How are likelihoods calculated when individual covariates are considered?

Consider a data set for 6 animals:

LD	Y	mass
11	0	45
10	1	75
10	1	80
11	0	32
10	1	42
11	0	50

A model containing an intercept and mass had the following estimated beta values: b0 = -5.935, b1 = .1146. To calculate the lnL for these parameter estimates, you'd use the following equation:

$$L = \left[\frac{e^{(-5.935+0.1146\times45)}}{1+e^{(-5.935+0.1146\times45)}}\right]^0 \times \left[1-\frac{e^{(-5.935+0.1146\times45)}}{1+e^{(-5.935+0.1146\times45)}}\right]^1 \times$$

$$\left[\frac{e^{(-5.935+0.1146\times75)}}{1+e^{(-5.935+0.1146\times75)}}\right]^1 \times \left[1-\frac{e^{(-5.935+0.1146\times75)}}{1+e^{(-5.935+0.1146\times75)}}\right]^0 \times$$

$$\left[\frac{e^{(-5.935+0.1146\times80)}}{1+e^{(-5.935+0.1146\times80)}}\right]^1 \times \left[1-\frac{e^{(-5.935+0.1146\times80)}}{1+e^{(-5.935+0.1146\times80)}}\right]^0 \times$$

$$\left[\frac{e^{(-5.935+0.1146\times32)}}{1+e^{(-5.935+0.1146\times32)}}\right]^0 \times \left[1-\frac{e^{(-5.935+0.1146\times32)}}{1+e^{(-5.935+0.1146\times32)}}\right]^1 \times$$

$$\left[\frac{e^{(-5.935+0.1146\times42)}}{1+e^{(-5.935+0.1146\times42)}}\right]^1 \times \left[1-\frac{e^{(-5.935+0.1146\times42)}}{1+e^{(-5.935+0.1146\times42)}}\right]^0 \times$$

$$\left[\frac{e^{(-5.935+0.1146\times50)}}{1+e^{(-5.935+0.1146\times50)}}\right]^0 \times \left[1-\frac{e^{(-5.935+0.1146\times50)}}{1+e^{(-5.935+0.1146\times50)}}\right]^1$$

Remember: if b0 = -5.935, b1 = .1146 are the maximum likelihood estimates, then no other combination of values can be found for b0 and b1 that will have a higher Likelihood value for this dataset and this model.

The calculation on the previous page is simply taking advantage of the following equations, right?

$$pi_i = \frac{e^{\beta_0 + \beta_1 \times X1_i}}{1 + e^{\beta_0 + \beta_1 \times X1_i}}$$

$$L = \prod_{i=0}^{5} \left(pi_i\right)^{y_i} \times \left(1 - pi_i\right)^{1-y_i}$$

$$lL = \sum_{i=0}^{5} \left[y_i \times \ln\left(pi_i\right) + \left(1 - y_i\right) \times \ln\left(1 - pi_i\right) \right]$$

For this example, L(b0 = -5.935, b1 = .1146|X) = 0.076
The lnL = -2.582, and –2lnL = 5.165
Given the preceding equations, do you see why having ***missing data for only some of the covariates of interest*** *can complicate model comparisons?*

Analysis of Mule Deer Fawn Data

Here, you've incorporated information about individual body characteristics. Use of individual covariates greatly expands your ability to model and should be considered for many problems.

Analyses indicated that the best model contains Sex and Length (deltaAIC=0.00)

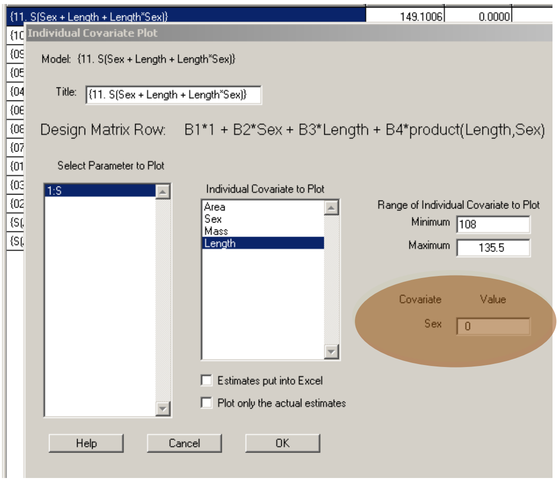
Ln(S/(1-S) = B0 + B1*Sex + B2*Length + B3*Sex*Length

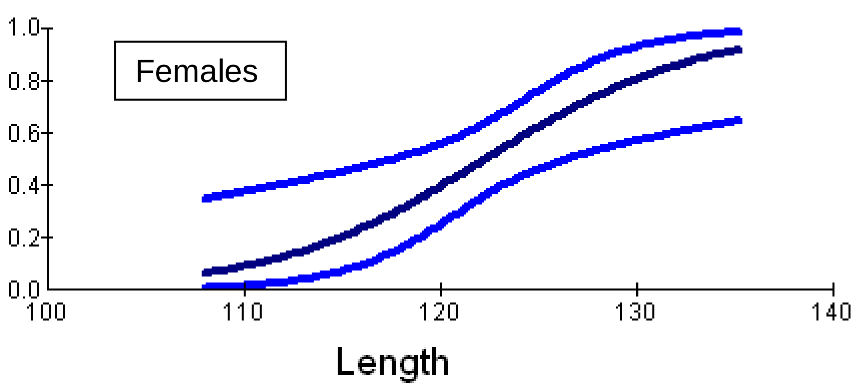
LOGIT Link Function Parameters of {11. S(Sex + Length + Length*Sex)}

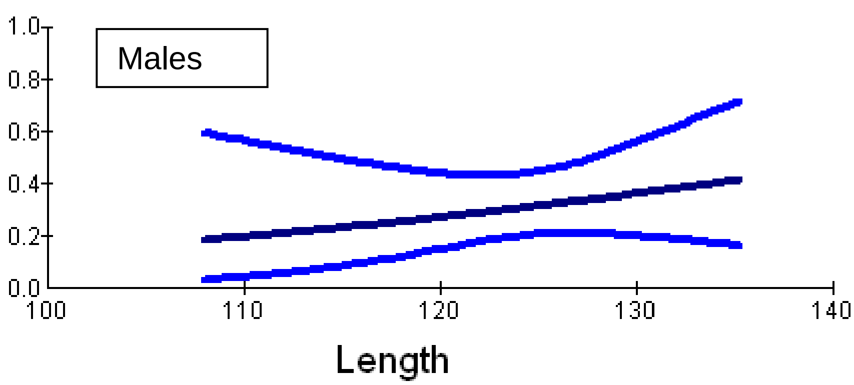
Parameter	Beta	Standard Error	95% Confidence Interval Lower	Upper
1: B0 (Intercept)	-22.724138	8.4190916	-39.225558	-6.2227183
2: B1 (sex, Male=1)	16.755701	10.833715	-4.4783812	37.989783
3: B3 (length)	0.1858881	0.0686643	0.0513060	0.3204701
4: B4 (sex x length)	-0.1443023	0.0877866	-0.3163641	0.0277595

From looking at the coefficients, we can see that males have a higher intercept (--22.72 + 16.76 = -5.97) and flatter slope (0.186 – 0.144 = 0.04) than do females (intercept = -22.72 and slope = 0.186). So, short females ought to have lower survival than do short males. And, females benefit more than males do from being longer, but … without calculating actual values for survival and examining them in a table or a figure, it's hard to know much more than that without some pretty hard thinking.

This is where MARK's "Plot Individual Covariates" tool can be very helpful. Choose the model of interest, choose the tool, work with length, and set the sex covariate to either 0 (female) or 1 (male)







So, the point estimates are lower for females at the shorter lengths (but very imprecisely estimated), and, at longer lengths, females appear to be surviving better. A table of values could easily be presented that would show this more concretely.

When we look at such a table, which I created quickly using MARK output, we find that the point estimates are lower for females up to a length of ~116 cm. Beyond that point the females are expected to survive at a higher rate.
We could put confidence intervals on the differences using one of several methods designed for such a task. One of these is the delta method, and we'll learn how to use it later in the semester.

	A	B	C	D	E	G	H	I	J	L
1		FEMALES				MALES				Difference
2	Length	1:S	SE	LCI	UCI	1:S	SE	LCI	UCI	Female-Male
3	108	0.066	0.064	0.009	0.352	0.186	0.144	0.034	0.595	-0.120
4	108.275	0.069	0.066	0.010	0.355	0.188	0.143	0.036	0.591	-0.118
5	108.55	0.073	0.068	0.011	0.359	0.189	0.141	0.037	0.587	-0.117
6	108.825	0.076	0.069	0.012	0.363	0.191	0.140	0.038	0.583	-0.115
7	109.1	0.080	0.071	0.013	0.366	0.193	0.139	0.040	0.579	-0.113
8	109.375	0.084	0.073	0.014	0.370	0.195	0.138	0.041	0.575	-0.111
9	109.65	0.088	0.075	0.015	0.374	0.196	0.136	0.043	0.571	-0.109
10	109.925	0.092	0.076	0.017	0.377	0.198	0.135	0.045	0.567	-0.106
11	110.2	0.096	0.078	0.018	0.381	0.200	0.134	0.046	0.563	-0.104
12	110.475	0.101	0.080	0.020	0.385	0.202	0.132	0.048	0.559	-0.101
13	110.75	0.106	0.081	0.021	0.389	0.204	0.131	0.050	0.555	-0.098
14	111.025	0.110	0.083	0.023	0.393	0.206	0.130	0.052	0.551	-0.095
15	111.3	0.116	0.084	0.025	0.397	0.208	0.128	0.054	0.547	-0.092
16	111.575	0.121	0.086	0.028	0.400	0.209	0.127	0.056	0.543	-0.088
17	111.85	0.126	0.087	0.030	0.404	0.211	0.125	0.058	0.539	-0.085
18	112.125	0.132	0.088	0.033	0.408	0.213	0.124	0.060	0.535	-0.081
19	112.4	0.138	0.090	0.035	0.413	0.215	0.122	0.062	0.531	-0.077
20	112.675	0.144	0.091	0.038	0.417	0.217	0.121	0.065	0.527	-0.073
21	112.95	0.151	0.092	0.042	0.421	0.219	0.119	0.067	0.523	-0.068
22	113.225	0.157	0.093	0.045	0.425	0.221	0.117	0.069	0.519	-0.063
23	113.5	0.164	0.094	0.049	0.429	0.223	0.116	0.072	0.515	-0.059
24	113.775	0.172	0.095	0.053	0.434	0.225	0.114	0.074	0.512	-0.053
25	114.05	0.179	0.096	0.057	0.438	0.227	0.113	0.077	0.508	-0.048
26	114.325	0.187	0.096	0.062	0.443	0.229	0.111	0.080	0.504	-0.042
27	114.6	0.194	0.097	0.067	0.447	0.231	0.109	0.083	0.500	-0.037
28	114.875	0.203	0.097	0.073	0.452	0.233	0.107	0.086	0.497	-0.030
29	115.15	0.211	0.097	0.078	0.456	0.235	0.106	0.088	0.493	-0.024
30	115.425	0.220	0.097	0.085	0.461	0.237	0.104	0.092	0.490	-0.018
31	115.7	0.228	0.097	0.091	0.466	0.239	0.102	0.095	0.486	-0.011
32	115.975	0.238	0.097	0.098	0.471	0.241	0.101	0.098	0.483	-0.004
33	116.25	0.247	0.097	0.106	0.476	0.243	0.099	0.101	0.479	0.004
34	116.525	0.257	0.096	0.114	0.481	0.246	0.097	0.104	0.476	0.011
35	116.8	0.267	0.095	0.122	0.486	0.248	0.095	0.108	0.473	0.019
36	117.075	0.277	0.095	0.131	0.492	0.250	0.093	0.111	0.469	0.027
37	117.35	0.287	0.094	0.141	0.497	0.252	0.092	0.115	0.466	0.035
38	117.625	0.298	0.093	0.151	0.503	0.254	0.090	0.119	0.463	0.043
39	117.9	0.308	0.092	0.161	0.509	0.256	0.088	0.122	0.460	0.052
40	118.175	0.319	0.090	0.172	0.515	0.258	0.086	0.126	0.457	0.061
41	118.45	0.331	0.089	0.183	0.521	0.261	0.085	0.130	0.455	0.070
42	118.725	0.342	0.088	0.195	0.527	0.263	0.083	0.134	0.452	0.079
43	119	0.354	0.086	0.207	0.534	0.265	0.081	0.138	0.449	0.088
44	119.275	0.365	0.085	0.220	0.541	0.267	0.079	0.141	0.447	0.098