Labs 4

RLC Resonant Circuits

Purpose

Highly tuned RLC tank circuits play a major role in electronics and communications. In these experiments you will investigate the frequency response of two RLC circuits that produce a low-pass and a bandpass filter. Through PSPICE simulation and actual laboratory experimentation, you will determine the natural frequency, damping ratio, 3-dB bandwidth, and quality factor of the circuits in terms of the circuit parameters.

Skills to develop

After completing this lab you should be able to:

- Choose the resistor value in a series or parallel RLC circuit to achieve a specified *BW* or *Q* value
- Use Pspice to simulate series or parallel RLC circuits in the frequency domain, determining ω_o, *BW* and *Q* from your plots
- Measure the resonance properties of RLC series and parallel circuits in the lab, including determination of the resonance frequency, bandwidth and *Q*

Equipment

For the laboratory experimentation, the standard laboratory equipment (function generator, oscilloscope, multimeter) will be used along with a capacitor (0.027 μF), an inductor (10 mH for the series RLC circuit and 1 mH for the parallel RLC circuit), and a resistor (to be determined in your prelab work).

Prelab Work

The particular RLC circuits that you will be investigating are shown in Fig. 1. They are often called "tank" circuits because they tend to have oscillatory stored energy in the LC "tank".

a. Find the transfer function $V_0(j\omega)/V_i(j\omega)$ for circuit (a) (lowpass filter) in terms of the circuit parameters R, R_a, L, and C. Put the denominator of the transfer function in standard form $[(j\omega)^2+(2\zeta\omega_o)(j\omega) + \omega_o^2]$ and find ω_o, ζ *(damping ratio)*, β *(BW)* $= 2\zeta\omega_o$ and *Q(quality factor)* $= \omega_o/BW$.

b. For circuit (b) (bandpass filter), first find the impedance $Z_1(j\omega)$ and use the "voltage divider rule" to show that

$$H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{(\frac{1}{RC})(\frac{R_a}{L} + j\omega)}{(j\omega)^2 + (\frac{1}{RC} + \frac{R_a}{L})j\omega + (\frac{R+R_a}{R})(\frac{1}{LC})} \quad (1)$$

The denominator of (1) is in the standard form given in part a., and when the damping ratio (ζ) is close to zero, the poles of (1) are complex conjugates and are close to the $j\omega$ axis in the complex ($\alpha + j\omega$) plane. This results in a frequency response that peaks near the natural frequency, ω_o of the circuit. Show that for circuit (b) the parameters ω_o, ζ, *BW, and Q* are as follows:

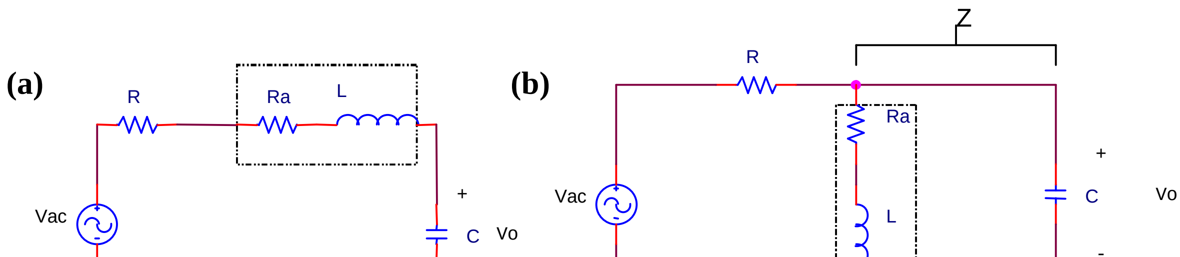

Fig. 1. RLC lowpass (a) and bandpass (b) filter circuits.

$$\beta = 2\zeta\omega_o = \frac{1}{RC} + \frac{R_a}{L} \text{ rad./sec (assuming } \zeta << 1), \qquad \omega_o = \sqrt{\frac{R+R_a}{R}(\frac{1}{LC})} ,$$

$$\zeta = \frac{1}{2}(\sqrt{\frac{R}{R+R_a}})(\sqrt{\frac{L}{C}}\frac{1}{R} + \sqrt{\frac{C}{L}}R_a), \text{ and } Q = \frac{\omega_o}{BW} = \frac{\sqrt{\frac{R+R_a}{R}}}{\frac{1}{R\sqrt{C/L}} + \frac{1}{(\frac{1}{R_a})\sqrt{L/C}}}$$

For a simple series RLC circuit (with R_a = 0), the Q factor can be shown to be $\frac{1}{R}\sqrt{\frac{L}{C}}$; and for a simple parallel RLC circuit, the Q factor can be shown to be $R\sqrt{\frac{C}{L}}$.

c. Find the value of R in each circuit to achieve a bandwidth β = 10 k rad/sec.
For circuit (a) (the series circuit), use R_a = 34 Ω, L = 10 mH and C = 0.027μF; and for circuit b, use R_a = 3 Ω ($R_a << R_{source}$), L = 1 mH and C = 0.027μF.

d. Using the known values of R, R_a, C, and L for each circuit, find the actual values of ω_o, $f_n = \omega_o/2\pi$, BW, ζ *and* Q.

Lab 4 PSPICE Simulation:

1 Using the circuit parameter values given in the prelab, model the circuits given in Fig. 1 using PSPICE. Drive the circuits with a unit amplitude (1 V) VAC sine wave, and obtain plots of the magnitude and the phase angle of the voltage across the capacitor (V_0) as a function of frequency. Plot from one decade below resonance to one decade above. Assume the input voltage has an angle of zero.

2. From your plots estimate the frequency at which the capacitor voltage is maximum and compare it with the circuit natural frequency of oscillation (f_o). Also, obtain the 3-db bandwidth and quality factor for each circuit and compare them with those you calculated in prelab.

3. Vary the value of R in each circuit to achieve *BW*=5000 r/s (double *Q*) and *BW*=20000 r/s (halve *Q*). Obtain magnitude and phase angle plots of the output voltage for these two new values of R and observe the effect of variation in R on the circuit *BW* and *Q* factor.