icarus* **130**, 125-139 (1997).
7. H. H. Kieffer and R. L. Wildey, "Establishing the moon as a spectral radiance standard," *J. Atm. Oceanic Technol.* **13**, 360-375 (1996).
8. K. J. Vogler, P. E. Johnson, and R. W. Shorthill, "Modeling the non-grey-body thermal emission from the full moon," *Icarus* **92**, 80-93 (1991).
9. W. L. Wolfe and G. J. Zissis, *The Infrared Handbook*, 3rd printing, Environmental Research Institute of Michigan, Ann Arbor, MI (available through SPIE, P.O. Box 10, Bellingham, WA 98227-0010), pp. 3-34 (1989).
10. M. L. Urquhart and B. M. Jakosky, "Lunar thermal emission and remote determination of surface properties," *J. Geophys. Res.* **102**(E5), 10959-10969 (1997).
11. M. A. Seeds, *Foundations of Astronomy*, Wadsworth Publishing Co., Belmont, CA, Chap. 3 (1984).
12. G. P. Anderson, Personal Communication, Air Force Research Laboratory, Hanscom Air Force Base, MA 01731 (currently visiting at the Climate Monitoring and Diagnostics Laboratory, 325 Broadway, Boulder, Colorado) (May 1999).
13. William M. Cornette, Personal Communication, National Imagery and Mapping Agency, Reston, VA (May 1999).