Calculating the likelihood value for a model and a dataset once you have the MLEs

For lab 01, weekly survival was monitored for 284 duck weeks. The results were that 265 of those 284 trials resulted in survival and 19 resulted in death. So, $y = 265$, $n = 284$, and the MLE for S in the $S(.)$ model is $y/n = 0.9330986$.

Let's work out the Likelihood and log-Likelihood values for this simple model.

$$p^y \cdot (1-p)^{(n-y)} = 0.9331^{265} \cdot (1-0.9331)^{19} = 5.17686e-31$$

The logLikelihood is just the natural logarithm of that very small number or -69.73594

Finally, -2lnL is $-2 \cdot (-69.73594) = 139.4719$ and $AIC = -2\ln L + 2(k) = 139.4719 + 2 = 141.4719$

What about the S(temp) model? Remember that now we find the MLE's for the intercept and slope terms and then calculate the predicted value of S for each week. That model estimated the following $\beta_0 = 0.7509420; \beta_1 = 0.2611644$. From those, we can use each week's temperature value to predict S for each week. And, from those values and the data (y & n), we can calculate the Likelihood, lnL, -2lnL, and AIC. Here, the product of the weekly values for contributions to the likelihood (values in the last column) is 1.57e-29, which yields -2lnL = 132.647. This model has 2 parameters (intercept & slope). Thus, AIC = 132.647 + 2(2) = 136.647.

Model=S(temp)							
Week	y	n	temp	predicted survival rate (p)	p^y	(1-p)^(n-y)	(p^y)*((1-p)^(n-y))
1	47	48	11	0.974011	0.29006912	0.025989	**0.007538606**
2	45	47	9	0.956951	0.13805198	0.001853191	**0.000255837**
3	39	41	8	0.944813	0.10926643	0.003045616	**0.000332784**
4	34	39	4	0.857612	0.00539381	5.85276E-05	**3.15687E-07**
5	28	32	6	0.910351	0.07208559	6.45925E-05	**4.65619E-06**
6	25	28	5	0.886629	0.0493801	0.001457144	**7.19539E-05**
7	24	25	10	0.966515	0.44157253	0.0334854	**0.014786233**
8	23	24	9	0.956951	0.36346805	0.0430487	**0.015646827**
					product		**1.57051E-29** Likelihood
					ln(product)		**-66.32356693** lnLik
					-2*LN(product)		**132.6471339** -2lnLik

Below you'll see the MARK output for 4 models applied to the black duck data. We've worked through how to get the -2lnL and AIC. To get the deviance values, we need the -2lnL value for the saturated model, i.e., a model where the number of parameters equals the number of data points. For this simple known fate example with data from 8 weeks for 1 group of ducks, we have 8 data points (# of survivors [*y*] given that *n* birds were studied each week). The key is that without knowing individual characteristics of the birds, this is the most complex model we can fit and so it serves as our baseline. In more general terms, S(g*t) is the saturated model for known fate if we don't have individual covariates. With one group, this collapses to S(t). Notice that each model's deviance value is simply the model's -2lnL value minus the -2lnL value for the S(t) model.

Given that you have seen how we work out MLE's and the associated -2lnL and AIC values for a given model and dataset, I hope you understand where the values in the table below come from. To help you interpret the ΔAICc values, AICc weights, and model likelihood values that come from -2lnL and AIC scores, read the assigned pages in the Cooch & White book (Ch. 1 and pages 4-29 through 4-50) and the multi-model inference handout for today. Finally, ask questions if you have them!

Model	AICc	Delta AICc	AICc Weights	Model Lik	Num. Par	Deviance	-2log(L)
{S(Temp)	136.6898	0	0.87172	1	2	0.6383	132.6471
{S(.)}	141.4861	4.7963	0.07923	0.0909	1	7.463	139.4719
{S(T)}	142.5427	5.8529	0.04671	0.0536	2	6.4911	138.5
{s(t)}	148.5325	11.8427	0.00234	0.0027	8	0	132.0089